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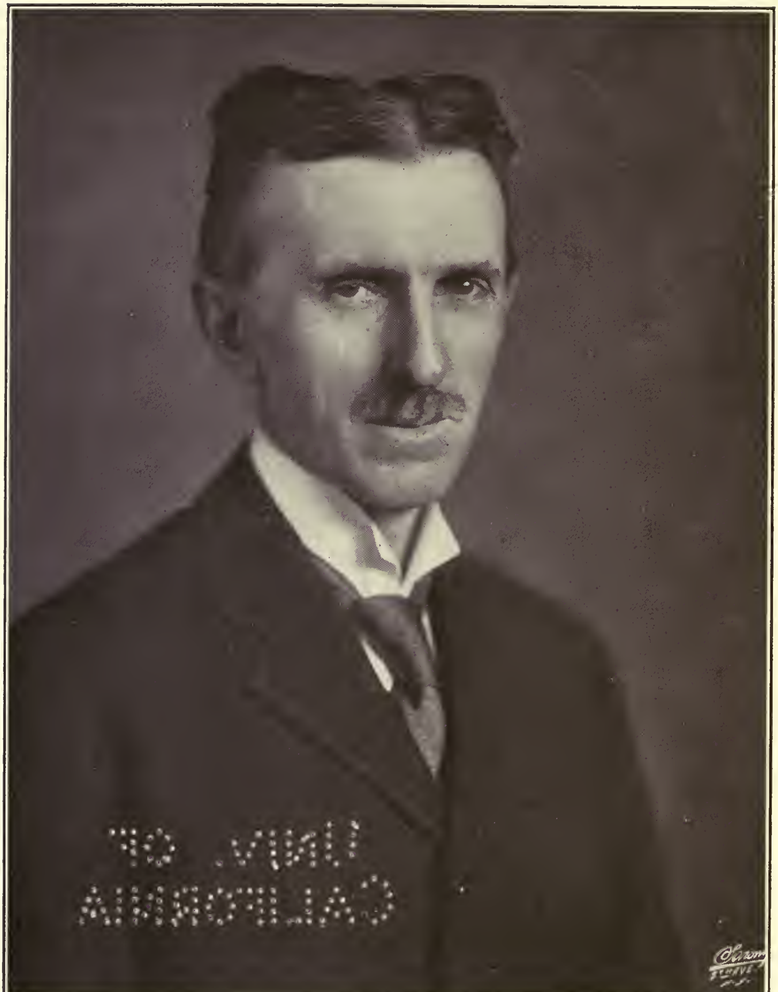
THE INDUCTION MOTOR
AND
OTHER ALTERNATING CURRENT MOTORS



To my friend B. A. Behrend with expressions of high regard
New York Jan. 16. 1920. Nikola Tesla

(Frontispiece)

THE INDUCTION MOTOR
AND
OTHER ALTERNATING CURRENT MOTORS

THEIR THEORY AND PRINCIPLES OF DESIGN

BY

B. A. BEHREND

FELLOW, AND PAST SENIOR VICE PRESIDENT, AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS; FELLOW, AMERICAN ACADEMY OF ARTS & SCIENCES; FELLOW, AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE; MEMBER, AMERICAN SOCIETY OF CIVIL ENGINEERS AND AMERICAN SOCIETY OF MECHANICAL ENGINEERS, ETC.

SECOND EDITION

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To

The Great Pioneers

who have been my friends

Nikola Tesla, Gisbert Kapp, André Blondel, C. E. L. Brown

this book is affectionately inscribed

"............Ignorance more frequently begets confidence than does knowledge."

CHARLES DARWIN,
"The Descent of Man," p. 3.

"It is particularly interesting to note how many theorems, even among those not ordinarily attacked without the help of the Differential Calculus, have here been found to yield easily to geometrical methods of the most elementary character.

"Simplification of modes of proof is not merely an indication of advance in our knowledge of a subject, but is also the surest guarantee of readiness for farther progress."

LORD KELVIN AND PETER GUTHRIE TAIT,
"Elements of Natural Philosophy," p. v.

"The simplicity with which complicated mechanical interactions may be thus traced out geometrically to their results appears truly remarkable."

SIR GEORGE HOWARD DARWIN,
"On Tidal Friction," in "Treatise on Natural Philosophy."
By KELVIN AND TAIT, p. 509.

"............the absence of analytical difficulties allows attention to be more easily concentrated on the physical aspects of the question, and thus gives the student a more vivid idea and a more manageable grasp of the subject than he would be likely to attain if he merely regarded electrical phenomena through a cloud of analytical symbols."

SIR JOSEPH JOHN THOMSON,
"Elements of the Mathematical Theory
of Electricity and Magnetism," p. vi.

"It is remarkable that such elementary cases of Newton's dynamics should require abstruse considerations for their explanation. But it is far worse in the more modern dynamics, with ignoration of coordinates, and modified Lagrangean functions. Dynamics as visible to the naked eye seems to disappear altogether sometimes, leaving nothing but complicated algebra."

OLIVER HEAVISIDE,
"Electromagnetic Theory," vol. iii, p. 401.

"Let them make the effort to express these ideas in appropriate words without the aid of symbols, and if they succeed they will not only lay us laymen under a lasting obligation, but, we venture to say, they will find themselves very much enlightened during the process, and will even be doubtful whether the ideas as expressed in symbols had ever quite found their way out of the equations into their minds."

"The Scientific Papers" of JAMES CLERK MAXWELL,
vol. ii, p. 328.

"The scientific career of Rankine was marked by the gradual development of a singular power of bringing the most difficult investigations within the range of elementary methods."

"The Scientific Papers" of James Clerk Maxwell,
vol. ii, p. 663.

"Lagrange came to grief over the small conical oscillations of the spherical pendulum, yet he could have saved himself and detected his error but for the self-imposed restraint of excluding the diagram from his "Mécanique analytique." So it is curious to find the same fashion coming again in the modern school of pure analytical treatment, of doing away with an appeal to the visual sense of a geometrical figure."

Sir George Greenhill,
Nature, April 17, 1919.

PREFACE TO SECOND EDITION

As indicated in the preface to the first edition of this little book, it owed its origin to a series of lectures delivered at the University of Wisconsin in January, 1900. These lectures were published in the Electrical World and appeared in 1901 in book form entitled "*The Induction Motor*." The book was translated into several languages among others into French and German. The American edition was soon exhausted and repeatedly attempts were made by myself and by assistants and associates of mine to revise it, but though several agreements were entered into between the publishers and Mr. A. B. Field and myself for a second edition, other and more urgent demands upon our time prevented the completion of the work.

Once more then, twenty years after its first appearance, this little book addresses itself to the engineering public. The first edition contained at the time almost entirely new matter and almost all of this originated with the author. Tested though it was by the most careful laboratory work, yet a certain diffidence prevented the author from pressing his claims to recognition. Rarely perhaps has any early work become so absorbed into the texture of thought of engineers as the substance of this little book. The kindly words of my friend Dr. Addams S. McAllister in a presentation copy of his own excellent treatise on "*Alternating Current Motors*," in which he says that to the present author "all writers on induction motors, and all students of induction motor phenomena, are indebted for the first presentation of the conception of the phenomena now considered modern," I would not here repeat—though I treasure them very highly—were it not for the fact that as a perusal of the introduction may indicate—my work is constantly quoted as done by others and these quotations are—as a dispassionate analysis indicates—not in accordance with the plain facts.

The circle diagram has become indispensable to the engineer. Its first demonstration and proof were developed by me in 1895. In its present form it is used exactly as given by me in "*The Induction Motor*," New York, 1901. The idea of the leakage coefficient and its characteristics have been found correct and have been universally adopted. The conception of the single-phase motor with the primary exciting belt resolved into two component motors simulated by two poly-phase motors in series with opposite torque, which conception I worked out quantitatively, has recently been commended as the best method for students in a paper by Mr. B. G. Lamme* (A. I. E. E., April 1918). Yet, here as elsewhere, "A prophet is not without honor, save in his own country, and in his own house." I think the explanation for this must be found in the tendency of mankind to prefer to give recognition to those remote from us rather than to associates or acquaintances. It is easier, for instance, to name some one whom the students do not know and will not come in contact with, as the originator of a certain theory, than a man whom they are likely to meet in their professional relations. Mankind, and especially professional mankind, is chary of praise of its fellow-workers.

An interesting example of this is furnished in the theory of the regulation of alternators. Numerous references in American textbooks are made to "Potier's Method" for determining the regulation of alternators. Now, I believe those who call a certain method by this name can never have referred to A. Potier's paper "Sur la Réaction d'Induit des Alternateurs," p. 133. L' Eclairage Electrique, 28th July, 1900. Prof. A. Potier was a great savant and a gentleman. His paper abounds in carefully selected references. He claimed no new method. He stated that Mr. Kapp many years before (about 1893 and since) used a method for the determination of the regulation of alternators in which he resolves the total effect of the armature currents into an "armature reaction" component and a "self-induction" component, forming a right angle triangle in the regulation curve in the case of zero power factor He then refers to copious data published by me in the *E. T. Z.*, in *L'Eclairage Electrique,* in the *Electrical World,* and other data which I sent him at his request privately, pointing out the corroboration of Kapp's method and indicating—and this is the only new point in the paper—that this method therefore implies identity of the zero power factor regulation curves as given by Dr. Behn-Eschenburg and myself for years, with the saturation curve, as

* See also F. W. Alexanderson, *Transactions A. I. E. E.*, 1918, Part I, p. 691, 692, 693.

the two have been proved by my tests to be equidistant and displaced from each other. After this interesting theoretical remark, he completes his paper by citing Kapp and giving the method of determining the regulation, upon my experimental data of which he bases his theoretical conclusion. Therefore, the method of two components is to be designated by no single name; while the intrinsic importance of zero power factor regulation was urged continually* since 1896 by myself until its final adoption by the American Institute in 1914!

As I once wrote to Mr. Oliver Heaviside, quoting Huxley, "*Magna est veritas et praevalebit!* Truth is great, certainly, but, considering her greatness, it is curious what a long time she is apt to take about prevailing." And to one with scholarly inclinations, eking out a livelihood by the practice of engineering, it is a matter of inward gratification to see one's work generally adopted "among the rubble of the foundations of later knowledge and forgotten." Remembering that I was twenty years old when I published the much referred to circle diagram I will say this to a young reader of the present generation by way of advice: Let him not trouble his head with recognition. "If truth does not prevail in his time, he will be all the better and the wiser for having tried to help her. And let him recollect that such great reward is full payment for all his labor and pains." (Huxley.)

In the treatment of the theory, the diagram of fluxes as developed in 1895 by A. Blondel and used ever since by me, has been continued throughout this book. Its simplicity and brilliant elegance is much to be preferred to the older method of Kapp's, so extensively adopted by Steinmetz and others. It is also greatly to be preferred to the "equivalent circuit" methods which are interesting as an exercise but somewhat artificial and removed from intimate contact with the physical phenomena. On account of their identity with Hopkinson's, I have adopted Blondel's stray coefficients, greater than one, which are the reciprocals of my former ones, smaller than one, in order to establish uniformity of notation.

It is opportune to say a few words on the subject of the absence of complex algebra in this little volume. I have given this question a great deal of thought. At first I intended to give in parallel chapters the results of the theory in complex algebraic

* "The Experimental Basis for the Theory of the Regulation of Alternators." By B. A. Behrend, Am. Inst. El. Engrs., May 19, 1903.

form. But I became discouraged in working out a number of problems. The *algebraization*, to borrow a term from Heaviside, is certainly cumbersome, and one may be happy indeed if one succeeds in avoiding algebraic or arithmetic errors. Page after page is covered with algebraic symbols at which the careful and conscientious calculator looks with much anxiety. It is indeed a beautiful method, this method of resolution of directed quantities into rectangular coordinates, but I doubt whether it is suitable for all types of engineering minds. Perhaps here as elsewhere, it is charitable to let men work out the methods best suited to themselves and not to press intolerance to the point of imposing one method upon all. This is particularly advisable as a graphical method can, and should, be checked by an algebraic process and then the graphical process is explanatory of, and elucidating, the physical process. I have therefore decided to omit the use of the symbolic method, and the reader should turn to other works if he desires algebraic treatment. It may be necessary to emphasize that the treatment of the phenomena loses nothing in accuracy or elegance by the adoption of graphic methods which have been used and advocated by Maxwell, Kelvin, Sir George Darwin, Sir. J. J. Thomson, and others.

The squirrel cage motor with two secondaries with different resistances and leakages is here treated graphically, and so is the theory of concatenation. A chapter on speed regulation of induction motors is also added.

On the subject of leakage in induction motors a great deal has been published, but I have found it inadvisable to embody much of it in this edition. Practical formulae and calculations based on them should be used sparingly, excepting in the workshop, and they should invariably be based on personal experience. One should not encourage begetting the formula habit.

The theory of the single-phase induction motor has been given in two ways. First, as originally given by me in 1897 with the assumption of two rotating fields, and the equivalence of two rotating field motors in series; and secondly, as first given by Potier and Goerges and beautifully completed by Prof. Sumec with the use of the cross-magnetization as used and advocated in this country by A. S. McAllister.

In the chapter on the poly-phase series motor, I have followed to some extent the brilliant work of André Blondel to whom we all are greatly indebted in every branch of electrical engineering.

The Heyland compensated motor has logically received its treatment in this chapter, as it is a poly-phase shunt motor as pointed out and proved by Blondel.

It seemed unnecessary to treat of the windings for induction motors in view of A. M. Dudley's treatise on "Connecting Induction Motors," McGraw-Hill Book Co., 1921, and equally unnecessary to retain the two chapters on design contained in the first edition as the books of Mr. H. M. Hobart and of my former assistant, Prof. Alexander Miller Gray,* have supplied this need better than I could have done.

A few chapters deal with such subjects as the improvement of power factor, as suggested by Leblanc and Kapp, the magnetic pull, and other allied subjects. Brevity in the text has been preferred to prolixity as the lesser of two evils.

This book is not meant to be a work of an encyclopedic character. Nothing that I could write could, in that respect, touch the work of Arnold and LaCour. Nor is it to take the place of such admirable work as Alexander Russell's which should be read by every electrical engineer. It is essentially the work of an engineer, who has had the good fortune to have been actively associated with the art of electrical engineering through almost three decades and who has had a part in the development of the machines about which he writes. He thus addresses himself to his fellow-engineers, revealing the methods which he has followed in the design and construction of alternating current motors, of which literally millions of horse-power were executed under his direction.

The design of electrical machinery, as of all machinery, is based upon intelligent comparison of empirical data, and the art of designing, therefore, cannot be taught without such data. The methods and principles taught in this book aim solely at creating means of effecting such comparisons. To "calculate" a machine, as the term is frequently employed, is not feasible and only principles and fundamentals can be taught in school.

No apology is made for the personal references which occur in this book. The tendency to write books without references is due largely to the desire to avoid the looking-up of other writers' papers. The reader is not benefited by such treatment, as he may frequently prefer the original to the treatment of the author

* "Induction Motor Design Constants." *Electrical World,* Dec. 30, 1911. "Electrical Machine Design," McGraw-Hill Book Co., 1913.

whose book he is reading. Besides, a knowledge of the literature of our profession is essential to an understanding of the art and to an honest interpretation of the part played therein by our fellow-workers.

My thanks are due to my secretary, Miss Gladys Naramore, A. B., Boston University, 1916, for much painstaking work, and to my friend Dr. Addams Stratton McAllister for his untiring aid, enthusiasm, and criticism. His friendship has been an inspiration and his labors in helping me to put the book through press are beyond the rendering of thanks. To the publishers thanks are due for the successful form of the book and to Mr. John Erhardt for his efficiency and for his patience with the author.

An entire chapter has had to be added to the book on account of a solution of certain problems of inversion solved in a very elegant manner by Dr. A. S. McAllister and communicated to me before publication by him. Thus has been solved a problem with which I have coped in vain these twenty-five years.

To Professor Miles Walker, of the University of Manchester, England, I am indebted for numerous suggestions.

This little book now goes forth as a sort of engineering testament of the author's work in connection with the motors invented thirty-three years ago by his friend Mr. Nikola Tesla. Great things have been done and illumined by these theories and gigantic engineering feats have been achieved.

> ....and tho'
> We are not now that strength which in old days
> Moved earth and heaven; that which we are, we are;
> One equal temper of heroic hearts,
> Made weak by time and fate, but strong in will
> To strive, to seek, to find, and not to yield. (*Tennyson*)

BOSTON, MASSACHUSETTS, B. A. BEHREND.
February, 1921.

PREFACE TO FIRST EDITION

The literature of electrical engineering has become so vast and extensive that it is impossible for any man to keep pace with all that is written on electrical subjects. He who produces a new book that adds to the swelling tide of new publications, may justly be asked for his credentials. My justification for writing this tract will be found in the fact that, though almost all branches of applied electricity have enlisted the industry of authors, the induction motor has received comparatively little attention from competent engineers. The few whose experience and knowledge would entitle them to speak with authority on this subject are deterred from publishing by commercial reasons.

I have made the induction motor the subject of early and special studies, and a comparison of my treatment of its theory with the purely analytical theories will show how far I have succeeded in simplifying and elucidating so complex a subject. The graphical treatment of abstruse natural phenomena is constantly gaining ground, and I quote with satisfaction the words of so great a mathematician as Prof. George Howard Darwin, Fellow of Trinity College, Cambridge, who says on p. 509 of the second volume of Lord Kelvin and Prof. Tait's Treatise on Natural Philosophy that "the simplicity with which complicated mechanical interactions may be thus traced out geometrically to their results appears truly remarkable."

All through this little book I have endeavored to let inductive method check at every step the mathematical or graphical deduction of the results. A wide experience with mono- and poly-phase alternating-current induction motors, gained at the Oerlikon Engineering Works, Switzerland, has enabled me to do so. Thus the careful reader who is willing to profit by the experience of others, will find many valuable hints and results which he can turn to account in his practice. Many induction motors have been designed on the principles laid down in this little treatise, and in no case has the theory failed to answer the questions suggested by observation.

The writing of this book has been mainly a labor of love. Those who know of the troubles, cares and labor involved in writing a book and bringing it through the press, not to mention the sacrifice of personal experience by publication, will doubtless be able to appreciate this thoroughly.

I wish to thank the editors of the *Electrical World and Engineer* for the pains they have taken with the publication of this book, and I must specially thank Mr. W. D. Weaver for the encouragement he has always given to me. To Mr. T. R. Taltavall, Associate Editor of *Electrical World and Engineer*, who has taken endless pains with the proofs of this book, I feel very much indebted.

The substance of this volume was delivered in January, 1900 in the form of lectures at the University of Wisconsin, Madison, Wis., and I wish to thank Prof. John Butler Johnson, Dean of the College of Mechanics and Engineering, for the invitation as non-resident lecturer which he extended to me. To him and to Prof. D. C. Jackson I am greatly indebted for the hospitality conferred upon the stranger within their gates.

SOUTH NORWOOD, OHIO, B. A. BEHREND.
January, 1901.

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THE INDUCTION MOTOR

CHAPTER I

INTRODUCTION AND BRIEF SKETCH OF THE HISTORY OF THE THEORY OF THE INDUCTION MOTOR

The Induction Motor, or Rotary Field Motor, was invented by Mr. Nikola Tesla, in 1888, a year so memorable for the experimental corroboration by Hertz of Maxwell's electromagnetic waves, a piece of work so shrewdly designated by Oliver Heaviside as "a great hit."[1] The Induction Motor was also "a great hit," though many people could not see it.

Engineers almost immediately seized upon its principles. Work was proceeding at Pittsburgh under Mr. George Westinghouse, Mr. Tesla, Mr. Shallenberger, Mr. Scott, and Mr. Lamme. The first successful motor, however, embodying in its design and construction those characteristic features which have marked the motor during its career of 30 years, was designed most probably by Mr. C. E. L. Brown at the Oerlikon Works in Switzerland in the year 1890. I said, "most probably" as it is not impossible that that brilliant engineer, whose untimely death we deplore, Mr. Michael von Dolivo-Dobrowolsky, whose company at that time cooperated with the Oerlikon Company, was as much responsible for its creation as Mr. C. E. L. Brown. Surely, both engineers deserve the utmost credit. A 20-hp. motor, built at the Oerlikon Works and designed by Mr. C. E. L. Brown, is shown in Figs. 1 and 2. This motor was exhibited in 1891 at the Electrical Exposition in Frankfort-on-the-Main.

A study of its features discloses the distributed stator winding, the small air-gap, and the squirrel-cage rotor, whose invention, I believe, is usually correctly credited to Mr. Dolivo-Dobrowol-

[1] "Hertz became quite Maxwellian after his great hit, save that, as I think, he attached rather too much importance, to the mere equations, as the representation of Maxwell's theory, to the comparative exclusion of the experimentative and philosophical basis." Oliver Heaviside, "Electromagnetic Theory," Vol. iii, p. 504, "The Electrician" Printing & Publishing Co. Ltd., London.

sky. This motor was exhibited in connection with the first alternating-current high-voltage power transmission plant in the world, the three-phase 30,000-volt experimental plant from

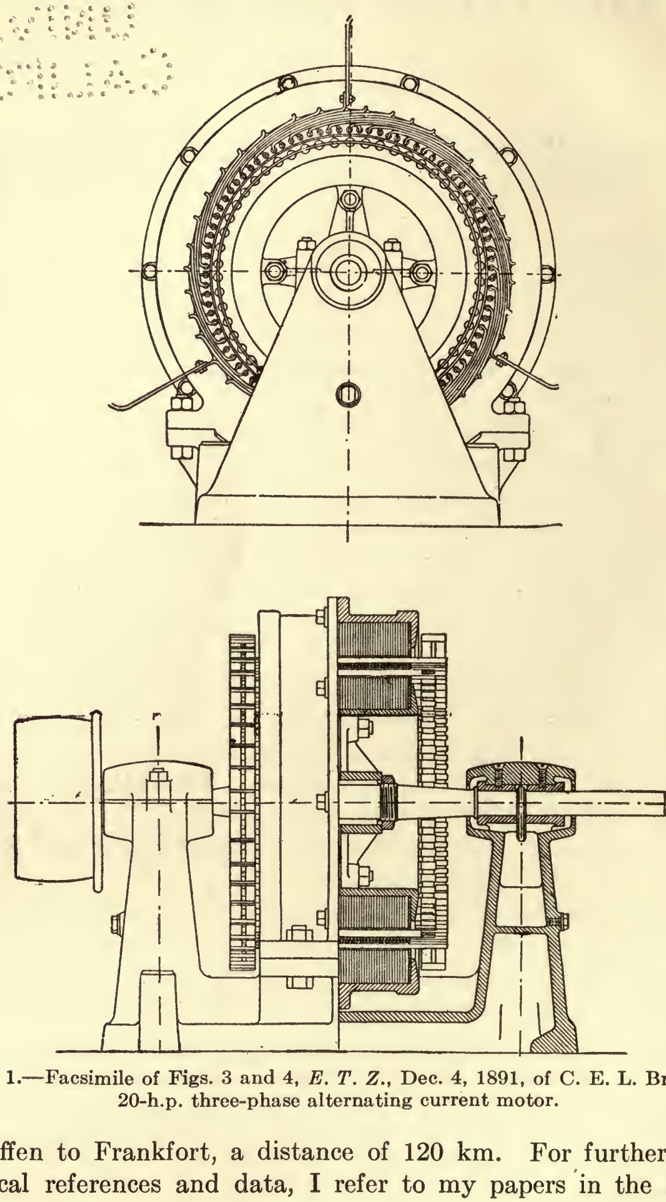

FIG. 1.—Facsimile of Figs. 3 and 4, *E. T. Z.*, Dec. 4, 1891, of C. E. L. Brown's 20-h.p. three-phase alternating current motor.

Lauffen to Frankfort, a distance of 120 km. For further historical references and data, I refer to my papers in the *Elec-*

To my friend B. A. Behrend
with most sincere regards
C. E. L. Brown

(Facing page 2)

trical World and Engineer, from Nov. 16, 1901 to March 1, 1902, entitled "The Debt of Electrical Engineering to C. E. L. Brown." Figure 1 is taken from these papers.

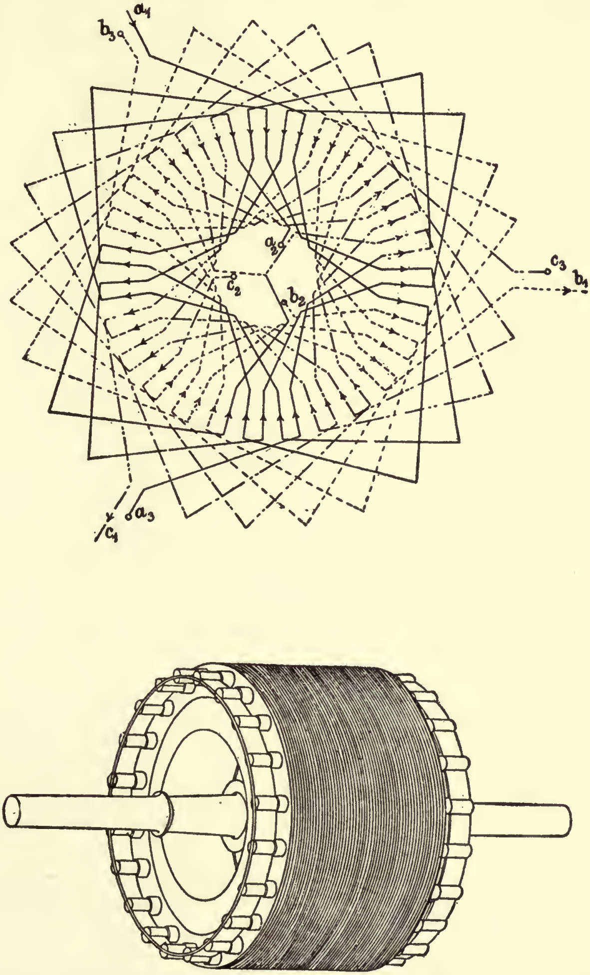

FIG. 2.—Facsimile of Figs. 1 and 2, *E. T. Z.*, Dec. 4, 1891, of C. E. L. Brown's 20-h.p. three-phase alternating current motor.

It is very interesting to observe that the industrial development of machinery, whose operation is based upon the correct interpretation of scientific theory, rarely proceeds rapidly and securely until a method of interpretation of such theory has been devised which enables the engineer to visualize the physical processes beyond the complex texture of a stream of mathematical symbols.[1]

However valuable the algebraization—to borrow a happy term from Heaviside—of physical phenomena may be, it does not supply ideas nor does it supply usually that symbolic skeleton into which the scientific imagination can weave the texture. Alternating-current phenomena are very complicated and, if quantitatively written out in equations, they appear indeed to be well-nigh incomprehensible. A clear comprehension of alternating-current theory was begun by a series of brilliant papers published in *The Electrician*, London, 1885, by Thomas H. Blakesley, entitled "Alternating Currents of Electricity." This series of 10 classical papers discussed for the first time alternating-current phenomena by means of polar diagrams, often perhaps erroneously called vector diagrams, as electromotive force and current are in these cases not vectors at all in the physical sense of the term. They are directive quantities only, because the maximum value of the harmonic wave is used in their construction.

I think the next landmark in the development of the theory was made by Mr. Gisbert Kapp, in two papers originally contributed to the British Institution of Civil Engineers and the Institution of Electrical Engineers, the latter being printed

[1] See also "The Story of the Induction Motor." By B. G. LAMME, *Journal of the A. I. E. E.*, March, 1921.

"The development of the Induction Motor being, in reality, an analytical problem, it did not make much headway in the 'cut and try' days of 1888 and 1889, when the Westinghouse Company was undertaking to put it into commercial form.

"This brings the Induction Motor up to the present. Its history has been a most interesting one to those who are at all familiar with it. To a certain extent this type of apparatus stands apart in that its development has been due, almost entirely, to the analytical engineer. It is almost impossible to conceive that the Induction Motor could have been developed to its present high stage by ordinary 'cut and try' methods. Some good motors might have been obtained in that way, but they would have been accidents of design, instead of the positive results of analysis, as the art now stands.

"New applications are continually leading to new developments which are worked out by the analytical designer with an assurance of success not exceeded in any other branch of the electrical art. And the result of all the elaborate theory and complicated analysis and calculation is a practical machine of almost unbelievable simplicity and reliability—a standing refutation of the too common idea that complexity in theory leads to complexity in results."

in *The Electrician*, Dec. 19, 26, 1890, London. This classical paper of Mr. Kapp's explained in a simple and graphical manner the interesting phenomenon observed by Sebastian Ziani de Ferranti on his 10,000-volt concentric cables from Deptford to London. It is true that neither the work of Blakesley nor that of Kapp contained new theories or new contributions to the science, but in a sense these papers accomplished more. The cumbersome mathematical processes with which these phenomena had been invested by mathematicians and physicists of the time made their utilization impracticable if not impossible. Their interpretation by means of the beautiful graphical methods

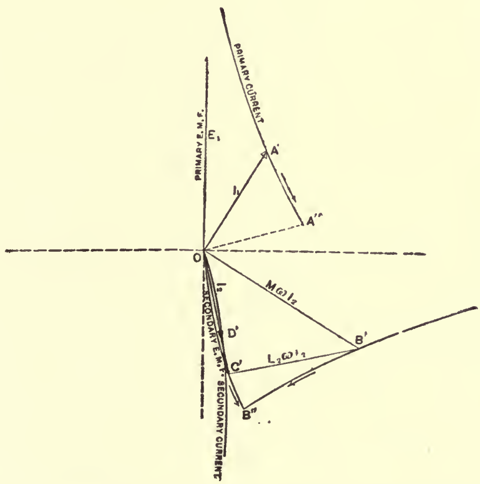

FIG. 3.—Fig. 12, p. 447, *The Electrical World*, "Theory of the Transformer," by F. Bedell and A. C. Crehore. Primary resistance, no leakage, constant primary voltage.

of Blakesley and Kapp gave an impulse to the entire field of electrical engineering which it could not have received without the labors of these men.

In 1892, F. Bedell and A. C. Crehore published a book entitled "Alternating Currents" (Electrical World Publishers) which was followed in 1893 by a series of articles in the *Electrical World*. In these works, the polar diagrams were used with extreme skill and lucidity, and they were applied to all manner of problems including the theory of the alternating-current transformer. In these papers the theory of the constant-current transformer was developed, showing the locus of the primary e.m.f. to be the periphery of a circle, and as Fig. 3 I here reproduce their Fig. 12, p. 447, *The Electrical World*, June 17, 1893, showing for constant primary e.m.f., the polar diagram, with variation of the primary current. This diagram takes account of primary resistance but it does not take account of what is now usually termed the leakage.

In 1894 came out the 4th edition of Gisbert Kapp's "Electric Transmission of Energy," in which a most brilliant elementary account is given of the phenomena in induction motors. The polar diagram was developed, including the primary resistance and the leakage. This diagram was given, however, only for each individual point of the load, showing no general solution of the variation of the different characteristic quantities with variation of load. It was based also on the method of representing leakage through internal self-inductive e.m.fs., which is rather cumbersome.

In 1895, André Blondel published in *Eclairage Electrique*, Aug. 10, 17, 24, 1895, his fundamental papers entitled "Quelques proprietés générales des champs magnétiques tournants." In these papers he developed the theory of the composition of magnetic fluxes, including the leakage fluxes, a method of conception which has since proved of tremendous value.

In the same year, the present author, utilizing the conception of magnetic fluxes as developed by Blondel, proved in a simple and direct manner that with variation in load through change in the non-inductive resistance of the secondary load of a transformer, or through change in load on the shaft of an induction motor, with constant primary e.m.f., the locus of the primary current is a circle in the polar diagram, *provided the primary resultant magnetic field is constant, which is the case if the primary resistance of the transformer can be neglected.* After delivering a lecture on the subject early in 1895, he published the theory later as a paper on Jan. 30, 1896, in *Elektrotechnische Zeitschrift*, Berlin. After the lecture, one of the learned professors present expressed his doubt as to my theory being an exact expression of the facts, as he said the theory was too simple to express the complicated facts. I, therefore, wished to test the results and I soon had an opportunity to do so on a 60-hp. Oerlikon motor. Testing in those days was not a very simple matter and running a brake test and watching Siemens dynamometers with zero reading and Cardew voltmeters was not as simple a procedure as perhaps the present-day generation may imagine, spoiled as they are by all manner of ingenious appliances for simple, direct measurements. When I felt reasonably sure that the theory was very likely correct, though corroborated by only one test, I embodied the record of the test in the paper and sent it to the *E. T. Z.*, which was at that time the central organ for discussing such topics and there it lay until Nov. 11, 1895, when I heard from Mr. Gisbert Kapp, who was then editor of the paper, that he had accepted it. As stated before, it was printed Jan. 30, 1896.

While it lay in the editorial offices, a letter came out in the *E. T. Z.*, p. 649, 1895, by A. Heyland, discussing a motor designed by Mr. Danielson and applying to it a circle locus diagram. In his letter, Mr. Heyland referred to his paper in the *E. T. Z.*, Oct. 11, 1894.

I immediately looked up Mr. Heyland's paper, expecting to find therein the same method of reasoning and proof which I considered novel in my paper. Instead, I found a rather formidable array of lines which I was quite unable to comprehend and which I reproduce herewith in facsimile, Fig. 4. When I received the proofs of my paper, I inserted a reference to Mr. Heyland's letter, *E. T. Z.*, p. 649, 1895. Immediately upon the publication of my paper, it was taken up by Prof. André Blondel in *L'Industrie Electrique,* Feb. 25, 1896, in a paper which begins as follows:

"Le diagramme fondamental des flux d'un moteur asynchrone que j'ai donné à diverses reprises, a été utilisé récemment d'une manière fort heureuse par M. Behrend, grace à la remarque qu'il a faite que si l'on suppose le F constant et fixe, l'extrémité du vecteur Φ décrit un *cercle.* Cet auteur n'a pas cependant donné encore la solution complète. C'est celle-ci que je me propose d'exposer ici en combinant mes propres remarques avec les siennes. La théorie qui résulte de cette collaboration à distance permet d'embrasser d'un coup d'oeil toutes les conditions de construction et de fonctionnement, et constitue à cet égard le meilleur résumé d'une étude détaillée que j'ai publiée récemment."

Mr. Heyland also addressed a letter to the *E. T. Z.*, p. 139, Feb. 27, 1896, which begins: "Mr. Behrend gives a very interesting derivation of my diagram . . . " and in this letter he claims his priority. In a communication to the *E. T. Z.*, p. 116, Feb. 13, 1896, Prof. A. Blondel writes:

"I have read with the greatest interest the paper by Mr. Behrend (*E. T. Z.*, 1896, No. 5, p. 63) in which he applies the diagram of magnetic fluxes developed by me two years ago in a very happy manner to the asynchronous motor . . ."

In 1895, F. Bedell and A. C. Crehore published a most interesting and important paper on "Resonance in Transformer

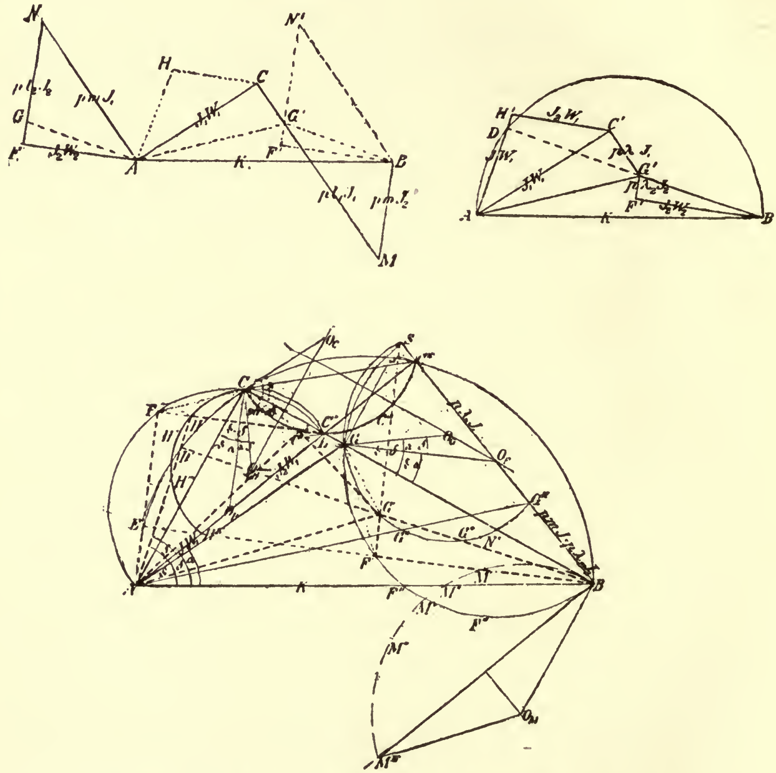

FIG. 4.—Facsimile of Figs. 1, 2, and 3 of A. Heyland's article, "A Graphical Method for the Predetermination of the Transformer and Polyphase Motors," Oct. 11, 1894, *E. T. Z.*

Circuits," in the *Physical Review*, May–June, 1895, Vol. ii, p. 442, in which the circle locus of the primary current of the transformer was clearly and fully treated by means of polar diagrams, including external inductance in the secondary. This paper contains the complete theory, but it does not make use of the identity of the case treated with that of a transformer with leakage. In 1896 came out "The Principles of the Transformer," by Frederick Bedell, in which Fig. 123, p. 226, shows the circular primary current locus of a constant-potential transformer, including leakage and primary resistance, obtained by the method of reciprocal vectors from the constant current transformer diagram, which is easier to derive than the constant potential diagram.[1]

Professor Blondel, in a letter dated at Paris, Sept. 19, 1903, and published in No. 40, *E. T. Z.*, 1903, writes:

"I have shown in the paper referred to[2] that Mr. Behrend and myself have a just claim to many parts of the circle diagram of the ordinary motor. In reference to Mr. Heyland's article, No. 41, *E. T. Z.*, 1894, so repeatedly brought forward, I may say that I have re-read it again, but unfortunately I found it impossible to discover a connection between his circles and the diagram under discussion."

[1] In his admirable "Direct and Alternating Current Manual," 2d Ed. New York, D. Van Nostrand Co., 1916, Dr. Bedell says on p. 288: "In any circuit or apparatus with constant reactance and variable power consumption the current will have a circle locus if the supply voltage is constant . . . This was first shown by Bedell and Crehore in 1892. That the induction motor nearly fulfills these conditions and that its current locus is practically the arc of a circle, was first shown by Heyland in 1894." A footnote states, "*E. T. Z.*, Oct. 11, 1894; published later in book form and translated into English by Rowe and Hellmund."

The book referred to is a little volume entitled "A Graphical Treatment of the Induction Motor" by Alexander Heyland; translated by G. H. Rowe and R. E. Hellmund, New York, McGraw Publishing Co., 1906. In this book Heyland uses an entirely different method from that given by him in 1894, using only a primary leakage coefficient and thus obtaining a simple diagram in contrast to the one using coefficients of mutual and self-induction in his paper of 1894. It must also be stated that Heyland by no means first pointed out the identity of the theory of the alternating current transformer and the induction motor but this was first done by Dr. Behn-Eschenburg and by Gisbert Kapp in 1893 and 1894.

[2] A. Blondel, *L'Eclairage Electrique*, p. 137, April 25, 1903.

"On me permettra de rappeler, à ce propos, que j'ai donné il y a plusieurs annèes la première épure graphique rigoureuse des flux, courants et forces électromotrices des moteurs asynchrones en fonction des coefficients de fuite de Hopkinson et des coefficients K et k. (Eclairage Electrique, 24 août 1894, p. 364, et 19 octobre, 1895, p. 100 et 254.) Cette épure, tenant compte de la résistance du stator, conduisait à une courbe représentative elliptique.

"La propriété indiquée sans démonstration par Heyland dans une lettre à l'Elektrotechnische Zeitschrift, de 1895, qu'en négligeant la résistance du stator, le lieu bipolaire de l'extrémité du triangle de I_1 et I_2 est un cercle, a été démontrée, au moyen du diagramme des flux, par Bernard Behrend

Other references are interesting as of historic importance. Henri Boy de la Tour, in his book "The Induction Motor," translation by C. O. Mailloux, p. 123, writes:

"This method, which is certainly one of the most beautiful applications of graphical methods to the solution of electrical problems, is due to the work of Messrs. A. Blondel, B. A. Behrend, and A. Heyland.

"Although M. Blondel may not have observed that a certain point of the diagram should move on a circumference, he has, nevertheless, in our opinion, contributed much to the discovery which was made independently and almost at the same time by Messrs. B. A. Behrend and A. Heyland, by his having given, ahead of all other authors, an exact analytical study of the operation of three-phase motors, based on a very simple diagram, which constitutes the starting-point of these two engineers."

G. Kapp says, p. 459 of "Dynamos, Motors, Alternators, and Rotary Converters," 3d edition, 1902:

"For practical purposes the so-called circle diagram, as elaborated by Heyland, is preferable. In the text I have chiefly followed Behrend's work."

And in the 4th edition he adds by way of consolation:

"In the previous chapter the circle diagram is obtained substantially as shown in the classical papers of Heyland, Behrend, et al."

Another reference is found in the following paragraph in Thomaelen-Howe's Textbook.[1]

"The historical development of the circle diagram is very interesting. Heyland published the diagram in the *E. T. Z.* on the 11th Oct. 1894, and gave further developments on pp. 649 and 823 for the year 1895. In the *E. T. Z.*, 1896, pp. 63, Behrend developed the diagram

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dans l'Elektrotechnische Zeitschrift, du 30 janvier 1896, p. 63, où se trouve aussi indiquée pour la première fois la représentation du travail et du couple moteur.

"Quant à la représentation du glissement par une échelle linéaire, elle a été indiquée pour la première fois dans mon article de l'Industrie Electrique, 25 février 1896, ainsi qu'un procédé de correction due à la résistance négligée.

"M. Heyland a indiqué plus récemment une correction graphique fort élégante, mais qui parait peu rigoureuse, comme je le montrerai prochainement."

[1] "A Textbook of Electrical Engineering," by Dr. A. Thomaelen, Translated by George W. O. Howe. 3d edition, 1912 (Longmans) p. 386.

analytically, but made a small error in the determination of the rotor current. The convenient determination of the slip and losses was given by Heyland in the *E. T. Z.* for 1896, p. 138. (See also Heyland's "Eine Methode zur experimentellen Untersuchung an Induktionsmotoren,' published in Voit's "Sammlung," Vol. ii, 1900). Emde corrected Behrend's error in a letter to the *E. T. Z.*, 1900, p. 781, which opened an interesting discussion. In the "*Z. für E.*," Vienna, for 1899, Ossanna gave the diagram, corrected for stator loss. (See also an article by Ossanna in the *E. T. Z.*, 1900, p. 712, and also by Thomaelen in the *E. T. Z.*, 1903, p. 972.) It is interesting to note, however, that Ossanna's circle was really included in Heyland's first publication."

References to the circle diagram being the work of A. Blondel, B. A. Behrend, and A. Heyland are to be found in the papers by J. Béthenod, *L. Éclairage Électrique*, Aug., 1904, and by M. Edouard Roth, of Belfort, France, in *L'Eclairage Electrique*, Apr. to June, 1909. The interesting small volume by Dr. K. Krug[1] on the circle diagram of the induction motor contains the following instructive historical reference:

"The circle diagram for the elucidation of the operation of induction motors, which was published almost simultaneously by Heyland and Behrend, took into account only approximately the iron losses and the primary copper loss. The accurate consideration of these losses was first given by Ossanna and his results were developed later in different ways by other authors.

"In most of these papers the methods employed consist in a reduction or adaptation of the Heyland circle diagram, so as to take account of the primary ohmic drop as well as of the iron losses in accordance with the facts.

"Original proofs of the accurate circle diagram which may be reduced to the problem of the so-called general alternating current transformer, have been given among others by Lehmann, by means of vector analysis, by La Cour by means of inversion, and by Petersen with the aid of a principle of superposition.

"The following shows a solution of the general alternating current circuit by means of complex algebra."

Another point of view is presented by Arnold and La Cour.[2]

[1] Dr. Karl Krug, "Das Kreisdiagramm der Induktionsmotoren." Berlin, J. Springer, 1909, p. 5.

[2] E. Arnold and J. L. La Cour, *Die Induktionsmaschinen.* Berlin, J. Springer, 1909, p. 65. Also French text, *Les Machines Asynchrones.* Première Partie. Les Machines d'Induction. Paris: Librairie Ch. Dela-

"A. Heyland showed first (*E. T. Z.*, 1895) that the locus of the current vector is a circle for constant main flux, and he gave a proof for it in *E. T. Z.*, 1896. Behrend also derived his relation from the transformer diagram, and Blondel has referred to some relations in this diagram. The diagram is sometimes called the Heyland diagram."

This brief reference to the history of the development of the theory is surely somewhat misleading, as it has been shown here that Heyland supplied no proof of the circle relation until considerably after the publication of my paper on Jan. 30, 1896, and then his proof neglected the secondary leakage. Also, it would appear that the fundamental labors of A. Blondel in this direction have been somewhat summarily put aside without the recognition due them.

In the treatise of Kittler and Petersen[1] we read on p. 486:

"Heyland developed this diagram bearing his name. Through their labors in giving final form and in clarifying the diagram, Emde and Behrend have achieved preeminent merit."

It is interesting to note a letter by Mr. A. Heyland, *E. T.* Z., p. 61, Jan. 21, 1904, in which, after submitting a lengthy apology for the use of a single leakage field, he proceeds to say: "In respect to the query why I introduced at one time the above simplifications into the diagram (meaning especially the single leakage field),[2] allow me to say that these simplifications (sic!) were thoroughly warranted at the time. The Circle Diagram in its more complex form found little recognition 9 years ago and *remained almost unknown.* (The italics are my own.) It

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grave, 1912, p. 64. "A. Heyland a signalé le premier (*E. T. Z.*, 1895) que le lieu de l'extrémité du vecteur du courant était un cercle si le flux principal est maintenu constant; il en a donné une explication en 1896 dans la *E. T. Z.*, Behrend a également déduit cette théorie (*E. T. Z.*, 1896); il l'a tirée du diagramme du transformateur; Blondel en a déduit à son tour quelques relations. On appelle souvent ce diagramme le diagramme d'Heyland."

[1] "Allgemeine Elektrotechnik." Edited by Dr. E. Kittler. Vol. II, "Einführung in die Wechselstromtechnik." By W. Petersen. Stuttgart: F. Enke, 1909. "Emde und Behrend haben sich durch ihre Arbeiten um die endgültige Formgebung und Klaerung des Diagrammes in hervorragender Weise verdient gemacht."

[2] "A Graphical Treatment of the Induction Motor." By Alexander Heyland. Translated by G. H. Rowe and R. E. Hellmund. McGraw Pub. Co., New York, 1906. The entire paper seems affected by this assumption.

became known only after the publication of the simplified construction of the circle, which I published in a letter to the *E. T. Z.*, 1895, p. 649, which represented an excerpt of a paper in *The Electrician* in Feb. 14, 1896."

I think Mr. Heyland is right that his paper of Oct. 1894 "remained almost unknown." I think I agree with Mr. Heyland that it was necessary to give a simple demonstration of the theory and a simple geometrical proof in order to introduce the Circle Diagram to the engineer. This simple demonstration and geometrical proof were first given by me, and Mr. Heyland's prior and later publications in no wise detract from this fact. When Mr. Heyland saw that I had succeeded in giving a treatment which was as accurate as his own paper in 1894 but a great deal simpler, and in which nothing was neglected which he there took into account, excepting the primary resistance, he endeavored to obtain an equally simple method of treatment and he tried to prove that the secondary leakage coefficient was non-existent[1] and that that which had been viewed as secondary leakage was merely part of the primary leakage and in time phase with the primary current. Then he introdued one of the most unhappy errors which, due to his authority, has not yet completely vanished. He also introduced the circular arcs for the representation of the copper losses in the primary and secondary windings, which were superseded a few years later by the straight lines as given in Fig. 5 reproduced from Fig. 56, p. 101, of the 1st edition of the present author's book, "The Induction Motor," 1901.

We have reproduced as Fig. 4, Figs. 1, 2 and 3 of Mr. Heyland's paper, p. 561, *E. T. Z.*, Oct. 11, 1894; as Figs. 6 and 7, Figs. 2 and 3 of my own paper, pp. 63 and 64, *E. T. Z.*, Jan. 30, 1896; and as Figs. 8 and 9, Figs. 226 and 227, pp. 224 and 228 of Silvanus P. Thompson's "Polyphase Electric Currents," 2d edition, 1900. We suggest to the reader a careful study of these figures and we may then leave it safely to his judgment whether I utilized in my early paper any of Mr. Heyland's work, or whether Mr. Heyland and the late Prof. S. P.

[1] F. Emde, *E. T. Z.*, p. 855, 1900, where we also read: "On this occasion I wish to refer to Heyland's paper, No. 41, *E. T. Z.*, 1894, which at any rate excels in accuracy his later papers, though it lacks the 'seductive' simplicity and it is therefore referred to only historically." Emde has been one of Heyland's strongest admirers, and surely one of the ablest.

Thompson used my early paper in their own work which came out after the appearance of my paper. It is true that Prof. S. P.

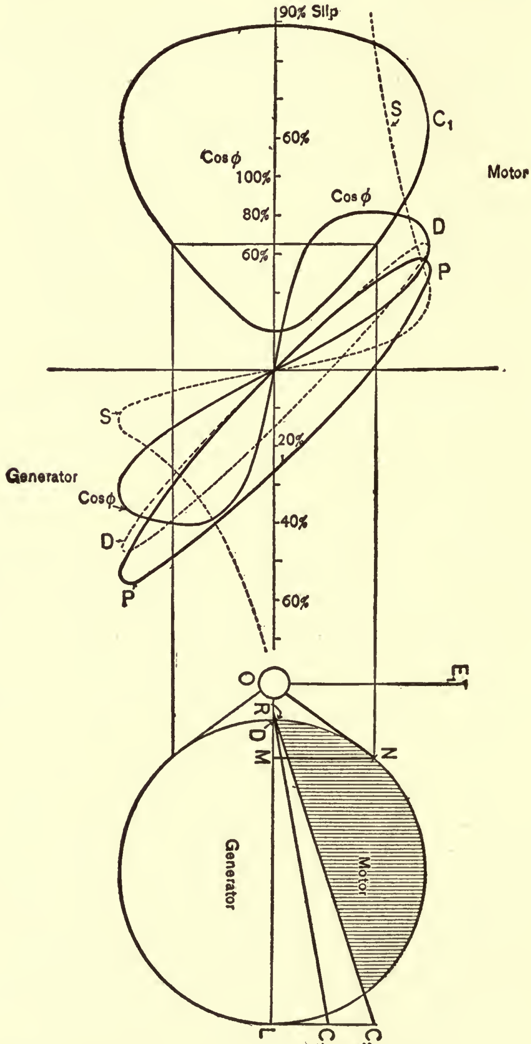


FIG. 5.—Facsimile of Fig. 56, p. 101, of the First Edition of B. A. Behrend's book, "The Induction Motor," New York, McGraw Publishing Co., 1901.

Thompson cited my paper in the bibliography in "Polyphase Electric Currents;" it is also true that he did me the honor of

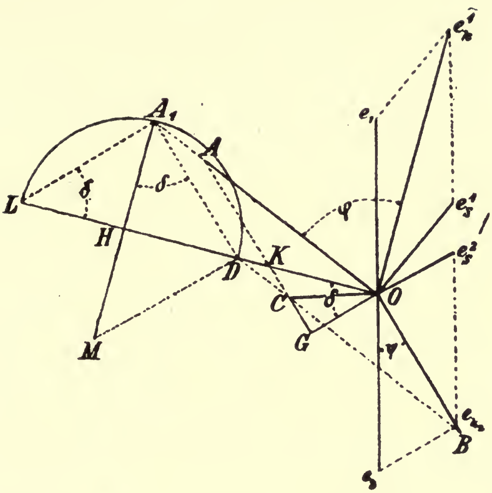


FIG. 6.—Facsimile of Fig. 2 of B. A. Behrend's paper "On the Theory of the Polyphase Motor," Jan. 30, 1896, *E. T. Z.*

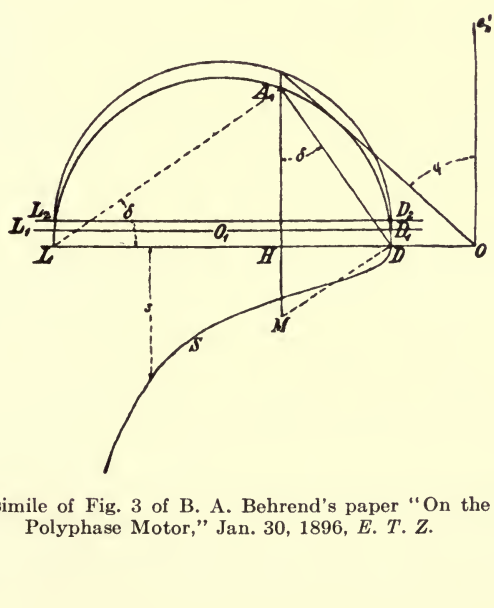


FIG. 7.—Facsimile of Fig. 3 of B. A. Behrend's paper "On the Theory of the Polyphase Motor," Jan. 30, 1896, *E. T. Z.*

naming[1] my little treatise "The Induction Motor" as one of the three books on the theory of alternating-current motors which he recommended to his readers, and I would fain refrain from showing how much my work has been used by these authors if it were not for the fact that during the last 25 years I have remained silent, trusting to the fairness of authors not to deprive a fellow author of the just credit due him.

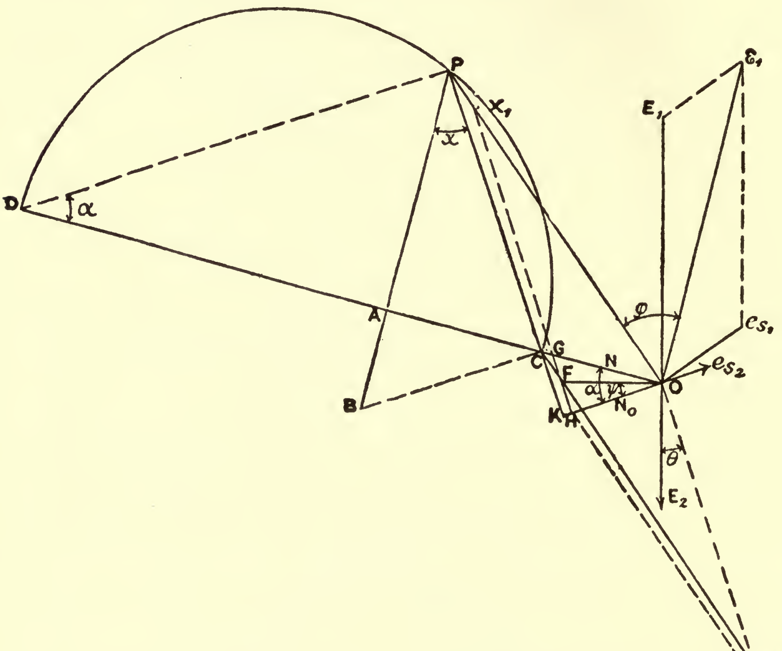


Fig. 8.—Facsimile of Fig. 226, p. 224, of Silvanus P. Thompson's book, "Polyphase Electric Currents," 2d Edition, 1900.

A typical case in question is the reference on p. 413 in my friend Miles Walker's "Specification and Design of Dynamo-

[1] Silvanus P. Thompson, "Dynamo-Electric Machinery," 7th ed., Vol. i, p. 36, London, E. & F. N. Spon, Ltd. 1904. "The theory of alternate current motors of the asynchronous and of the synchronous types has of late received much attention from various writers. The reader is referred to the author's book "Polyphase Electric Currents" (2d ed., 1900) to Steinmetz's "Alternating Current Phenomena" (3d ed., 1900), or to Behrend's "The Induction Motor" (1901).

Electric Machinery," Longmans, Green & Co., 1915, which shows the circle diagram as given on p. 101 of "The Induction Motor," New York, 1901, but with the following comment:

"It is, therefore, convenient to reproduce here a form which is found to be very convenient in workshop use, and to give results which check sufficiently with those obtained in practice."

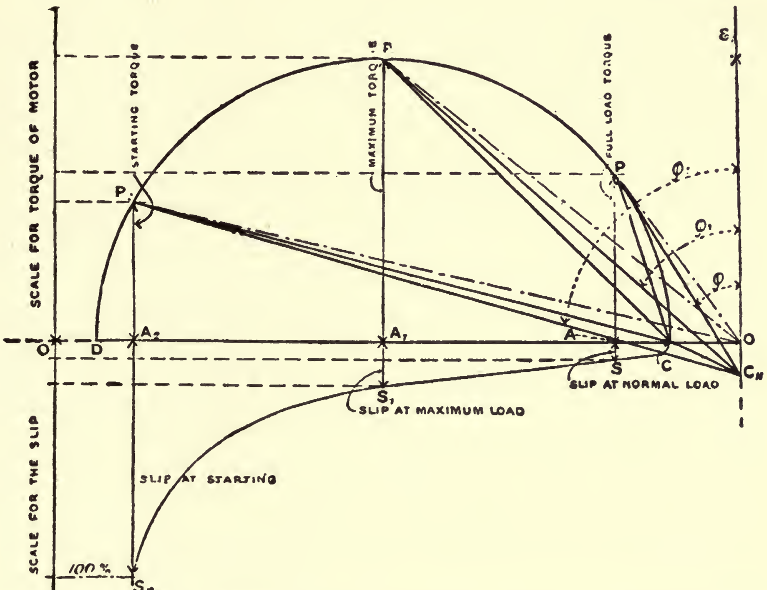


Fig. 9.—Facsimile of Fig. 227, p. 228, of Silvanus P. Thompson's book, "Polyphase Electric Currents," 2d Edition, 1900.

Reference is made to a footnote which begins:

"See Karapetoff's 'Experimental Electrical Engineering,' Vol. ii, p. 166; Cramp and Smith, 'Vector Diagrams;' Graphical Treatment of the Rotating Field, R. E. Hellmund, *A. I. E. E. Proceedings,* p. 927, 1918, etc., etc. . ."

The present form of the circle diagram, as applied to the solution of induction motor and transformer problems, and the methods of its demonstration and proof, are based upon Blondel's diagram of the composition of fluxes and upon the proof of the circular locus as developed by the present author. Mr. Heyland's contributions to the subject, however interesting and suggestive they have been, have not survived.

THE DEVELOPMENT OF THE THEORY OF THE SINGLE-PHASE INDUCTION MOTOR

The analytical theory of the single-phase induction motor owes much to the labors of Potier, Dr. Behn-Eschenburg, Goerges, Steinmetz, and McAllister. As the analytical theory has always been somewhat abstruse, an attempt was made by the author as early as 1896 to represent the locus of the primary current through graphical analysis, and it was found that the primary current in the polar diagram could be represented by vectors drawn from a pole to the circumference of a circle. This was proved, however, only for a limited case, viz. for a motor in which the secondary resistance was partially negligible.

This analysis of the operation of the single-phase induction motor by means of a proof that the primary current locus is also a circle, was given by the author in the *E. T. Z.*, March 25, 1897. The analysis was carried through by dissolving the single oscillating field into two equal and oppositely rotating fields. It was assumed that the rotor resistance of the second motor with reverse torque was negligible. With these assumptions a circle represents correctly the locus of the primary current.

The same analysis was repeated in the first edition of the author's book on "The Induction Motor."

Utilizing the able papers of H. Goerges in the *E. T. Z.*, 1895 and 1903, on the single-phase induction motor, in which Goerges introduced the cross field, Prof. J. K. Sumec gave a comprehensive and elegant graphical solution which remains perfectly simple in spite of its accuracy. His first paper was published in the "*Zeitschrift für Elektrotechnik*," Vienna, No. 36, 1903. The subject is most admirably treated in a little pamphlet entitled "Der einphasige Induktionsmotor," by J. K. Sumec, Nov. 20, 1904, reprinted from the "Archiv der Mathematik und Physik," Leipzig, B. G. Teubner.

Treating the theory of the single-phase induction motor by means of a resolution of the oscillating field into two equal oppositely rotating fields, Dr. A. Thomaelen, in *E. T. Z.*, 1905, p. 1,111 *et seq.*, arrives at the same result as that given by Sumec without neglecting the rotor resistance which was the new element in Sumec's work. Dr. Thomaelen's treatment, however, is rather complex and its value consists in proving that the two methods lead to the same result.

This has again been proved by Arnold and LaCour in "Les Machines d'Induction," Part I, p. 149, of the French edition, Paris, Ch. Delagrave. The authors have used the method of equivalent circuits which they have employed throughout their work. It may, therefore, be safely assumed that both methods of analysis give identical results.

Reference must here be made to the seventh edition, 1918, of Dr. A. Thomaelen, "Kurzes Lehrbuch der Elektrotechnik," which has just come to our attention. Throughout the treatment of the theory of the induction motor, both poly-phase and single-phase, Dr. Thomaelen has used the author's leakage coefficients, assuming apparently that they are novel and expressing his satisfaction that they give results easily and clearly. Since the present author introduced these coefficients in his first monograph of 1896 and as they have been used since with full credit by Messrs. Kapp and Sumec, he likes to point out again that in this work he is using the reciprocals of the coefficients which he used in his early monographs and in the first edition of this book. Thus, the coefficients are the same as those of Hopkinson, as they were adopted in 1894 by our great master, André Blondel.

CHAPTER II

THE THEORY OF FLUXES AND STRAY FIELDS

The problem of problems, in the solution of which the electrical engineer is deeply interested, and which underlies all others, is set before us in the form of the alternating-current transformer possessing considerable leakage and a relatively large magnetizing current.

A choking coil of n turns or a transformer with an open secondary, takes from the primary mains just so much current as is necessary to produce a magnetic field F, which balances the primary voltage e_1. The induced voltage e, opposite in time-phase to the impressed voltage e_1 is

$$e = -n \frac{dF}{dt} \cdot 10^{-8} \text{ volts} \tag{1}$$

The magnetizing current—neglecting for the moment hysteresis and eddy currents—*lags* behind the primary-impressed voltage by a quarter of a time-phase. It leads by a quarter of a time-phase over the *induced* counter e.m.f. We say, "It is in quadrature with the impressed e.m.f." The product of this current into the impressed e.m.f., integrated over the time of one complete period, *i.e.*, the work done by this current, is zero. Currents in quadrature with the e.m.f. have been called by M. Dobrowolsky "watt-less" currents.

Any transformer, induction motor, or other alternating-current device of any sort or description, under load or under no load, has one and only one primary magnetic field resulting from the actions, and interactions, of its current-carrying coils. The magnitude of this resultant magnetic field is such that its variation produces a counter e.m.f. in phase with the impressed e.m.f., and of such magnitude as will permit the flow of the primary current through the ohmic resistance of the primary coils.

The fundamental importance of this statement must be emphasized as it is applied throughout this book.

A choking coil of sectional area A, of magnetic reluctance ρ, of ohmic resistance r, and number of terms n, placed in a circuit of frequency $\sim$, with effective (square root of mean square) current i, carries a maximum flux F, of maximum induction B:

$$F_{max} = A \cdot B_{max} \tag{2}$$

$$= \int \frac{Hds}{\rho} \tag{3}$$

$\int Hds$ is the line-integral of the magnetic force H

$$\int Hds = 0.4\pi I \tag{4}$$

which, for any closed circuit, is equal to 0.4π times the entire magnetizing ampere-conductors. If the integral taken around a closed circuit, viz. the "magneto-motive force," is zero, no flux can result from the currents around which the integral was taken. A neglect of this fundamental conception of the theory of electromagnetism has led to false diagrams of stray-fields.

Therefore,

$$B = \frac{0.4\pi ni\sqrt{2}}{\rho} \tag{5}$$

We assume the time variation of the flux to follow a simple sine law

$$F = F_{max} \cdot \sin \omega t \tag{6}$$

$$\omega = 2\pi \sim \tag{7}$$

Therefore from (1)

$$e_t = -n\frac{dF}{dt} \cdot 10^{-8} \tag{1}$$

$$= -\omega . F_{max} \cdot \cos \omega t \cdot 10^{-8}$$

$$= -2\pi \sim F_{max} \cos \omega t \cdot 10^{-8} \tag{8}$$

$$e_{max} = e\sqrt{2} \tag{9}$$

$$\therefore e = -4.44 \sim nF_{max} \cdot 10^{-8} \tag{10}$$

From (8) it is apparent that e, the induced e.m.f., lags 90 time degrees behind the inducing current. The impressed e.m.f. therefore leads the current by 90 time degrees. (See Fig. 10.)

All our polar diagrams rotate in the positive direction, which is counter-clockwise by international agreement.

The ohmic drop requires the addition of ir in time-phase with the flux to the impressed voltage e_1, requiring E_1 as the final resultant impressed voltage.

The placing of a secondary coil on the magnetic circuit makes the device a transformer.

If we assume that both primary and secondary coils embrace the entire flux, there being no stray or leakage fluxes, and if we assume the number of turns to be the same in both circuits, then

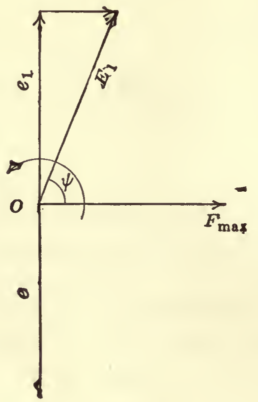


Fig. 10.

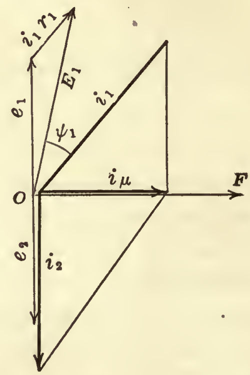


Fig. 11.

e_1 and e_2 are the e.m.fs. impressed upon both the primary and secondary circuits, respectively. If the load on the secondary circuit is non-inductive, the current i_2 is in phase with e. The primary current must be such, in phase-direction and magnitude, that the resultant m.m.f. of the primary and secondary ampere-turns produces the field F in magnitude and time-phase.

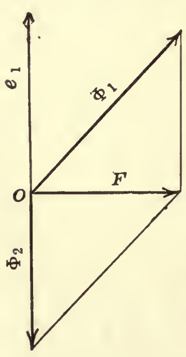


Fig. 12.—Flux quantities in the transformer. No leakage. Fig. 12.

Knowing e_1, we find F; knowing ρ we find the primary ampereturns for magnetization, represented by i_μ in Fig. 11. Adding $i_1 r_1$ to e_1 gives E_1, the resultant impressed voltage.

There is another, and a safer way, proposed by Prof. André Blondel in a famous paper entitled, "Quelques propriétés générales des champs magnétiques tournants" (*Eclairage Electrique*, 10, 17, 24 Aug., 1895), in which the magnetic fluxes are composed as follows: If acting alone i_2 would produce a flux Φ_2 equal to $X_2 \div \rho$, where X_2 represents the m.m.f. of i_2, and ρ the reluctance of the magnetic circuit in common to both primary and secondary circuits; if acting alone i_1 would produce a flux Φ_1 equal to $X_1 \div \rho$, where X_1 represents the m.m.f. of i_1, and ρ again the reluctance

To Mr. B. A. Behrend
with the best compliments of
his faithfully
Blondel

(*Facing page 22*)

of the magnetic circuit in common to both primary and secondary circuits. Φ_2 vectorially subtracted from Φ_1 must leave F in magnitude and direction (Fig. 12).

The great advantage of this method becomes apparent in the treatment of the theory of the transformer with leakage and in the more complex problems of double squirrel-cage motors, concatenation, etc. Its disadvantage lies in the danger of looking upon the fictitious Φ_1 and Φ_2 as fluxes actually in existence and having physical entity. We shall use both methods wherever they represent closely the physical phenomena.

It is well known in dynamo design, as first taught us by Dr. John Hopkinson, that the flux threading the primary does not reach the secondary without leakage or stray fields. If we assume with Prof. A. Blondel that the ratio of the flux of the primary to that which reaches the secondary is v_1, where v_1 is greater than 1, and the ratio of the secondary flux to that which reaches the primary is v_2, where v_2 is also greater than 1, then $(v_1 - 1)\Phi_1$ and $(v_2 - 1)\Phi_2$ represent the stray fluxes or leakage fluxes, which are in time-phase with their respective m.m.fs. or currents.

In my paper *E. T. Z.*, Jan. 30, 1896, and in the first edition of this book, I used the reciprocals of Blondel's *v's*. Though unfortunately most authors have since followed my use of these coefficients, as Silvanus P. Thompson, Gisbert Kapp, Alexander Gray, J. K. Sumec, A. Thomaelen, and others, after very careful consideration, I have become convinced that it is better, in the interest of uniformity and clearness, to give up my coefficients, which were smaller than 1, and instead to adopt Blondel's, which are larger than 1, and this practice also conforms to the dispersion coefficients of Dr. Hopkinson's which are also greater than 1. This matter is solely a convention and in no manner affects the accuracy or correctness of our arguments or of previous papers. There is much to be said for the retention of my old coefficients as they are logical in viewing the deviation of the ratio of transformation at no load from the ideal ratio as the measure of the leakage. In my early notation, the primary and secondary leakage fields were:

$$\left.\begin{aligned} f_1 &= \left(\frac{1}{v_1} - 1\right)\Phi_1 \\ f_2 &= \left(\frac{1}{v_2} - 1\right)\Phi_2 \end{aligned}\right\} \quad \text{Behrend's old Notation} \qquad (11)$$

In Blondel's notation:

$$f_1 = (v_1 - 1)\Phi_1 \qquad \text{Blondel's Notation} \qquad (12)$$
$$f_2 = (v_2 - 1)\Phi_2$$

The utmost care is essential to avoid confusion and I believe a service is rendered by the adoption of a uniform notation.

The diagram of fluxes can now be drawn directly (Fig. 13 and 14).

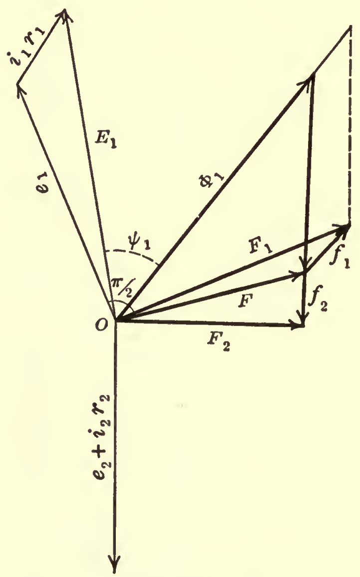


Fig. 13.—The flux diagram of the induction motor or transformer, including leakage.

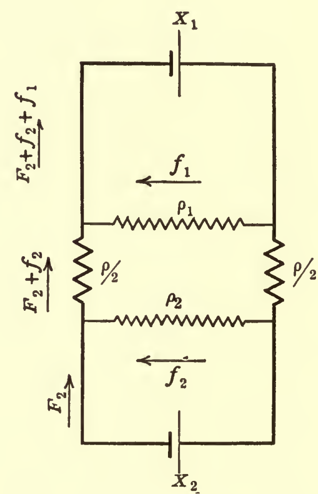


Fig. 14.—Electric circuits simulating the leakage paths of the magnetic circuit of the induction motor.

F_2 induces $e_2 + i_2r_2$
$f_2 = X_2 \div \rho_2 = (v_2 - 1)\Phi_2$ secondary leakage flux
$f_1 = X_1 \div \rho_1 = (v_1 - 1)\Phi_1$ primary leakage flux
$F_1 =$ resultant primary flux
$\Phi_2 = X_2 \div \rho$ fictitious secondary flux
$\Phi_1 = X_1 \div \rho$ fictitious primary flux
$E_1 = \overline{e_1 + i_1r_1}$ primary impressed voltage

It is very desirable to keep in mind a picture of the corresponding electric currents with their e.m.fs. and distribution of resistances. In Fig. 14, X_1 and X_2 represent the primary and secondary m.m.fs., ρ, ρ_1, ρ_2, the reluctances of the common and leakage paths. The fluxes are entered and the diagram shows clearly how the leakage fluxes become cumulative by vectorial addition.

Figures 13 and 14 should always be kept together before the mind, with the underlying assumption that the reluctance of the iron is assumed as negligible, in fact zero.

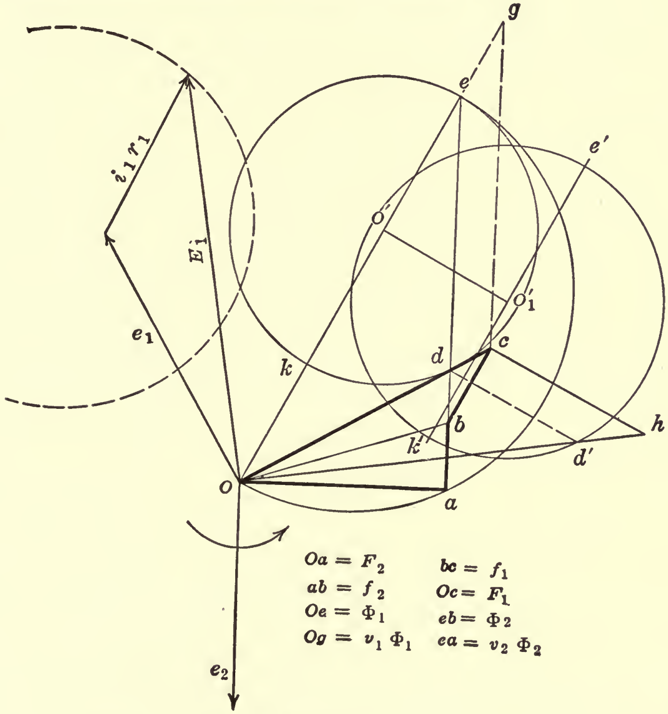


FIG. 15.—The polar diagram for constant current.

Mr. G. Kapp originated a method before the advent of the Blondel flux diagram, which is still adhered to by Dr. Steinmetz, in which the e.m.fs. induced by the leakage fields f_1 and f_2 are represented lagging by a quarter phase the primary and secondary currents. We refer to the author's Fig. 6 from his original paper of Jan. 30, 1896, in which both methods are shown in the diagram, and from which it is apparent that the flux method of Blondel is both more nearly in keeping with the physical facts and a great deal simpler in its geometrical interpretation. The results obtained by both methods are, of course, identical, though there seems now little warrant for retaining the older method of Mr. Kapp's as done throughout in the works of Dr. Steinmetz and in the recent textbook of Prof. R. R. Lawrence, "Principles of Alternating Current Machinery," McGraw Hill Book Co., 1916. At least the flux method should be considered beside the older conventional one.

We are still concerned with the transformer. We wish to know how the magnitude and phase of the primary e.m.f. vary with *constant primary current* and varied secondary resistance. As Oe is proportional to the primary current, Fig. 15, we shall assume it to remain constant, neglecting for the present the primary resistance which is easy to take into account. The angle Oaw is a right angle, hence, describe a semi-circle over Oe as diameter, then by varying the secondary resistance we vary $F_2 = Oa$, which is in quadrature with and proportional to e_2, the secondary voltage. Remember that to obtain the secondary terminal voltage we must deduct i_2r_2 the ohmic drop in the transformer windings from e_2. As $Od = Oc \div v_1$, it follows that Od is a measure of the primary voltage. The point d divides ae in the same ratio for all configurations of the diagram, as is easily shown.

$$ab = (v_2 - 1)\Phi_2$$
$$bd = be - ed$$
$$= \left(1 - \frac{1}{v_1}\right)\Phi_2$$
$$\therefore ab + bd = \left(v_2 - \frac{1}{v_1}\right)\Phi_2$$
$$ad = ab + bd = (v_1v_2 - 1)\frac{\Phi_2}{v_1}$$
$$\therefore ad \div ed = v_1v_2 - 1 \tag{13}$$
$$\sigma = v_1v_2 - 1 \tag{14}$$

We shall call $v_1v_2 - 1$ by the Greek letter σ, which we shall see later is the most characteristic constant of a transformer or induction motor and it is usually called the Leakage Factor. [In my former notation, my old coefficients being the reciprocals of the Blondel coefficients here used, the Leakage Factor was equal to $\sigma = \frac{1}{v_1v_2} - 1$.] Describing now the semi-circle edk so that $Ok \div ke = \sigma = v_1v_2 - 1$, then draw ae at its intersection with the semi-circle edk, at d we obtain the polar ray Od, which is $1 \div v_1$ times the primary field F_1.

Taking now into account the primary resistance, we consider that i_1r_1 is produced by a field proportional to i_1r_1 and in quadrature with its time-phase. If Oc measures e_1, to an arbitrary scale, then ch measures to the same scale i_1r_1, and as i_1 is constant in a constant current transformer (primary current constant), i_1r_1 is constant also, and therefore, ch constant, which is the field corresponding to it. Draw Oh, then dd' parallel to ch, and so on for any number of points "d." Obviously, the point d' lies on the semi-circle $e'd'k'$, whose center O_1 is determined by the distance $O'O_1' = dd'$.

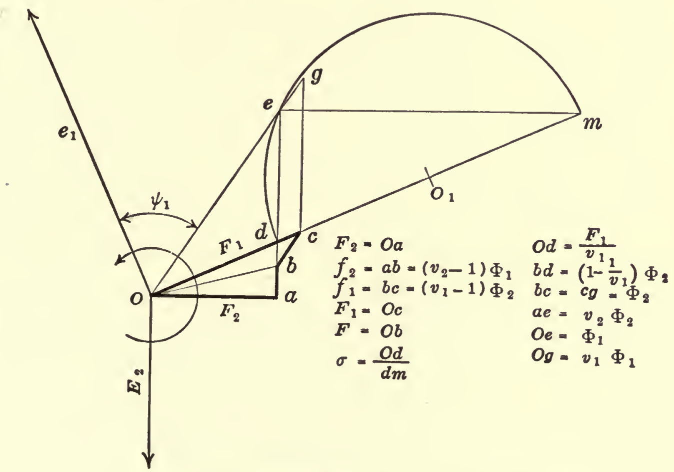


FIG. 16.—The polar diagram for constant voltage.

The reader should plot from this simple polar diagram in Cartesian coordinates the primary e.m.f., the cos φ, and the secondary current, relations ordinarily obtained by long formulas. Of course, if we like these long formulas, we can have them by reading them off the diagram in complex algebra or in trigonometric functions. He should see what is evident by inspection of the diagram—that large leakage will make the transformer into a choking coil.

What happens if the *primary voltage* is kept *constant* instead of the primary current? The flux diagram yields a direct and beautifully simple answer (Fig. 16). Neglecting for the present the primary resistance, F_1 is constant with constant impressed e.m.f., therefore, Od is constant, if the secondary resistance is varied in any manner. We found above in equation (13) that

$ad \div ed = v_1v_2 - 1$, and as $\angle\, Oae$ is a right angle, draw em and triangles $\Delta\, Oad \sim \Delta\, med$, and

$$Od \div dm = v_1v_2 - 1 \tag{15}$$

Hence, the point e describes a semi-circle over dm as diameter. This is the identical proof given by the author in his paper Jan. 30, 1896, and the first proof ever given of this remarkable relation.

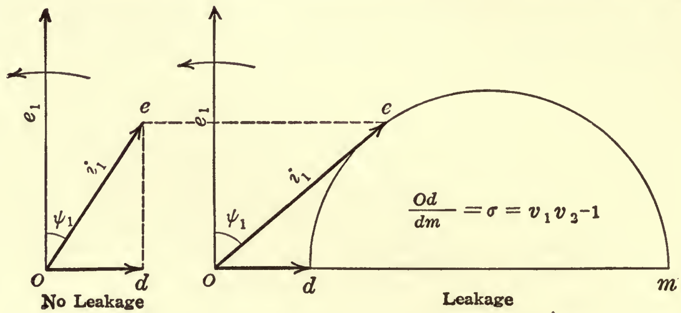


FIG. 17.—Transformer with and without leakage.

The effect of the leakage fields consists in turning the polar ray of the primary current from its locus on the line dy which it would have without leakage, counter-clockwise upon the semi-circle into which the line dy has been transformed. The phase lag is therefore increased and the power factor diminished by the presence of leakage in the transformer. Figure 17 shows this once more.

CHAPTER III

THE GENERAL ALTERNATING-CURRENT TRANSFORMER

A. THE TRANSFORMER WITH NON-INDUCTIVE LOAD

In an alternating-current transformer with resistance and leakage, a resultant primary field F_1, which is the *real* field and which is produced by the m.m.f. resulting from the interaction of the m.m.fs. of the primary and secondary windings, induces in the primary windings a counter e.m.f. which, added vectorially to the impressed e.m.f., leaves a resultant e.m.f. which is equal to the ohmic drop in the primary windings.

If the secondary of the transformer is open we are led to the simple classic statement due to G. Kapp that,

> "A transformer working on open secondary circuit must take from the primary mains sufficient current to produce that field which will just balance the primary voltage. This current is called the open circuit current."[1]

The component of the open circuit current in phase with the e.m.f. supplies the losses incident to the ohmic loss in the primary circuits, eddy currents, and hysteresis, and has therefore aptly been called by M. Dolivo-Dobrowolsky the "watt-component," in contradistinction to the "watt-less" component, which is in quadrature with the e.m.f. and which magnetizes the core, and lags behind the impressed e.m.f. by a quarter-time period. As seen in Chap. II, Eq. (10) the impressed e.m.f. e_1 ("effective," or "square root of mean square") is:

$$e_1 = 4.44 \sim n F_{max} 10^{-8} \text{ volts} \tag{16}$$

where $\sim$ is the primary frequency, n the number of complete turns around the core, and $F_{max} = F_1$ the maximum value of the flux.

The secondary current i_2 and the secondary impressed e.m.f.

[1]Gisbert Kapp, "Dynamos, Alternators, and Transformers," p. 443. London: Biggs & Co., 139–140 Salisbury Court, Fleet Street, E. C., 1893.

e_2, which is also the induced e.m.f. of the secondary circuit, will now be assumed to be in time-phase, in other words the transformer is closed upon a non-inductive resistance R_2. Then

$$e_2 = i_2 R_2 \tag{17}$$

In order to obtain a clear diagram we assume the leakage fluxes $(v_1 - 1)$ and $(v_2 - 1)$ to be large, say, $v_1 = 1.04$ and $v_2 = 1.06$, then the leakage factor σ becomes (Chap. II, Eq. (14)),

$$\sigma = v_1 v_2 - 1 \tag{14}$$
$$\sigma = 0.1$$

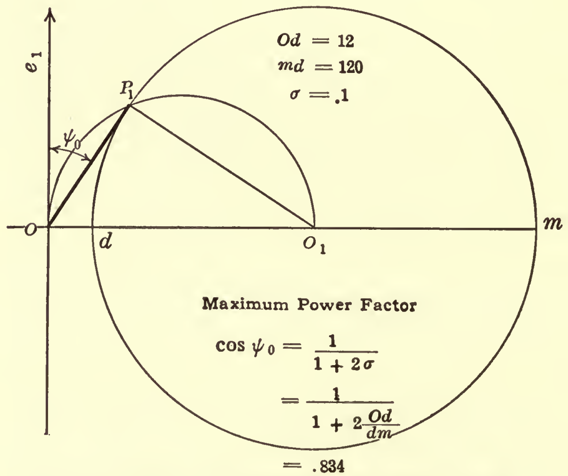


FIG. 18.—The circle diagram for constant voltage.

Assume, $F_1 = 12$, and $e_1 = 120$ volts

$$D = \frac{F_1}{\sigma} = 120 \tag{18}$$

With these values we construct the polar diagram, Fig. 18, whose values are shown in Cartesian coordinates in Fig. 19.

The maximum power factor obtainable is

$$\cos \psi_0 = \frac{O_1 P_1}{O_1\, O} = \frac{\dfrac{i_0}{2\sigma}}{\dfrac{i_0}{2\sigma} + i_0}$$

$$\therefore \cos \psi_0 = \frac{1}{1 + 2\sigma} \tag{19}$$

This simple relation gives at a glance the highest possible power factor for a given amount of leakage for non-inductive load. For our numerical case, we have

$$\cos \psi_0 = \frac{1}{1.2} = 0.834$$

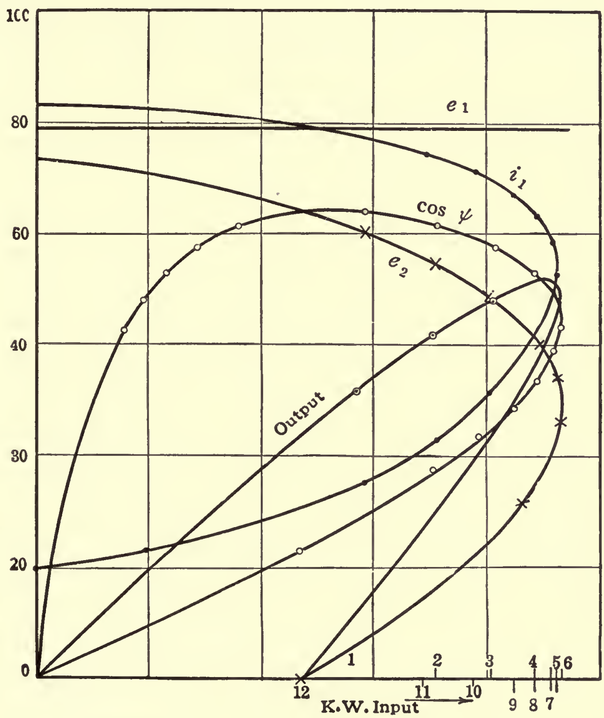


FIG. 19.—Characteristic curves of transformer in Cartesian coordinates.

THE AUTHOR'S METHOD OF ACCOUNTING FOR PRIMARY RESISTANCE

The primary resistance adds to the impressed e.m.f. e_1 the component i_1r_1 in phase with i_1. Before we proceed further it may be advisable to call attention to the fact that, in going from the *flux* diagram to the *current* diagram, the primary current is proportional to *Oe*, the secondary current to *be*, and the resultant magnetization to *Oe*. In Fig. 16, therefore, if *Oe* is drawn to represent i_1, *ed* represents, not i_2, but $\frac{i_2}{v_1}$, while *Od* represents the open circuit current (neglecting losses) which produces the total primary flux $Oc = F_1$.

Instead of adding i_1r_1 to e_1, and obtaining E_1, and then turning the current vector $Oe = i_1$ in the positive or counter-clockwise direction so that E_1 coincides again in phase with e_1, apply the following simple geometrical device (Fig. 20).

Make triangle Oeg similar to triangle Ohk, then

$$eg : Oe :: hk : Oh$$

$$eg = \frac{i_1 \cdot i_1 r_1}{e_1}$$

$$eg = i_1^2 \cdot \frac{r_1}{e_1} \qquad (20)$$

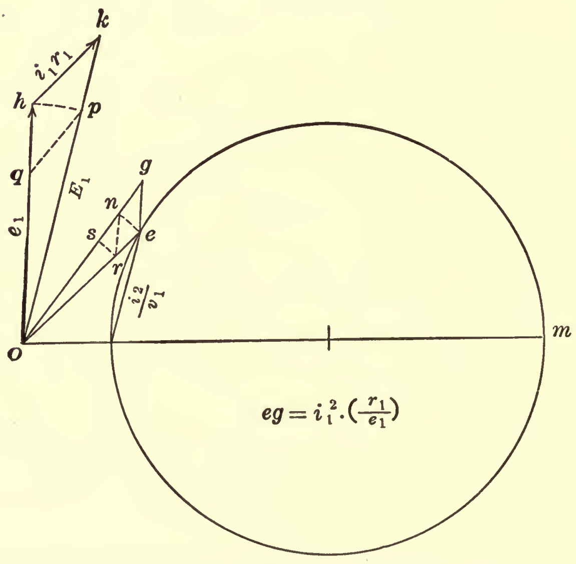


FIG. 20.—The author's method of taking into account the primary resistance.

The new current i_1' must be in the time-phase Og and its magnitude remains equal to $On = Oe$ if the impressed e.m.f. is raised from e_1 to E_1.

If, however, we assume the impressed e.m.f. E_1 to be reduced to Op, the value of e_1, then the current $Oe = i_1$ is to be reduced in the ratio $Og \div Oh = e_1 \div E_1$, or $Or \div Oe$. The magnitude of the real primary current is therefore represented by Or and its phase by Og, its magnitude and phase, therefore, by Os.

The method here described is rapid and easy. It is carried out in the figure. Its advantage lies in its convenient and ready application to all sorts of alternating-current problems.

ANOTHER METHOD OF ACCOUNTING FOR PRIMARY RESISTANCE

A simple and elegant geometrical method applicable to the circle diagram has been given by Prof. J. Sumec,[1] of the Czecho-Slovac University of Brünn, which we shall now proceed to explain.

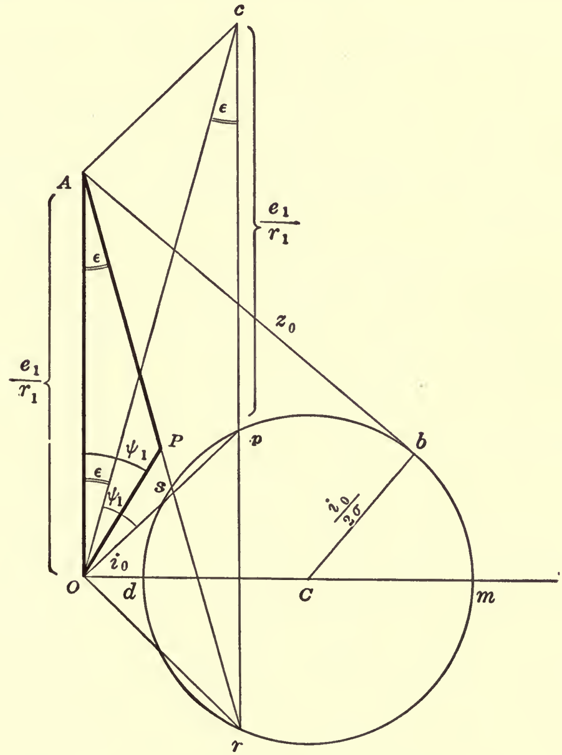


Fig. 21.

$$AP \,.\, AR = OA^2 \qquad As \,.\, Ar = Ab^2 = z_0^2$$

$$AP = As\left(\frac{OA}{z_0}\right)^2$$

p is a point on the original circle
P is a point on the new circle corresponding to p.

Instead of adding i_1r_1 to e_1, add i_1 to $\frac{e_1}{r_1}$, so that Oe represents

[1] J. Sumec, *E. T. Z.*, Feb. 3, 1910. The method is due to Messrs. Stehr and Pichelmeyer.

both i_1 and the resistance component to be added to $\frac{e_1}{r_1}$. Then Fig. 21,

$$\Delta\, OPA \sim \Delta\, OPc \sim \Delta rOA$$

$$AP : AO :: AO : Ar$$

$$\therefore AP \cdot Ar = \left\{\frac{e_1}{r_1}\right\}^2 \tag{21}$$

$$As \cdot Ar = \overline{AB}^2 = z_0^2$$ from a well-known property of the circle.

$$AP = \left\{\frac{e_1}{r_1}\right\}^2 \frac{As}{z_0^2} \tag{22}$$

Hence, P lies on the radius vector from A to c intersecting the circle in s, and P lies again on a circle as any arbitrary radius vector AP is always proportional to the radius vector As.

Now, we have

$$z_0^2 = \overline{AC}^2 - \left(\frac{i_0}{2\sigma}\right)^2$$

$$z_0^2 = \left(\frac{e_1}{r_1}\right)^2 + \left\{i_0 + \left(\frac{i_0}{2\sigma}\right)\right\}^2 - \left(\frac{i_0}{2\sigma}\right)^2$$

$$z_0^2 = \left(\frac{e_1}{r_1}\right)^2 + \frac{(1+\sigma)}{\sigma}\, i_0^2 \tag{23}$$

Now, from Fig. 22, we have

$$Ag : AO :: AO : Ak$$

$$\therefore\ Ag \cdot Ak = AO^2$$

But, from the properties of the circle,

$$Ad \cdot Ak = z_0^2$$

$$\therefore \frac{Ag}{Ad} = \frac{AO^2}{z_0^2}$$

However, $$Ag : Ad :: AC_0 : AC$$

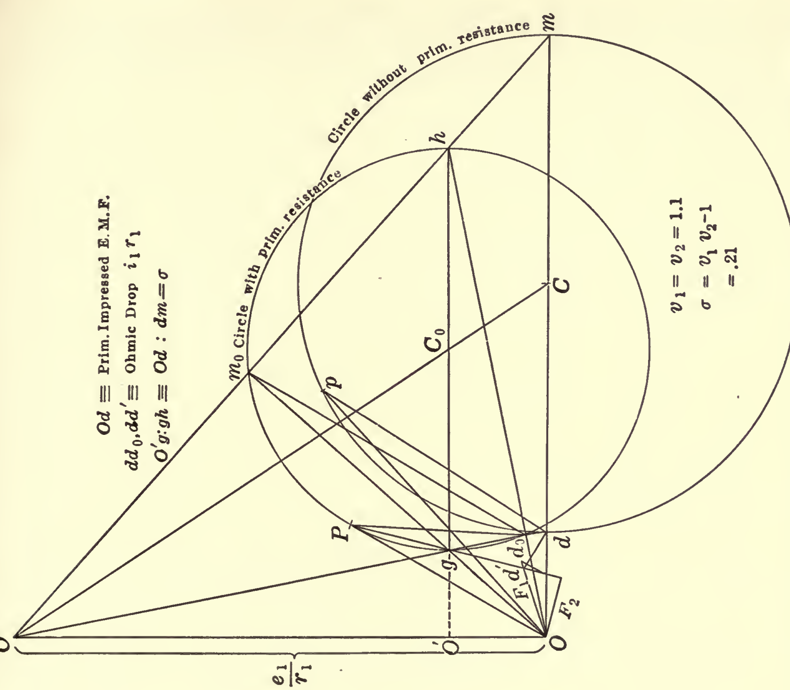
$$\therefore AC_0 = AC \cdot \frac{AO^2}{z_0^2} \tag{24}$$

$$AC_0 = AC \cdot \left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2 \tag{25}$$

From inspection of Fig. 22, calling D the diameter of circle with C as center, and D_0 the diameter of circle with C_0 as center,

$$D_0 : D :: AC_0 : AC$$

$$\therefore D_0 = D\left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2 \tag{26}$$



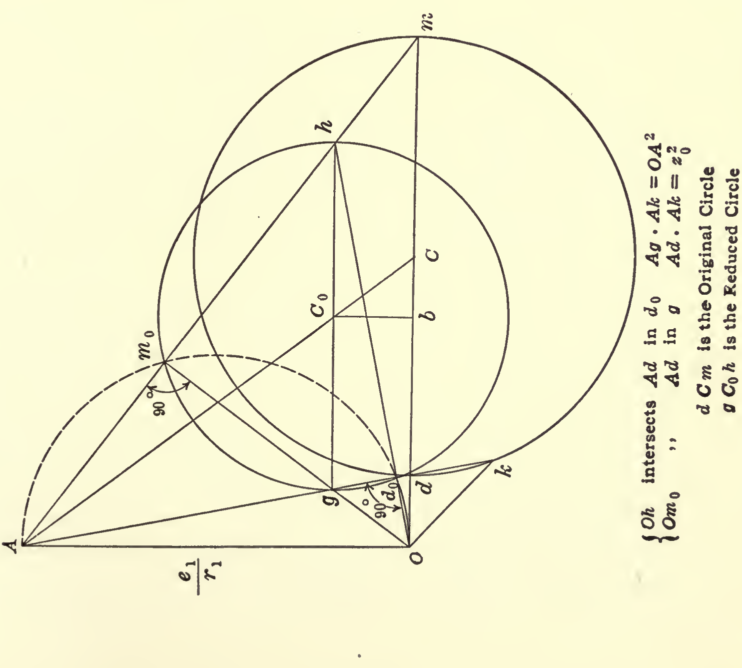


FIG. 22.—The primary and secondary current loci of the induction motor including the resistance of the primary. Point g is a fixed point through which all the secondary current vectors may be drawn.

To obtain the coordinates of the center of the circle of diameter D_0, which is the new locus of the primary current with full consideration of the primary resistance of the transformer, we proceed as follows:

$$C_0b : AO :: (AC - AC_0) : AC$$

$$C_0b : \left(\frac{e_1}{r_1}\right) :: AC \left\{ 1 - \left(\frac{\frac{e_1}{r_1}}{z_0}\right) \right\} : AC$$

$$\therefore C_0b = \zeta = \frac{e_1}{r_1} \left\{ 1 - \left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2 \right\} \tag{27}$$

The abscissa $Ob = OC - bC$ is found,

$$bC : OC :: CC_0 : AC$$

$$bC : OC :: (AC - AC_0) : AC$$

$$\therefore bC = OC \left\{ 1 - \left(\frac{\frac{e_1}{r_1}}{z_0}\right) \right\}^2$$

$$\therefore Ob = \xi = OC\left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2$$

It is interesting to note that point g in the two diagrams of Fig. 22 remains a fixed point through which *all* vectors of the secondary currents may be drawn from the corresponding points of the vector locus of the primary current with C_o as center. The proof of this must be left to the reader.

This method has been used to consider a most instructive case, viz., that of a transformer with leakage and resistance in primary and secondary operating upon a circuit whose impressed voltage varies proportional to the frequency. At a frequency 60 cycles per second, the voltage is six times that at 10 cycles, etc. If there were no primary resistance, the locus of the primary current would be the same circle through the entire range of change of frequency and voltage, the resultant flux remaining the same. If there is primary resistance, however, its effect will be greater with reduced primary impressed voltage for the same current.

The characteristic circles for this case are drawn in Fig. 23 for the conditions given in the following table:

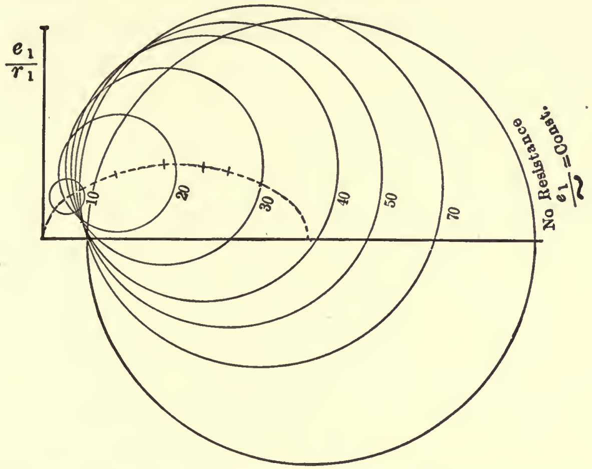


FIG. 23.—Variable frequency transformer. Primary current loci. Impressed voltage varies with frequency.

VARIABLE FREQUENCY TRANSFORMER

$\frac{e_1}{r_1}$	$\left(\frac{e_1}{r_1}\right)^2$	z_0^2	D_0	ζ	ξ
50	2,500	3,600	69.50	15.20	41.7
40	1,600	2,700	59.30	16.30	35.6
30	900	2,000	45.00	16.50	27.0
20	400	1,500	26.65	14.70	16.0
10	100	1,200	8.35	9.17	5.0
0	0	1,100	0.00	0.00	0.0

$i_0 = 10$ $\qquad$ $D = 100$

$\sigma = .1$ $\qquad$ $r_1 = 1$

$$D_0 = D\left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2$$

$$z_0^2 = \left(\frac{e_1}{r_1}\right)^2 + \frac{1+\sigma}{\sigma}\cdot i_0^2$$

$$\zeta = \frac{e_1}{r_1}\left\{1 - \left(\frac{\frac{e_1}{r_1}}{z_0}\right)\right\}^2$$

$$\xi = i_0\left(1 + \frac{1}{2\sigma}\right)\left(\frac{\frac{e_1}{r_1}}{z_0}\right)^2$$

These results will be found of great help in understanding Chap. XII on "Concatenation of Induction Motors." It is also of importance where induction motors are started with their generators from standstill by gradually raising the speed with constant field excitation.

We shall now investigate whether it is allowable to neglect the primary resistance in practice so far as it extends to its influence upon the circle locus of the primary current.

We shall calculate five values for the percentage of voltage consumed by resistance and tabulate the errors in the coordinates of the centers of the circles. We shall assume a leakage coefficient $\sigma = 0.06$, corresponding to a maximum obtainable power factor of 0.893. We will assume the transformer or motor to operate at this point of maximum power factor, and therefore the normal current will be 50 amp. for $i_0 = 12$ amp.

Table of Errors Introduced by Neglecting Primary Resistance

$\frac{i_1 r_1}{e_1}$ per cent	r_1	$\frac{e_1}{r_1}$	$\left(\frac{e_1}{r_1}\right)^2$	z_0^2	D_0	ζ	ξ
2	0.2	2,500	$625 \cdot 10^4$	$625.255 \cdot 10^4$	200.0	0.00	112.0
3	0.3	1,670	$280 \cdot 10^4$	$280.255 \cdot 10^4$	200.0	0.00	112.0
5	0.5	1,000	$100 \cdot 10^4$	$100.255 \cdot 10^4$	199.5	3.00	111.6
7	0.7	715	$51 \cdot 10^4$	$51.250 \cdot 10^4$	199.0	3.58	111.3
10	1.0	500	$25 \cdot 10^4$	$25.255 \cdot 10^4$	198.0	4.95	109.0

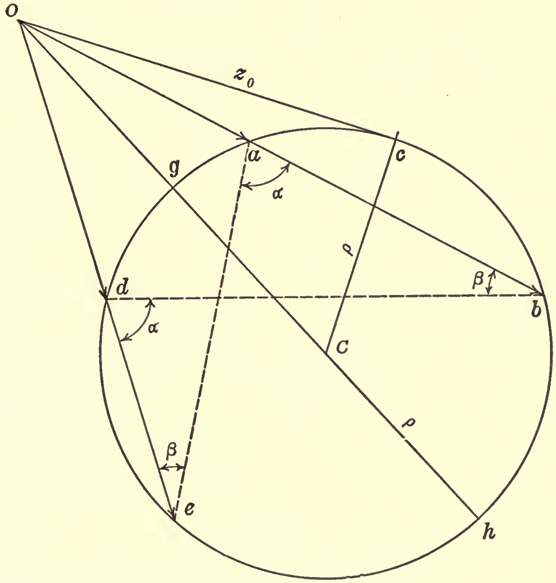
$\sigma = 0.06$ $\quad i_1 = 50$ amp.
$i_0 = 12$ amp. $\quad v_1 = 0.2$ ohm
$D = 200$ amp. $\quad e_1 = 500$ volts

From this table it appears that up to 5 per cent the effect of resistance upon the circle is negligible without a question of a doubt, while from 5 per cent to 10 per cent it seems negligible for all practical purposes, errors from other sources being vastly greater than from the neglect of primary resistance, so far as the magnitude and location of the circle are concerned.

Needless to say, the *losses* due to primary resistance must not be neglected. How this is done is shown below.

ACCOUNTING FOR PRIMARY RESISTANCE BY THE METHOD OF RECIPROCAL VECTORS

In Chap. VIII of the first edition of this book the circle diagram of the alternating-current transformer was developed, including the effect of primary resistance, using the well-known method of reciprocal vectors first applied to this problem by Prof. F. Bedell and A. C. Crehore.[1] Though principally of academic interest, we repeat here the demonstration as a useful exercise in the geometric interpretation of alternating-current phenomena.



$Oa \,.\, Ob = z_0^2$
$Od \,.\, Oe = z_0^2$
$Oa \,.\, Ob = Od \,.\, Oe$
$Oa \,.\, Od :: Oe : Ob$

FIG. 24.—Reciprocal vectors.

Figure 24 from similar triangles Δ *Odb* and Δ *Oae* we have

$$Oa : Oe :: Od : Ob$$

$$\therefore \quad Oa \cdot Ob = Od \cdot Oe = \text{constant} \tag{28}$$

Also

$$Og \cdot Oh = \text{constant}$$

$$(Oc - \rho)(Oc + \rho) = Oc^2 - \rho^2 = z_0^2$$

$$\therefore \quad Oa \cdot Ob = Od \cdot Oe = z_0^2 \tag{29}$$

where z_0 is the length of the tangent to the circle from O to c.

[1] (1) F. BEDELL and A. C. CREHORE, "Resonance in Transformer Circuits," *Physical Review*, Vol. ii, No. 12, May–June, 1895.

(2) FREDERICK BEDELL, "The Principles of the Transformer," p. 223 *et. seq.* New York, The Macmillan Company, 1896.

Now, imagine a circle (Fig. 25) about C as center representing the locus of the primary impressed e.m.f. for a constant-current transformer whose current vector coincides with the ordinate OA as proved in Chap. II. If, instead of a constant current, we keep constant the impressed e.m.f., then the current of the transformer will maintain the same phase relation to its impressed e.m.f., but its magnitude will be increased in the ratio of $\frac{e_1}{Oa}$, or $\frac{e_1}{Oc}$ and its phase will remain ψ, so that the variable voltages for

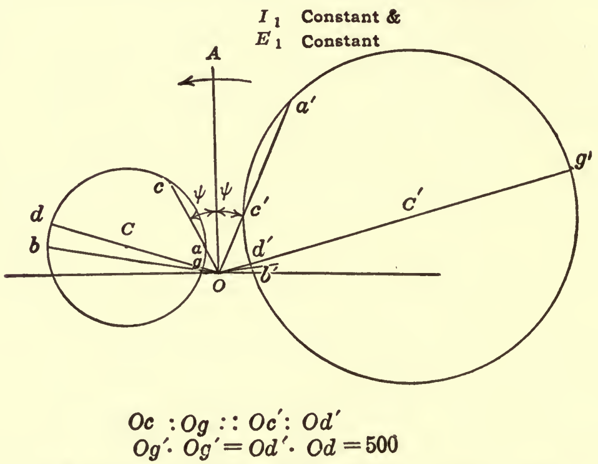


FIG. 25.—Transformation from constant current to constant voltage by means of reciprocal vectors.

the constant-current transformer will be represented by vectors on the left of OA, while the variable currents for the constant potential transformer will be represented by vectors on the right of OA.

Now,

$$\frac{e_1}{Oa} \cdot I_1 = Oa'$$

$$\frac{e_1}{Ob} \cdot I_1 = Ob'$$

$$\frac{e_1}{Oc} \cdot I_1 = Oc'$$

$$\frac{e_1}{Og} \cdot I_1 = Og'$$

$$\frac{e_1}{Od} \cdot I_1 = Od'$$

where I_1 is the primary current of the constant-current transformer.

Add the last two equations,

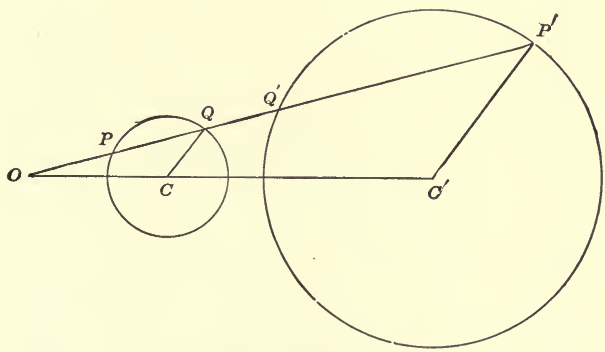
$$\frac{e_1 I_1 (Og + Od)}{Od \cdot Og} = Od' + Og'$$

$$\frac{e_1 I_1 \cdot 2OC}{Od \cdot Og} = 2OC' \qquad (30)$$

This equation (30) can also be written by substituting the value for $\frac{1}{Od} = \frac{Od'}{e_1 I_1}$.

$$OC \cdot Od' = OC' \cdot Og \qquad (31)$$

Either equation can be used to calculate the center of the derived circle.



$\frac{C'P'}{CQ} = \frac{OC'}{OC} = \frac{k^2}{z^2} =$ const.

FIG. 26.—Reciprocal vectors.

We will now prove, for the sake of completeness, that the inverse of a circle is another circle. Let P, Fig. 26, be any point on the circle, P' its inverse. Let OP cut the circle again in Q. Let C be the center of the circle. Then $OP \cdot OP' = k^2$, where k^2 is $e_1 I_1$ in the preceding argument. Now $OP \cdot OQ = z^2$, where z is tangent to circle C from O.

$$\therefore \frac{OP'}{OQ} = \frac{k^2}{z^2}$$

Take C' on OC such that $\frac{OC'}{OC} = \frac{k^2}{z^2}$, then C' is a fixed point and CP' is parallel to CQ.

Therefore, $$\frac{C'P'}{CQ} = \frac{OC'}{OC} = \frac{k^2}{z^2} = \text{constant}$$

Therefore, P' describes a circle round C' and we have proved that the inverse of a circle is another circle. This proof, it will be noticed, is similar to that given previously.

The Losses and their Representation by Straight Lines

The first and second methods showing the transformation of the resistance-less circle locus into a circle locus taking account of primary resistance, yield a better insight into the transformation than does the third method.

In order to render an account of the *loss* due to primary resistance we shall first consider the circle diagram with the

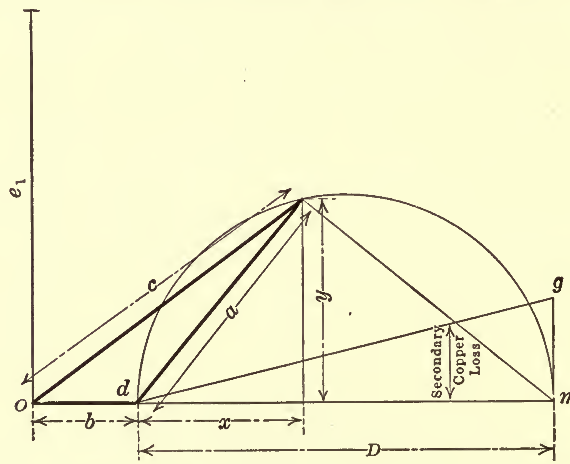


Fig. 27.—Accounting for secondary copper loss by means of the loss line.

center of the circle on the abscissa axis, *i.e.*, we shall assume, as we have demonstrated in Chap. III, that the circle locus is changed only immaterially by the presence of primary resistance of such an order as is encountered in practical apparatus.

The primary copper loss is equal to $i_1{}^2r_1$, the secondary copper loss is equal to $e_2{}^2r_2$. The vector $i_1 = c$, $i_0 = b$, and $\frac{i_2}{v_1} = a$, Fig. 27. Obviously,

$$a:x::D:a$$

$$\therefore x = \frac{a^2}{D}$$

or $$a^2 = x \cdot D$$

or $$a^2r = x \cdot (D \cdot r)$$

or $$i_2{}^2r_2 = x \cdot (D \cdot r_2 v^2{}_1) \tag{32}$$

In other words, $\frac{i_2^2 r_2}{e_1}$, which is the watt-component corresponding to the loss in the secondary copper, may be represented by the ordinates of the straight line dg.

This representation is evidently applicable only so long as $a = \frac{i_2}{v_1}$, or a proportional to $\frac{i_2}{v_1}$, which relation can be proved to hold even for the case in which primary resistance r_1 is taken account of.

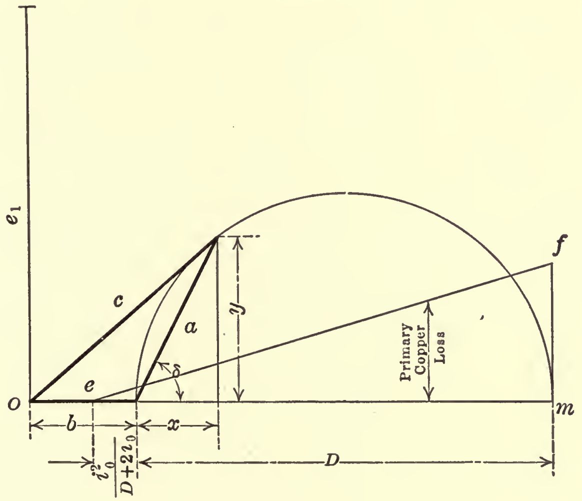


FIG. 28.—Accounting for primary copper loss by means of the loss line.

The primary copper loss is found as follows, Fig. 28,

$$c^2 = a^2 + b^2 + 2ab\cos\delta$$
$$c^2 = x \cdot D + b^2 + 2bx$$
$$\therefore\ c^2 = x(D + 2b) + b^2 \qquad (33)$$

Also
$$i_1^2 r_1 = x(D + 2i_0)r_1 + i_0^2 r_1 \qquad (34)$$

Also
$$i_1^2 \cdot \frac{r_1}{e_1} = x(D + 2i_0)\frac{r_1}{e_1} + i_0^2 \cdot \frac{r_1}{e_1} \qquad (35)$$

In words, the primary copper loss is proportional to x plus the constant $i_0^2 r_1$, and it may therefore be represented by a straight line whose ordinates, measured from the x-axis are equal to the watt-component of the primary copper loss.

For $i_1 = i_0$ we obtain

$$x(D + 2i_0)r_1 = 0$$
$$\therefore x = 0$$

For $i_1 = 0$ we obtain

$$x = -\frac{i_0^2}{D + 2i_0} \qquad (36)$$

These simple methods for determining the copper losses were originally given by the author 20 years ago in Appendix III of the first edition of this book.

If the position of the circle is such that the center does not lie on the abscissa, then the watt-components of the losses are no longer to be measured parallel to e_1 but as proved above, normal to the diameter of the circle.

The Iron Losses Due to Hysteresis and Eddy Currents

The iron losses, or the core loss, of an induction motor are due to hysteresis and eddy currents. There has been discussion regarding the most accurate manner in which to take them into account in the diagram.

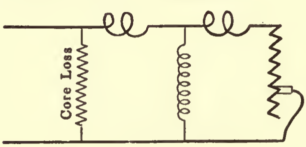


Fig. 29.—Equivalent circuits showing position of core loss circuit.

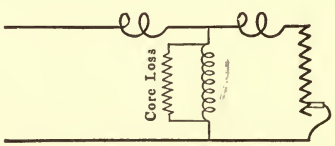


Fig. 30.—Equivalent circuits showing position of core loss circuit. (Alternate.,)

In view of the fact that the hysteresis loss is likely to be proportional to the resultant primary field, this loss may well be assumed constant. The loss due to the eddy currents generated by this field may also be assumed constant at all loads. Losses due to stray fields are apt to be very considerable and these, therefore, would increase with increasing current load. As it is not practicable to take all these factors into account, I proposed *first* in 1896, and I was seconded by Prof. Blondel, to look upon these losses as though produced in a resistance shunted across the primary potential, Fig. 29. Other writers, like Steinmetz, Arnold, LaCour, McAllister, and Bragstad have, however, used a *second* method of an equivalent circuit, as shown in Fig. 30, in which at standstill the core loss is a minimum, gradually increasing with decreasing load or increasing speed. This theory does not appear very reasonable, as the hysteresis loss is more likely to be dependent upon the *total* field rather than the *common* field. It is true that it is much more difficult to take into account the core loss if it depended upon the potential at the terminals of the exciting shunt as it is indicated in the equivalent circuit. But the fact that it is more difficult to take it into account with this assumption, though this assumption is farther removed from the actual conditions, should be no reason why it should be considered necessary to do it. "Error which is not pleasant, is surely the worst form of wrong."

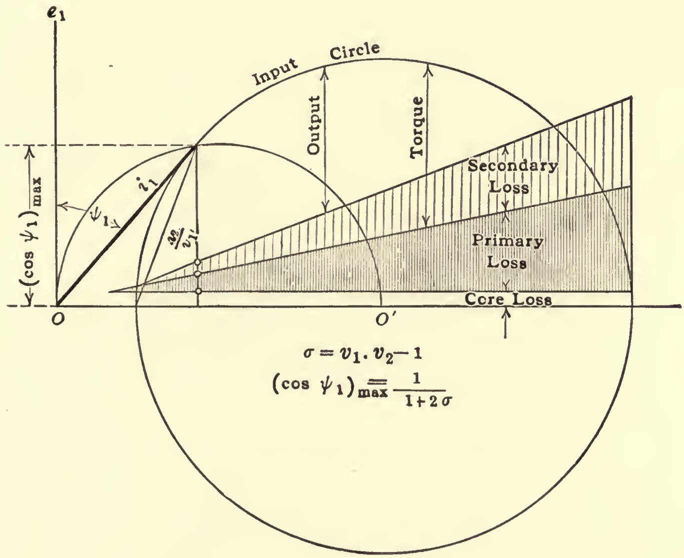


Fig. 31.

A *third* method which I have followed from time to time since 1900 assumes that the circle diagram is derived without taking into account the core losses, and that this loss is later accounted for by deducting it from the secondary output. This procedure is likely to be about as accurate as any of the previous methods and it has the merit of greater simplicity. I believe it was first suggested by my friend Heyland.

Load losses should doubtless be accounted for by an increase in the primary and secondary resistances. If this is done, then it appears altogether illogical to take into account the core loss by making it depend upon the common field, which goes through the air-gap, and this strange conception would never have arisen but for the equivalent circuit methods which make it appear as though the voltage drop in the primary leakage reactance occurred outside the machine while, in reality, the flux which produces the reactance voltage, is vectorially added to the common air-gap flux.

If we assume the core loss which is made up of hysteresis and eddy-current losses to be constant at all loads, which is a very problematical assumption, and justifiable only on account of our profound ignorance of the causes of core loss and the magnitude of these losses, then we may assume its effect to be equivalent to a constant watt loss, whose watt-component is constant and may be represented in our diagram by a line parallel to the diameter of the circle. By this amount the available secondary power will be diminished. We will recur to this matter in subsequent chapters, while Fig. 31 shows these losses graphically.

B. THE TRANSFORMER WITH INDUCTIVE LOAD

Assume the secondary of a transformer to be closed by a circuit with resistance and inductance. Assume inductance and resistance to vary in such a manner that the power of the secondary external circuit remains constant. Then, Fig. 32, we have:

F_2 the secondary resultant magnetic field which induces $E_2 = e_2 + i_2 r_2$

i_2 the secondary load current lagging in time by the angle ψ.

$f_2 = (v_2 - 1)\Phi_2 = ab$, secondary leakage field in phase with the secondary load current.

$\Phi_2 = be$ the fictitious secondary flux proportional to the secondary m.m.f.

$v_2\Phi_2 = ae$

$\Phi_1 = Oe$ the fictitious primary flux proportional to the primary m.m.f.

$f_1 = (v_1 - 1)\Phi_1 = bc$ the primary leakage flux.

$$bd = \left(1 - \frac{1}{v_1}\right)\Phi_2$$

$$dc = v_1 \cdot Od$$

ΔOad and Δmed are similar triangles. Angle Oae is equal to a right angle plus ψ_2, it is therefore a constant angle for a variation of i_2. Hence point a moves on the arc of a circle, and point e does equally so.

$$Od : ad :: md : de$$

$$\frac{F_1}{v_1} : \left(v_2 - \frac{1}{v_1}\right) \Phi_2 :: D : \frac{\Phi_2}{v_1}$$

$$\therefore \quad \frac{F_1}{v_1} \cdot \frac{\Phi_2}{v_1} = (v_1 v_2 - 1) \frac{\Phi_2}{v_1} \cdot D$$

$$\frac{F_1}{v_1} : D = v_1 v_2 - 1 \tag{37}$$

$$\sigma_i = v_1 v_2 - 1 \tag{38}$$

$$\frac{Od}{md} = \sigma_i \tag{39}$$

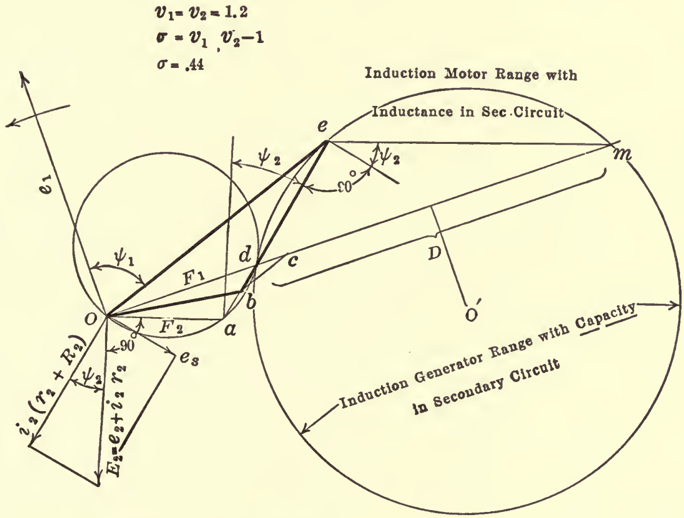


FIG. 32.—Inductance in the secondary of constant potential transformer (motor range) and capacity in secondary for induction generator range.

The effect of the inductance in the secondary circuit consists in greatly reducing the maximum watt-component of the primary current, and in reducing the power factor. The capacity of a transformer is thus greatly reduced by an inductive load.

C. THE TRANSFORMER WITH CAPACITY LOAD

Assume the secondary of a transformer to be closed by a circuit with such condenser capacity that both resistance and capacity may be varied so as to keep the power factor $\cos \psi_2$ of the external circuit constant. Then Fig. 33 shows that triangle

Oad is similar to triangle med and $eb = \Phi_2$, in phase with the secondary current i_2.

$$Od : ad :: D : de \tag{39}$$

$$\frac{F_1}{v_1} : \left(v_2 + \frac{1}{v_1}\right)\Phi_2 :: D : \frac{\Phi_2}{v_1}$$

$$\frac{F_1}{v_1} : D = v_1 v_2 - 1 \tag{40}$$

$$\sigma_c = v_1 v_2 - 1 \tag{41}$$

$$\frac{Od}{md} = \sigma_c \tag{42}$$

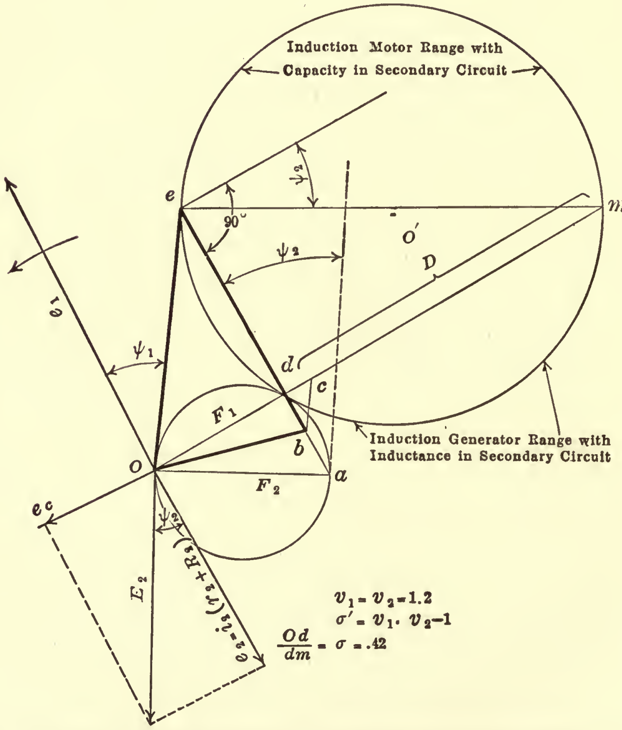


FIG. 33.—Capacity in the secondary of constant potential transformer (motor range) and inductance in secondary for induction generator range.

The effect of variable capacity in the secondary of a constant-potential transformer consists therefore in greatly raising its receptive capacity and in increasing the power factor.

How this effect can be obtained by means of rotating apparatus is shown in Chap. XV. The effect of such apparatus upon the characteristics of the induction generator is discussed in Chap. IX.

CHAPTER IV

THE McALLISTER TRANSFORMATIONS

It is well known that geometrical figures can be "transformed" by means of a complex function used as an operator and that the "transformation" is frequently a solution of a problem otherwise impossible of solution. There are certain partial differential equations in physics, especially in hydrodynamics and in electricity, to which these transformations have been applied successfully. In fact, the entire fascinating subject of "conformal representation" of functions including the problem of "Mercator's Projection" in geography and solutions for the electrostatic capacity of conductors of different shape, is intimately connected with this subject

In connection with vector diagrams as used in this book, the transformation by "reciprocal vectors" occurs most often and it has therefore been discussed in the previous chapter. It consists in simultaneously shrinking and turning a vector, and it transforms invariably one circle into another.

As a general rule "point for point" reductions have been made both in the graphical treatment and in the analytical treatment by means of complex algebra. In a few cases, the graphical treatment has had the advantage as the circle locus property permits an easy representation of the entire set of complex algebraic equations. Where, however, a "point for point" method has to be resorted to, the advantage of the graphical method is less apparent.

It has been shown in the previous chapter that the addition of resistance in series with the induction motor, the primary resistance of which has been neglected, leads again to a circle for the locus of the primary current. The center of this circle is raised above the abscissa and its diameter is smaller than that of the circle representing the performance of the motor without primary resistance. However, the process had been limited to a special case and no generalization of it had been developed.

Dr. A. S. McAllister has succeeded recently in applying the same process of reasoning through which we have taken the reader in the development of the general circle diagram, to the general case which comprises resistance in series, reactance in series, or resistance and reactance in series with the motor. He has discovered a method, simple and direct, of determining the "center of inversion" from which the new circle can be located by drawing a few lines to the image of the original circle. It may be remarked in passing that it is curious that so simple a solution has taken 25 years to be brought to light.

In the accompanying figures there are treated the following cases:

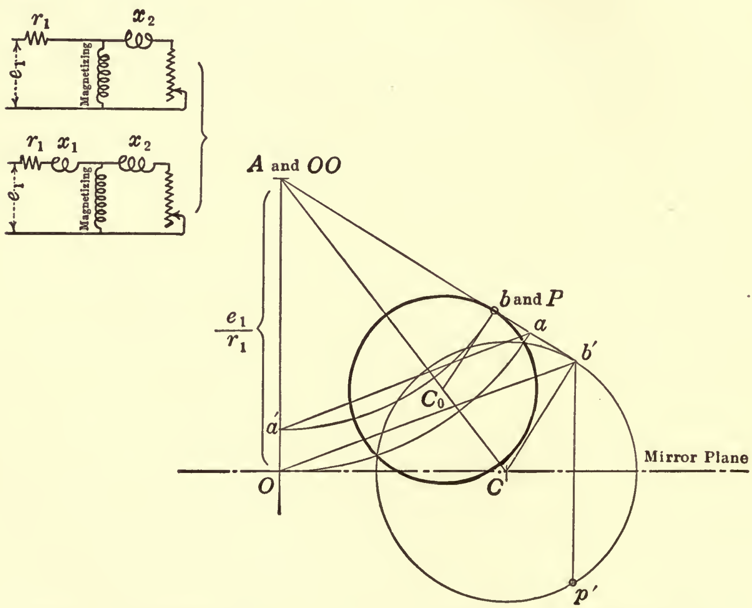


FIG. 34.—The McAllister transformations. Resistance in series with the motor, *without* core loss.

A. Resistance in series with the motor *without* core loss (Fig. 34).
B. Reactance in series with the motor *without* core loss (Fig. 35) .
C. Impedance in series with the motor *without* core loss Figs. 36 and 37).
D. Resistance in series with the motor *with* core loss (Fig. 38).
E. Reactance in series with the motor *with* core loss (Fig. 39).
F. Impedance in series with the motor *with* core loss (Fig. 40).

It is interesting to observe that, if we assume the core loss to depend upon the air-gap field F only, the cases D, E, and F show that the resultant primary locus remains a circle. If we assume the core loss to depend upon the total primary flux F_1, which seems to the writer the more rational assumption, then the resultant primary locus is also a circle. These circles, however, differ in magnitude and location.

The proof of the McAllister method is very simple. It is suggested in all the diagrams. The start is made with the original circle whether this is the circle of the locus of the primary current as used for the performance of the induction motor in this book, or whether it is the locus of the primary current if the

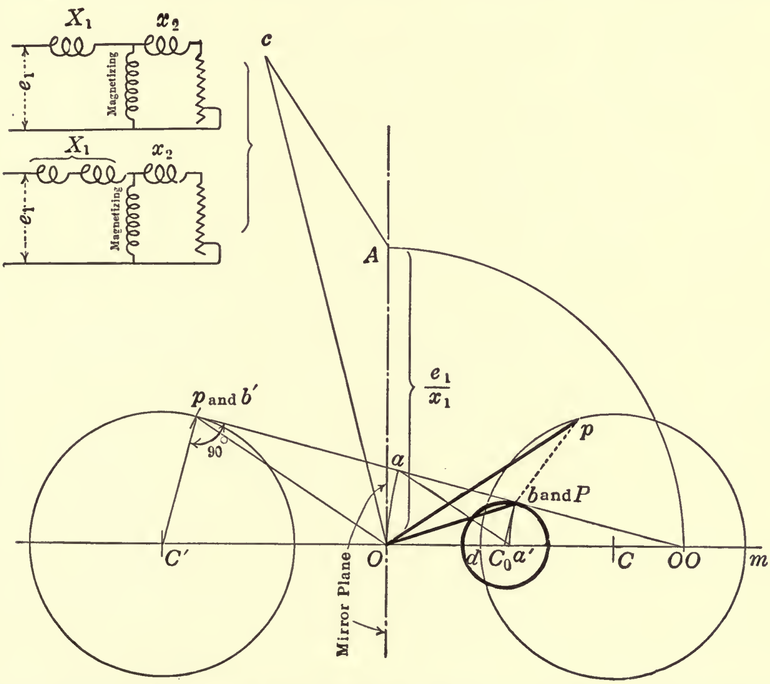


FIG. 35.—The McAllister transformations. Reactance in series with the motor, *without* core loss.

voltage on the magnetizing circuit in the "equivalent" diagram is kept constant.

By reflecting this circle for the cases A and D below the abscissa; for B and E to the left of the ordinate; and for cases C and F below a mirror plane which forms with the abscissa an angle whose tangent is equal to X/R, similar triangles are formed by drawing rays from the "center of inversion" to the *reflected* point. In these rays lie the transformed points cutting the ray in the inverse proportion.

The procedure then is the same in all six cases.

Referring now specifically to Fig. 34 illustrating case "A," the circle with Cb' as radius represents the locus of the primary current if the potential on the "magnetizing circuit" were kept constant. As we make OA equal to $\frac{e_1}{r_1}$ any radius vector measuring the primary current measures also the ohmic drop in the primary. Thus, as we have seen in Figs. 22 and 26, Chap. III, the point A or OO becomes the center of inversion from which,

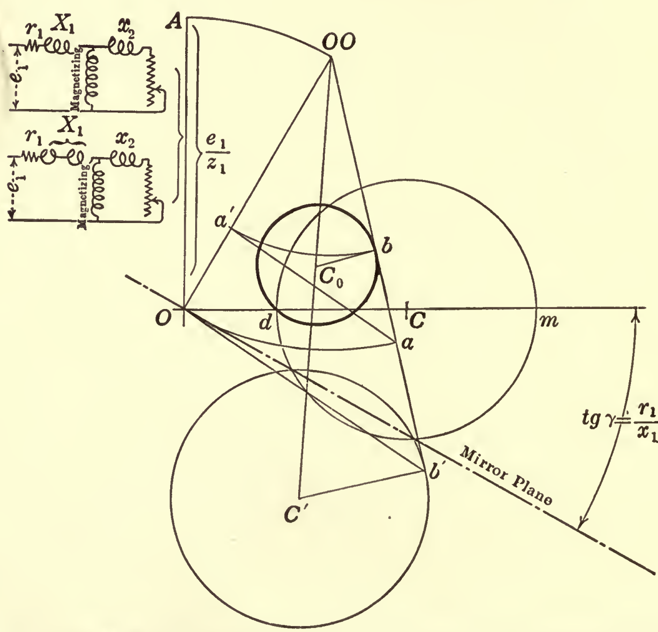


FIG. 36.—The McAllister transformations. Impedance in series with the motor, *without* core loss.

by means of similar triangles as used before, we obtain the new locus for the primary currents with center C_0 in a manner indicated geometrically in the figure. The vectors drawn from O to the circle whose center is C_0 represent, in the scale adopted, both the primary current and the primary ohmic drop.

In the next figure (Fig. 35), which represents the case "B" in which reactance is placed in series with the motor, the process is shown in greater detail. The circle with C as center represents in Op the primary current if the potential on the magnet-

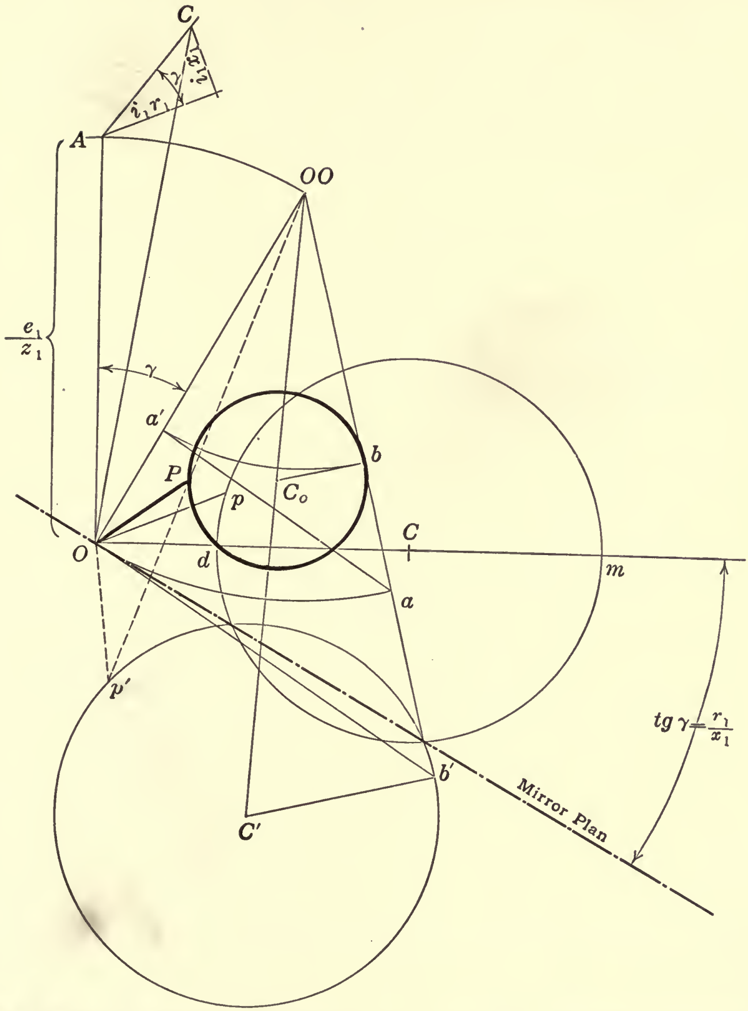


FIG. 37.—The McAllister transformations. Impedance in series with the motor, $AC = Op$.

izing circuit were kept constant. As OA represents the primary impressed voltage divided by the additional reactance placed in series with the motor, $Ac = Op = i_1 \cdot X_1$ represents the reactance drop in the circuit connected in series with the motor.

Now draw the circle with C' as center which is theimage circle of C. Note that $\Delta OpOO = \Delta AcO$ which triangles are similar to $\Delta CO - OO$. From this follows that

$$Op : O - OO :: Ob : OO - b$$

or

$$OO - b \cdot Op = \frac{e_1}{X_1} \cdot i_1 X_1$$
$$= e_1 \cdot i_1$$

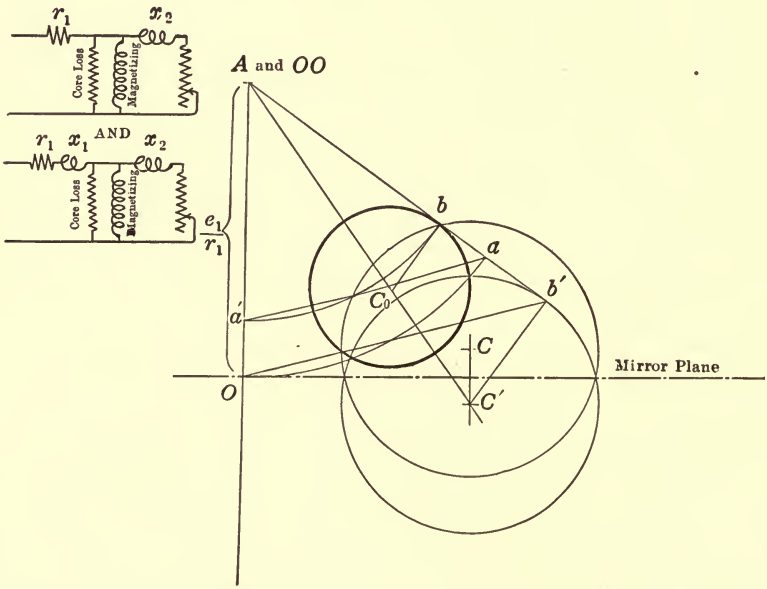


Fig. 38.—The McAllister transformations. Resistance in series with the motor, *with* core loss.

This proves our proposition as it shows that the point b lies invariably upon the ray drawn from the "center of inversion OO to the "reflected" point p or b' which is the image of the original point p in the mirror plane OA. Point b divides $OO - p$ in such a ratio that the product of the vectors drawn from OO is a constant. Hence the new locus of i_1 is a circle with C_0 as center $C_0 b$ being perpendicular to $OO - p$. The reader would

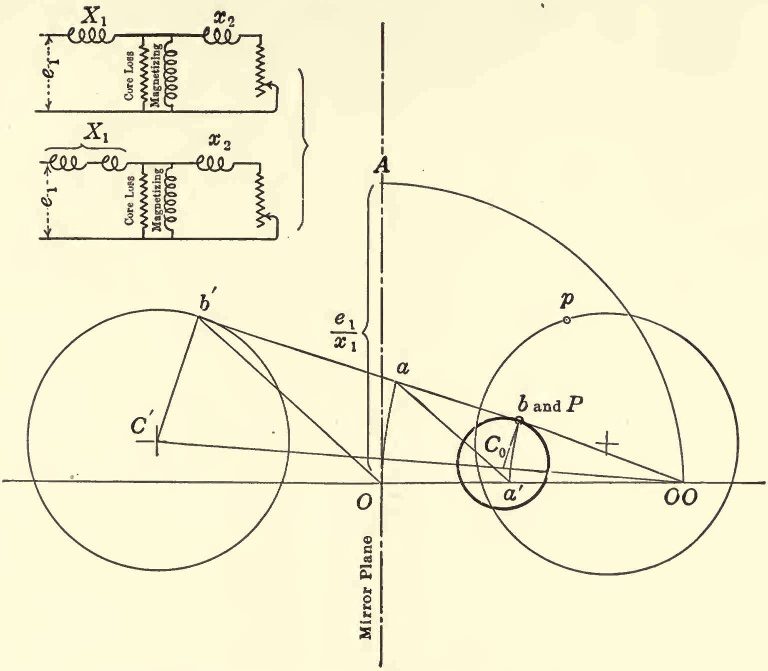


FIG. 39.—The McAllister transformations. Reactance in series with the motor, *with* core loss.

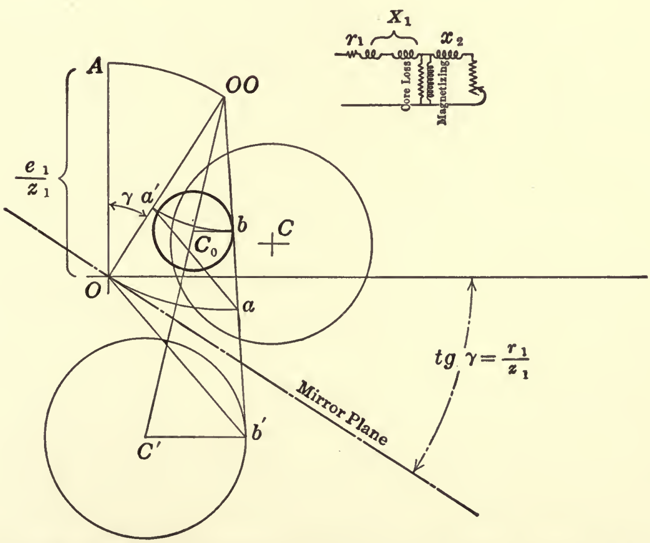


FIG. 40.—The McAllister transformations. Impedence in series with the motor *with* core loss.

do well to draw for himself a few points and to transform them into their new positions in the vector diagram.

The remaining figures referring to the six cases enumerated are sufficiently clear to require no further explanations as to their mode of derivation, provided the reader will take the pains to draw a few points in order to comprehend the principles on which this method is based. This has been done in detail in Fig. 37 illustrating with Fig. 36, the case C.

Similar results to those arrived at above by the McAllister method have been obtained by successive applications of the method of reciprocal vectors, discussed in Chapter III–A, as shown by Messrs. LaCour and Bragstad, who have transformed the admittance circle diagram into an impedance circle diagram to which they have added the primary impedance, re-transforming the new impedance diagram back into the final admittance diagram. This method is described at length in the great work of these authors frequently referred to in this book.

CHAPTER V

THE ROTATING FIELD AND THE INDUCTION MOTOR

A. THE AMPERE TURNS AND THE FIELD BELT

In the induction motor, as invented by Mr. Nikola Tesla, two, three, or more windings lodged in slots, usually located in the stationary part or stator, are fed with alternating currents of the same frequency and voltage but of different time-phase. A

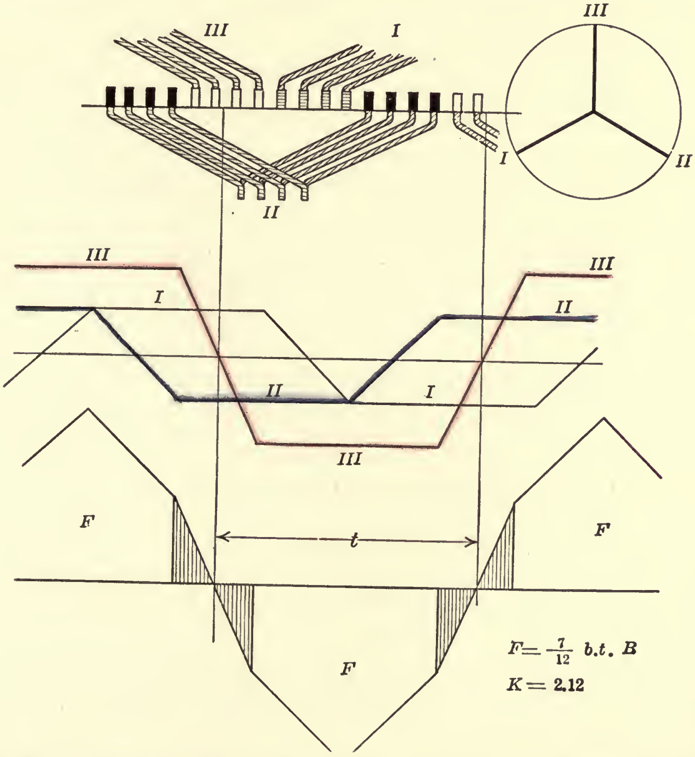


FIG. 41.—Distributed three-phase winding. The belt of ampere-turns and flux.

belt of these windings produces a rotating field. In a short-circuited rotor winding currents are induced whose interaction with the field results in the production of torque. We shall briefly examine the manner in which such a field is produced.

Let I, II, and III be the three phases of the stator. We shall assume that the reluctance of the iron is negligible and that the induction is proportional to the line integral of the m.m.fs., and inversely proportional to the length of the gap (Fig. 41). We shall assume the current to vary according to a simple sine law. Then the intensity of the currents in I and II is each one-half that of III. The m.m.fs. of each phase are represented by the ordinates of the curves I, II, and III respectively. Each ordinate measures the m.m.f. produced in that point of the circumference where it is drawn. The adding together of the ordinates of the three curves yields the heavy line curve which is the sum of the m.m.fs. at the particular moment of time over the circumference of the air-gap.

If the magnetic reluctance is the same at every point of the circumference, and the reluctance of the iron is negligible, then the magnetic induction B, produced by the m.m.f. belt shown in the heavy line in the Fig. 41, is proportional to this m.m.f. and, therefore, also represented by the ordinates of the heavy line curve. We call the total flux F, and we assume that the time variation of this flux follows a simple sine law. We shall now calculate the e.m.f. induced by this flux belt in each of the three phases.

We shall assume arbitrarily that the distribution of conductors per phase is uniform, in other words, that there is an infinite number of slots.

B. THE E.M.FS. INDUCED IN THE WINDINGS

If all the conductors of each phase were concentrated in one slot, then the e.m.f. induced—remembering that two conductors equal one complete turn—would be according to Eq. (10) equal to $2.22 \sim \cdot z \cdot F \cdot 10^{-8}$ volts, where z is the total number of effective conductors per phase equal to $2n$. On account of the distribution of the windings over one-third of the pole-pitch, only the parts of the flux not covered with hatchings can induce an e.m.f. according to this formula, while the hatched parts of the field will have a considerably smaller effect. Let the width of the coil be $2b$, and n conductors in the coil spread over $2b$. Per unit length there are, therefore, $\frac{n}{2b}$ conductors, hence the element dx contains $dx \cdot \frac{n}{2b}$ conductors. The number of lines of induction threading all the conductors in the element dx is equal to F_x represented by the hatched area. Hence,

$$de = 2.22 \cdot \sim \cdot \frac{n}{2b} \cdot dx \cdot F_x \cdot 10^{-8} \text{ volts}$$

$$F_x = B \cdot \frac{b}{2} - B_x \cdot \frac{X}{2} = B \cdot \frac{b}{2} - \frac{Bx^2}{2b}$$

$$\therefore \quad de = 2.22 \cdot \sim \cdot \frac{n}{2b} \left[\frac{B \cdot b \cdot dx}{2} - \frac{B \cdot x^2 \cdot dx}{2b} \right] 10^{-8}$$

$$e = 2.22 \cdot \sim \cdot \frac{n}{2b} \left[\int_o^b \frac{B \cdot b \cdot dx}{2b} - \int_o^b \frac{B \cdot x^2 \cdot dx}{2b} \right] 10^{-8}$$

$$e = 2.22 \cdot \sim \cdot \frac{n}{2} \cdot B \left[\frac{b}{2} - \frac{b}{6} \right] 10^{-8}$$

$$e = 2.22 \cdot \sim \cdot \frac{n}{2} \cdot B \cdot \frac{b}{3} \cdot 10^{-8}$$

With $F = \dfrac{B \cdot b}{2}$

$$e = 2.22 \cdot \sim \cdot \frac{n}{2} \left(\frac{2}{3} F \right) 10^{-8} \qquad (43)$$

In words, the e.m.f. induced by the field F upon a coil of the width b is two-thirds as great as the e.m.f. which would be induced by the same field upon a coil whose conductors are not distributed but lodged in one slot. Such a coil would not produce a triangular field but a rectangular field. Therefore, the inductance of the flat coil, *i.e.*, the number of lines or tubes of force per unit current, is one-third as large as the inductance of a coil lodged in one slot.

The e.m.f. generated by the field belt in Fig. 41 can now readily be calculated. The flux of the white area is

$$\left(\frac{2}{3} \cdot t \cdot b \cdot B \right) \frac{3}{4}$$

The e.m.f. induced by this flux is equal to

$$e_a = 2.22 \cdot \sim \cdot z \cdot \left(\frac{3}{4} \cdot \frac{2}{3} \cdot t \cdot b \cdot B \right) 10^{-8}$$

The hatched areas represent a flux equal to

$$\frac{1}{6} \cdot t \cdot b \cdot \frac{B}{2}$$

The e.m.f. induced by this flux is

$$e_b = 2.22 \cdot \sim \cdot z \cdot \frac{2}{3} \left(\frac{1}{12} \cdot t \cdot b \cdot B \right) 10^{-8}$$

Hence $$e = e_a + e_b = 2.22 \cdot \sim \cdot z\left(\frac{20}{36} \cdot t \cdot b \cdot B\right) 10^{-8}$$

The total flux is

$$F = \frac{7}{12} \cdot b \cdot t \cdot B$$

$$\therefore e = 2.22 \cdot \sim \cdot z \cdot \frac{20}{21} \cdot F \cdot 10^{-8}$$

$$e = 2.12 \cdot \sim \cdot z \cdot F \cdot 10^{-8} \tag{44}$$

The amperė-turns in each phase which are needed to produce the induction B in the air-gap are determined by the consideration, which follows immediately from Fig. 41, that

$$2(.4 \cdot n \cdot i_\mu \sqrt{2}) = B \cdot 2\Delta$$

where Δ is the single distance, or length, between rotor and stator separated by the air-gap, n the number of conductors per pole per phase, and i_μ the magnetizing current. The reluctance of the iron has been neglected.

Hence,

$$i_\mu = \frac{B \cdot 1.6\Delta}{2n\sqrt{2}} \text{ amp.} \tag{45}$$

If the reluctance of the iron is not negligible, then the magnetic induction B has to be determined point for point, which can be done with the aid of a magnetizing curve. It is of importance to note that the maximum induction does not extend over a very large part of the pole-pitch, hence very high induction in the teeth may be resorted to without materially raising the magnetizing current, although increasing the losses which are dependent upon B_{max} in the teeth.

There are numerous modes of distribution of the conductors per pole per phase. We have considered above that each phase covers one-third of the pole-pitch in a three-phase motor. The conductors may, however, spread over two-thirds of the pole-pitch. Figure 42 shows the m.m.f. belt at the time when phase *III* is a maximum.

In order to obtain quickly an expression of the e.m.f. which such a field induces, we avail ourselves of the simple and direct method given by the author in Appendix II of the first edition of this book.

Consider a winding *ag*, Fig. 43, spanning an arc of 180 electrical degrees, *i.e.*, extending over the pole-pitch. In each small element *ab*, or *bc*, there is induced an e.m.f. *de*, represented graphically by the small vector *AB* or *BC*, which we arbitrarily

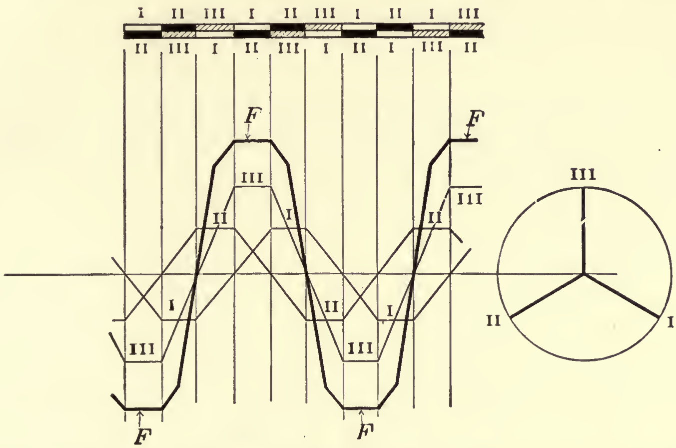


Fig. 42.—Field belt of three-phase motor. Each phase spread over two-thirds of pole pitch.

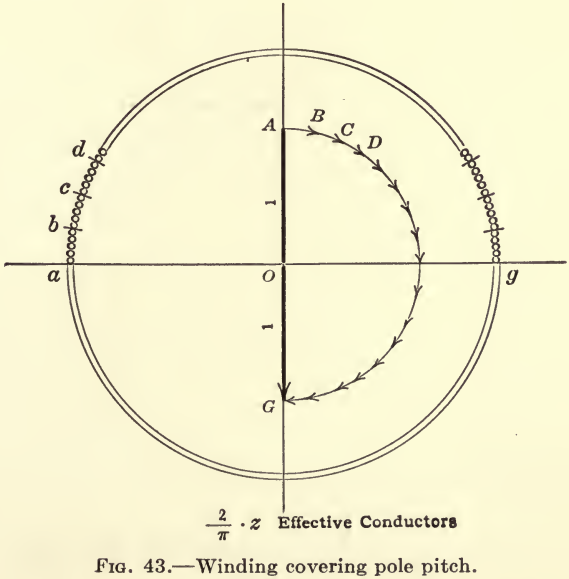


Fig. 43.—Winding covering pole pitch.

represent at right angles to the element. Then *ABCD—G* is, so to speak, the hodograph of the induced voltages, whose vector sum is AG. If x is the total number of conductors in the winding ag, then there are, as a result of distribution,

$$\frac{2}{\pi} \cdot z \text{ effective conductors only.}$$

Hence,
$$e = \sqrt{2} \cdot \sim \cdot z \cdot F \cdot 10^{-8} \text{ volts} \tag{46}$$

If the winding is distributed over two-thirds the pole-pitch, or over 120 electrical degrees, Fig. 44, then the number of effective conductors is

$$(\sqrt{3} \div \tfrac{2}{3}\pi)\, z$$

which is equal to $0.825\, z$

$$\therefore \quad e = 1.836 \sim z \cdot F \cdot 10^{-8} \text{ volts} \tag{47}$$

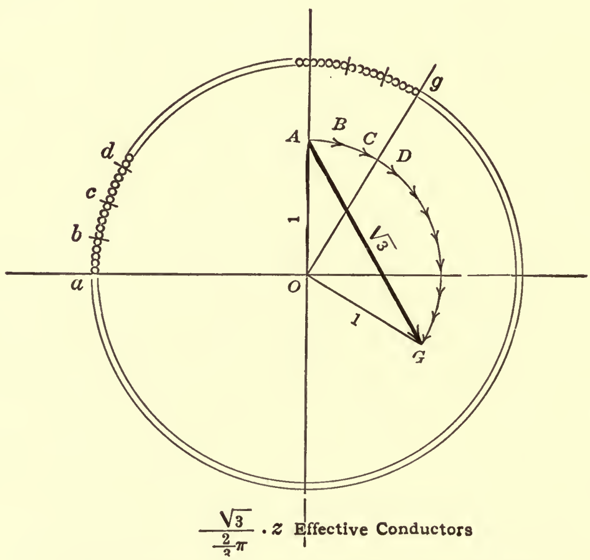


FIG. 44.—Winding covering two-thirds of pole pitch.

In a quarter-phase system one-half the pole-pitch is covered by the coil, Fig. 45, therefore there are

$$\left(\sqrt{2} \div \frac{\pi}{2}\right) z \text{ effective conductors per phase}$$

$$\therefore \quad e = 2 \sim \cdot z \cdot F \cdot 10^{-8} \text{ volts} \tag{48}$$

In a three-phase motor, the coils usually cover one-third of

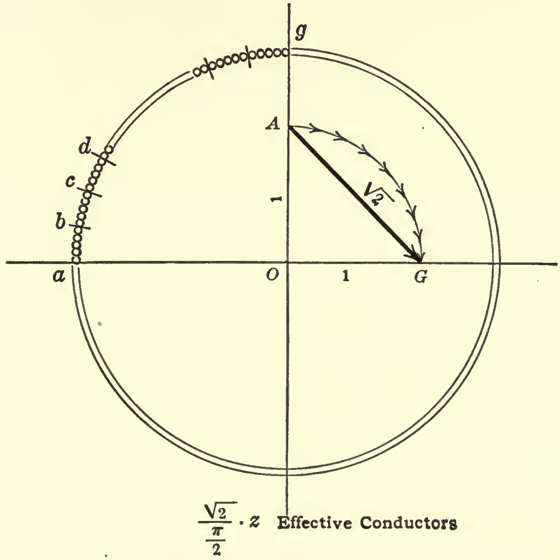


FIG. 45.—Winding covering one-half of pole pitch.

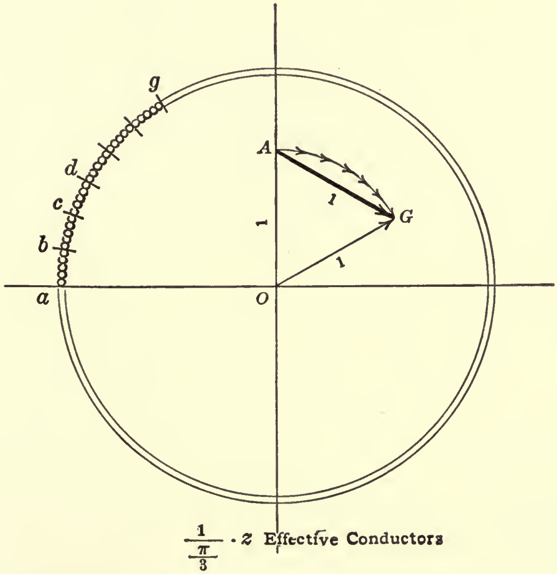


FIG. 46.—Winding covering one-third of pole pitch.

the pole-pitch, or 60 electrical degrees (Fig. 46). Therefore, there are

$$\left(1 \div \frac{2\pi}{6}\right) z \text{ effective conductors per pole per phase.}$$

$$\therefore \quad e = 2.12 \cdot \sim \cdot z \cdot F \cdot 10^{-8} \text{ volts} \tag{49}$$

These methods are correct whatever the shape of the field belt, as long as F equals the total flux produced.

C. THE ELEMENTARY THEORY OF THE INDUCTION MOTOR

The elementary phenomena of the functioning of an induction motor has been stated with admirable clearness by Prof. Gisbert Kapp, to whom we owe so much in the interpretation of the theory of electrical apparatus. As his account is also of historical interest it is here reprinted in full, as was done in Appendix I of the first edition of this book.

Excerpt from Gisbert Kapp: "Electric Transmission of Energy, and its Transformation, Subdivision, and Distribution." Fourth Edition, Thoroughly Revised. London, Whittaker & Co., Paternoster Square, 1894, p. 301 to p. 311.

"In order to be able to deal by means of simple mathematics with the working condition of a rotary field motor, we assume that the induction within the interpolar space between field and armature varies according to a simple sine law. Whether this induction is due to the current in the field coils alone, or to the combined effect of field and armature currents, we need at present not stop to inquire; all we care to know is that such an induction does actually exist when the motor is at work, and that the sinusoidal field which it represents revolves with a speed corresponding to the frequency of the supply currents. Thus, if there be four field coils, and the frequency is 50, we would have a two-pole field revolving 50 times a second, or 3,000 times a minute, round the centre of the armature, and if there were no resistance to the movement of the latter it would be dragged round by the field at a speed of 3,000 r.p.m. It is obvious that the actual speed must be smaller. If the speed of the armature coincided exactly with that of the field, then the total induction passing through any armature coil, or between any pair of conductors on the armature would remain absolutely constant, and there would be no e.m.f., and, therefore, no current induced in the armature wires. Where there is no current there can be no mechanical force, and the armature could, therefore, not be kept in rotation. In order that there may be a mechanical force exerted, it is obviously essential that there shall be a variation in the magnetic flux passing

(*Facing page* 64)

through any armature coil, and that necessitates a difference in the speed of rotation between field and armature. This difference is called the 'magnetic slip' of the armature. If, for instance, the speed of the field in our two-pole motor is 50 revolutions per second, and the speed of the armature 48 revolutions per second, we would have a magnetic slip of two revolutions out of 50, or 4 per cent. In modern machines the slip at full load averages about 4 per cent, and rarely reaches as high as 10 per cent, so that good rotary field motors are in point of constancy of speed under varying load about equal to continuous shunt motors.

"It was mentioned above that the motor would have a frequency of 50 revolutions at a speed only by 4 per cent short of 3,000 r.p.m. This is an inconveniently high speed for any but very small sizes. To reduce the speed is, however, quite easy. We need only increase the number, and proportionately reduce the length of the field coils. Thus, if instead of 4 coils, each spanning 90° of the circumference, we use 8 coils, each spanning 45°, and connect them so as to produce two rotary fields, the speed will be reduced to one-half of its former value. By using 12 coils we obtain a six-pole motor, in which the speed will be reduced to one-third, or about 1,000 r.p.m.; with 16 coils we get down to 750 revolutions, and so on. In order to avoid unnecessary complexity we shall, however, commence the investigation on a two-pole machine, having only one revolving field, and leaving the transition to a multi-polar machine running at lower speed until the more simple case has been dealt with.

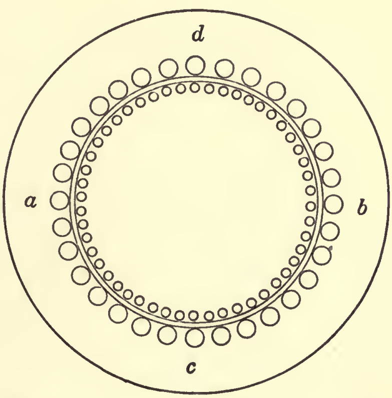


Fig. 47.—Diagram of rotor and stator of induction motor.

"Such a machine is shown in Fig. 47. The field consists of a stationary cylinder, composed of insulated iron plates, and provided close to the inner circumference with holes through which the winding passes. The armature is also a cylinder made up of insulated iron plates provided with holes near its outer circumference for the reception of the conductors. The use of buried conductors, although not absolutely necessary, has two important advantages—first, mechanical strength and protection to the winding; and, secondly, reduction of the magnetic resistance of the air-gap, which, it will be seen later on, is an essential condition for a machine in which the difference between the true watts and apparent watts shall not be too great. The armature conductors may be connected so as to form single loops, each passing across a diameter, or they may all be connected in parallel at each end face by means of circular conductors, somewhat in the fashion of a squirrel cage. Either system of winding does equally well, but as the latter is mechanically more simple, we will assume it to be adopted in Fig. 47. The circular end connections are supposed to be of very large area as compared with the bars, so that their resistance may be neglected. The potential of either connecting ring will then remain permanently at zero, and the current passing through each bar from end to end will be that due to the e.m.f. acting in the bar divided by its resistance. It is important to note that the e.m.f. here meant is not only that due to the bar cutting through the lines of the revolving field, but that which results when armature reaction and self-induction are duly taken into account.

"Let us now suppose that the motor is at work. The primary field produced by the supply currents makes $\sim_1$ complete revolutions per second, whilst the armature follows with a speed of $\sim_2$ complete revolutions per second. The magnetic slip is then

$$s = \frac{\sim_1 - \sim_2}{\sim_1} \tag{50}$$

If the field revolves clockwise, the armature must also revolve clockwise, but at a slightly slower rate. Relatively to the field, then, the armature will appear to revolve in a counter clockwise direction, with a speed of

$$\sim\ = \ \sim_1 - \sim_2 \tag{51}$$

revolutions per second. As far as the electro-magnetic action within the armature is concerned, we may therefore assume that the primary field is stationary in space, and that the armature is revolved by a belt in a backward direction at the rate of $\sim$ revolutions per second. The effective tangential pull transmitted by the belt to the armature will then be exactly equal to the tangential force which in reality is transmitted by the armature to the belt at its proper working speed, and we may thus calculate the torque exerted by the motor as if the latter were worked as a generator backward at a much slower speed, the whole of the power supplied being used up in heating the armature bars. The object of approaching the problem from this point of view is of course to simplify as much as possible the whole investigation. If we once know what torque is required to work the machine slowly backward as a generator, it will be an easy matter to find what power it gives out when working forward as a motor at its proper speed.

"Let, in Fig. 48, the horizontal *a*, *c*, *b*, *d*, *a*, represent the interpolar space straightened out, and the ordinates of the sinusoidal line, *B*, the induction in this space, through which the armature bars pass with a speed of $\sim$ revolutions per second. We make at present no assumption as to how this induction is produced, except that it is the resultant of all the currents circulating in the machine. We assume, however, for the present that no magnetic flux takes place within the narrow space between armature and field wires, or, in other words, that there is no magnetic leakage, and that all the lines of force of the stationary field are radial. The rotation being counter clockwise, each bar travels in the direction from *a* to *c* to *b*, and so on. The lines of the field are directed radially outwards in the space *dac*, and radially inward in the space *cdb*. The e.m.f. will, therefore, be directed downwards in all the bars on the left, and upwards in all the bars on the right of the vertical diameter in Fig. 47. Let *E* represent the curve of *e.m.f.* in Fig. 48, then, since there is no magnetic leakage, the current curve will coincide in phase with the e.m.f. curve, and we may represent it by the line *I*. It is important to note that this curve really represents two things. In the first place, it represents the instantaneous value of the current in any one bar during its advance from left to right; and in the second place it represents the permanent effect of the current in all the bars, provided, however, the bars are numerous enough to permit the representation by a curve instead of a line composed of small vertical and horizontal steps. The question we have now to investigate is: What is the magnetising effect of the currents which are collectively represented by the curve *I*? In other words, if there were no other currents flowing but those represented by the curve *I*, what would be the disposition of the magnetic field produced by them? Positive ordinates of *I* represent currents flowing upwards or towards the observer in Fig. 47, negative ordinates represent downward currents. The former tend to produce a magnetic whirl in a counter clockwise direction, and the latter in a clockwise direction. Thus the current in the bar which happens at the moment to occupy the position *b*, will tend to produce a field, the lines of which flow radially inwards on the right of *b*, and radially outwards on the left of *b*. Similarly the current in the bar occupying the position *a* tends to produce an inward field, *i.e.*, a field the ordinates of which are positive, in Fig. 48, to the left of *a*, and an outward field to the right of *a*. It is easy to show that the collective action of all the currents represented by the curve *I* will be to produce a field as shown by the sinusoidal line *A*. This curve must obviously pass through the point *b*, because the magnetizing effects on both sides

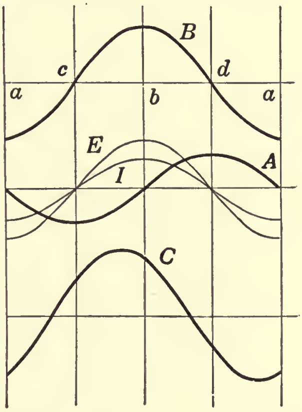


FIG. 48.—The interpolar space of the induction motor. The m.m.f. and flux belts.

of this point are equal and opposite. For the same reason the curve must pass through a. That the curve must be sinusoidal is easily proved, as follows: Let i be the current per centimeter of circumference in b, and let r be the radius of the armature; then the current through a conductor distant from b by the angle α, will be $i \cos \alpha$ per centimeter of circumference. If we take an infinitesimal part of the conductor comprised within the angle $d\alpha$, the current will therefore be

$$di = ir \cos \alpha \, d\alpha$$

and the magnetizing effect in ampere-turns of all the currents comprised between the conductor at b, and the conductor at the point given by the angle α will be

$$\int_o^\alpha di = -ir \sin \alpha \tag{52}$$

and since the conductors on the other side of b act in the same sense, the field in the point under consideration will be produced by $2ir \sin \alpha$ ampere-turns, i being the current per centimeter of circumference at b.

"Since for low inductions, which alone need here be considered, the permeability of the iron may be taken as constant, it follows that the field strength is proportional to ampere-turns and that consequently A must be a true sine curve.

"When starting this investigation, we have assumed that the field represented by the curve B is the only field which has a physical existence in the motor; but now we find that the armature currents induced by B would, if acting alone, produce a second field, represented by the curve A. Such a field, if it had a physical existence, would, however, be a contradiction of the premise with which we started, and we see thus that there must be another influence at work which prevents the formation of the field A. This influence is exerted by the currents passing through the coils of the field magnets. The primary field must therefore be of such shape and strength, that it may be considered as composed of two components, one exactly equal and opposite to A, and the other equal to B. In other words, B must be the resultant of the primary field and the armature field A. The curve C in Fig. 48 gives the induction in this primary field, or as it is also called, the "impressed field," being that field which is impressed on the machine by the supply currents circulating through the field coils. It will be noticed that the resultant field lags behind the impressed field by an angle which is less than a quarter period.

"The working condition of the motor, which has here been investigated by means of curves, can also be shown by a clock diagram. Let in Fig. 49, the maximum field strength within the interpolar space (*i.e.*, number of lines per square centimeter at a and b of Fig. 47), be represented by the line OB, and let OI_a represent the total ampere-turns due to armature currents in the bars to the left or the right of the vertical, then OA represents to the same scale as OB the maximum induction due to these ampere-turns. We need not stop here to inquire into the exact relation between OI_a and OA, this will be explained later on. For the present it is only necessary to note that under our assumption that there is no magnetic leakage in the machine, OA must stand at right angles to OI_a, and therefore also to OB, and that the ratio between OI_a and OA (*i.e.*, armature ampere-turns and armature field) is a constant. By drawing a vertical from the end of B and making it equal to OA, we find OC the maximum induction of the impressed field. The total ampere-turns required on the field magnet to produce this impressed field are found by drawing a line from C under the same angle to CO, as AI_a forms with AO, and prolonging this line to its intersection with a line drawn through O at right angles to OC. Thus we obtain OI_c, the total ampere-turns to be applied to the field. The little diagram below shows a section through the machine, but instead of representing the conductors by little circles as before, the armature and field currents are shown by the tapering lines, the thickness of the lines being supposed to indicate the density of current per centimeter of circumference at each place."

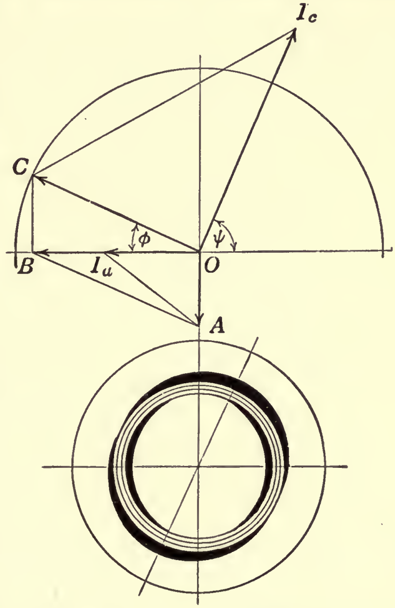


FIG. 49.—The vector diagram of the induction motor in the elementary theory.

NOTE.—This is the only figure in this book in which clockwise rotation has been assumed. It was taken from Mr. Kapp's book published in 1894, before the international agreement had been reached on positive counter-clockwise rotation.

D. THE SQUIRREL CAGE

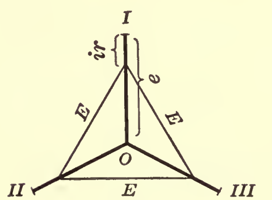
A rotor winding consisting of a number of bars connected in parallel, or short-circuited, by circular end rings, was invented by the late M. von Dolivo-Dobrowolsky. It is called a "squirrel-cage"[1] winding. It is a true poly-phase winding and its theory and understanding are based on the same principles as those of other poly-phase windings.

[1] French, "cage d'écureuil;" German, "Kurzschlussanker," or "Käfig Wicklung."

To fix ideas, let us look at a star-connected generator closed through a delta connected load, Fig. 50. In each phase a voltage e is induced. The delta voltages are I-II, II-III, and III-I. Calling the delta voltages E, we have

$$E = \sqrt{3}.e \tag{53}$$

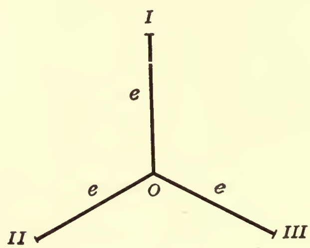
At any point I, II, or III, the algebraic sum of the currents which meet is zero, according to Kirchhoff's First Law. This, extended to our case, may be expressed in vector terms and we



(a)

Non-inductive Voltage Drop in Balanced Three-phase System

Fig. 50.



Three-phase System

Kirchoff's First Law Applied to Point I

Fig. 51.

substitute "vector sum" for "algebraic sum." In Fig. 51 this vector sum has been drawn, from which we see that

$$I = \frac{1}{\sqrt{3}}\cdot i \tag{54}$$

It is very important to watch directions and it is easy to commit errors.

If each phase of the star-connected generator has a resistance r_1 and no reactance, then there will be an ohmic drop ir in each phase, and in time-phase with e, as shown in Fig. 50. Then

$$E = \sqrt{3}(e - ir) \tag{55}$$

in accordance with Kirchhoff's Second Law that in any closed circuit the algebraic sum (vector sum) of the products of the current and resistance in each of the conductors in the circuit is equal to the e.m.f. in the circuit.

In a four-phase (two-phase) system, Fig. 52, we have, if the phase voltages between neutral and outside equal e,

$$E = \sqrt{2}.e \tag{56}$$

for the voltages I–II, II–III, III–IV, and IV–I.

The currents are

$$I = \frac{1}{\sqrt{2}} \cdot i \tag{57}$$

from Kirchhoff's First Law.

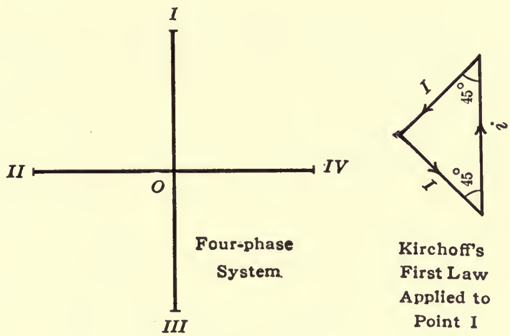


FIG. 52.

Consider, next, a six-phase system (Fig. 53). The line voltages are

$$E = e \tag{58}$$

and the currents

$$I = i \tag{59}$$

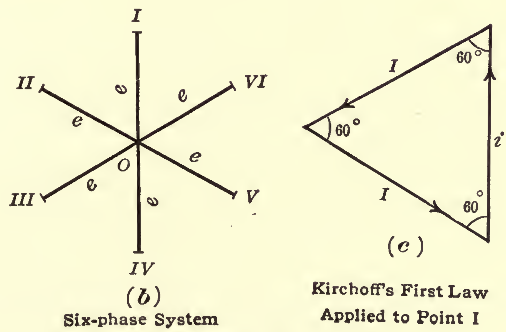


FIG. 53.

Now, consider a true poly-phase network in which there are n phases (Fig. 54). Then the time difference between two phases is $\frac{2\pi}{n}$, and the voltage (with no current) between phases is

$$In = E = 2e \sin \frac{\pi}{n} \tag{60}$$

Apply Kirchhoff's First Law and draw the current polygon, Fig. 54, whence, directly

$$I = \frac{i}{2 \sin\left(\frac{\pi}{n}\right)} \tag{61}$$

The squirrel-cage winding is such a poly-phase winding as Fig. 55 indicates, in which e is the induced e.m.f. per bar, e its current,

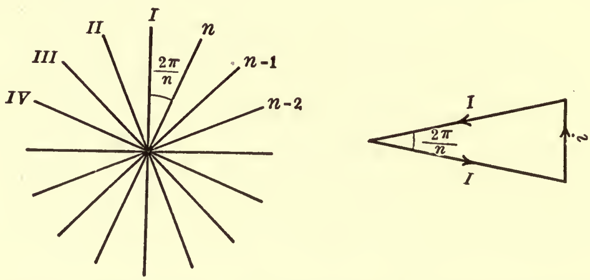


Fig. 54.—n-phase system. Kirchoff's first law applied to point I.

and r its resistance, while $E = 2(e - ir) \sin\left(\frac{\pi}{n}\right)$ is obviously the ohmic drop on the *two* sections $2R$, we have

$$E = 2IR = \frac{iR}{\sin\left(\frac{\pi}{n}\right)}$$

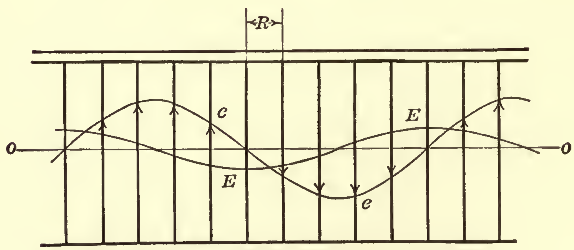


Fig. 55.—The squirrel cage.
e = Voltage induced in bars.
E = Voltage drop in end ring.

$$E = 2(e - ir) \sin\left(\frac{\pi}{n}\right)$$

$$\therefore \quad e = i\left(r + \frac{R}{2 \sin^2\left(\frac{\pi}{n}\right)}\right) \tag{62}$$

The end rings act, therefore, in such a manner as to increase the resistance r of each individual bar by the amount

$$\frac{R}{2\sin^2\left(\frac{\pi}{n}\right)}$$

A similar argument leads to a similar relation in regard to the reactance of the rings, but its application is of small importance.

The relation (62) can also be obtained from the consideration that in the squirrel-cage winding the total loss is

$$\text{Total Loss} = n(i^2r + 2I^2R)$$

$$I = \frac{i}{2\sin\left(\frac{\pi}{n}\right)}$$

$$\therefore \text{Total Loss} = ni^2\left(r + \frac{R}{2\sin^2\left(\frac{\pi}{n}\right)}\right) \tag{63}$$

from which it follows that the effect of the end rings consists in raising apparently the resistance of each bar by the amount

$$\frac{R}{2\sin^2\left(\frac{\pi}{n}\right)} \tag{64}$$

as obtained before.

Assuming again the distribution of the flux belt in the air-gap to follow a simple sine law, the e.m.fs. and the currents follow a sine law distribution at any moment of time. The e.m.fs. and currents in the end rings also are in magnitude represented by the ordinates of the curve E. The assumption of sine and cosine curves implies, of course, the tacit assumption of an infinite number of phases. The relations obtained previously are correct for any number of phases.

E. THE TORQUE AND SLIP AND THE EQUIVALENCE OF MOTOR AND TRANSFORMER

The theory of the Induction Motor is the theory of the General Alternating-current Transformer. Credit for this important relation is due to Dr. H. Behn-Eschenburg, of Oerlikon, Switzerland, who demonstrated this relation in 1893. It has become fundamental.

It is evident that this relation pertains so long as the rotor is standing still. As was shown by Mr. Kapp, the theory of the field belt and its interactions, if represented by vectors, leads to a polar diagram in which time-phase and space-phase are interchangeable. If the armature runs in synchronism, there is no current induced in the rotor, the no-load current corresponding to the open circuit current of the transformer. If the armature lags behind the field in angular velocity, then, if $\sim_1$ is the impressed frequency, and $\sim_2$ the frequency corresponding to ω_2, the angular velocity of the rotor, the currents induced in the rotor windings are of frequency $\sim_1 - \sim_2$. If the armature resistance per phase is r_2, then a current will flow

$$i_2 = \frac{e_2}{r_2} = 2.12 \frac{(\sim_1 - \sim_2)}{r_2} z_2 F_2 10^{-8} \tag{65}$$

$$i_2 = 2.12 \frac{(\sim_1 - \sim_2)}{\sim_1} \frac{\sim_1}{r_2} z_2 F_2 10^{-8}$$

$$i_2 = 2.12 \left(\frac{s}{r_2}\right) \sim_1 z_2 F_2 10^{-8} \tag{66}$$

The same current will be obtained with the secondary at rest, if the external resistance is equal to $\frac{r_3}{s}\cdot$ Therefore, substitute for the motor an equivalent transformer with a total internal and external resistance equal to $R_2 = \frac{r_2}{s}\cdot$

The Torque.—Imagine the rotor to be turned with angular velocity $(\omega_1 - \omega_2)$ against the magnetic field, which is supposed to be at rest. If T is the torque in *mkg*, then we have

$$9.81 T \cdot (\omega_1 - \omega_2) = (3 i_2^2 r_2) \tag{67}$$

where i_2 is the current, r_2 the resistance in each phase, the rotor to be assumed three-phase. If it is n-phase, substitute n for 3.

$$\omega = 2\pi \frac{\sim_1}{p} \tag{68}$$

where p is the number of north or south poles. Also

$$9.81 \cdot T \cdot \omega_2 = P \text{ watts.}$$

From these equations follows

$$9.81 \cdot T_{mkg} \left(2\pi \frac{\sim_1}{p} - \omega_2\right) = 3 i_2^2 r_2 \tag{69}$$

$$9.81 \cdot 2\pi \cdot T_{mkg} \frac{\sim_1}{p} - P = 3 i_2^2 r_2 \tag{70}$$

$$61.6 \cdot T_{mkg} = \frac{p}{\sim_1} (3 i_2^2 r_2 + P_{watts}) \tag{71}$$

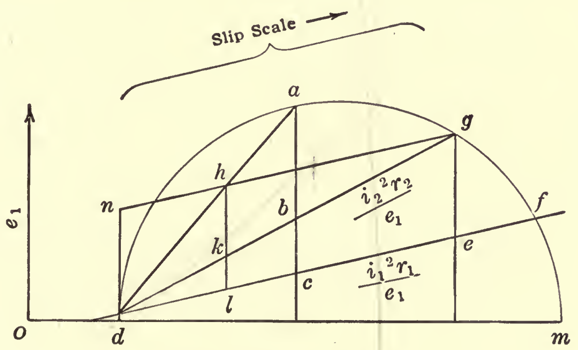
If we want to obtain the torque in foot-pounds, the formula is

$$8.5 \cdot T_{ft.\ lbs.} = \frac{p}{\sim_1}(3i_2^2 r_2 + P_{watts}) \qquad (72)$$

The torque is, therefore, proportional to the algebraic sum of the output of the motor plus the energy dissipated in the armature. Subtract, therefore, the primary copper loss and the core loss from the input as given by the ordinates of the circle and we obtain directly the torque T *in synchronous kw.*

The Slip.—The slip of the motor equals the loss in the rotor, divided by the output plus the loss in the rotor,

$$s = \frac{3i_2^2 r_2}{P + 3i_2^2 r_2} \qquad (73)$$



Determination of the Slip from the Loss Lines

FIG. 56.—Torque, slip, and loss lines for the induction motor.

André Blondel[1] suggests a beautiful mode of representing the slip by extending *me*, Fig. 16, to an intersection with vector e_1. Then it can easily be proved that the segment OX, X being the point of intersection, is a measure of the slip. This method has to be modified if the copper loss in the primary winding is to be taken into account as diminishing the receptive capacity of the motor.

Perhaps the simplest way to show graphically the slip of the motor is the following. Let *ab* be the output of the motor, *bc* the rotor loss, then slip $= \frac{bc}{ac}$.

Draw through *g*, the standstill position of the motor in which

[1] ANDRÉ BLONDEL, "Théories Graphiques des Moteurs Polyphasés." *L'Industrie Electrique*, 1896, p. 77.

the output is zero, gn parallel to df, the primary copper loss line. Then

$$s = \frac{nh}{ng}$$

$$bc:ac::kl:hl$$

$$kl:hl::dl:de::nh:ng$$

$$bc:ac::nh:ng \ldots\ldots\ q.e.d. \qquad (74)$$

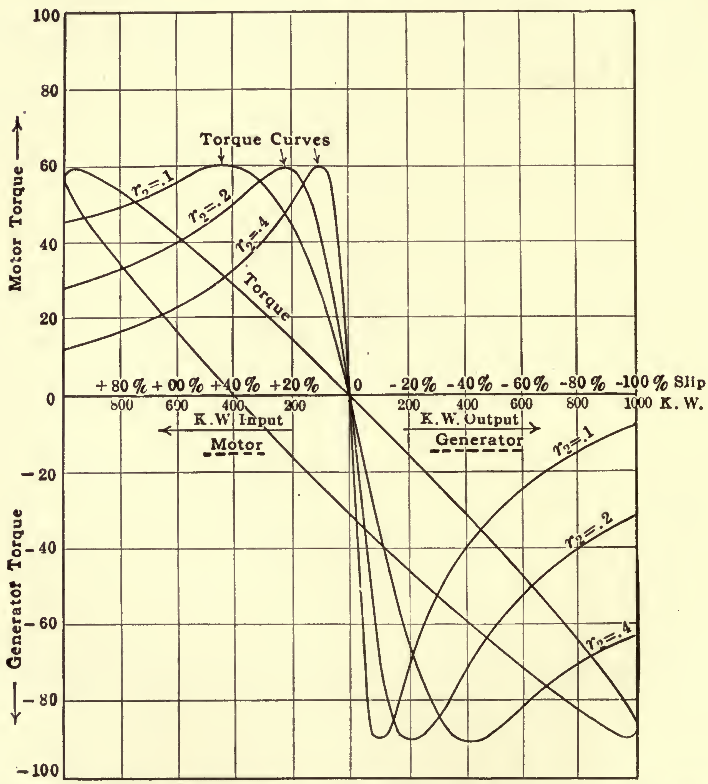


Fig. 57.—Torque as a function of rotor resistance, slip, and primary input.

Divide ng into a percentage scale and the slip may be read off for each current. At g the slip is 100 per cent, at n it is zero.

Torque Curves.—The results obtained in polar coordinates shall now be represented in Cartesian coordinates. We shall first use as abscissa the watts input of the motor, and secondly the slip, representing all characteristics as functions of these two parameters (Fig. 57).

It is interesting to note, and obvious from the diagrams, that there is a maximum torque for a given motor frame. This torque may occur at starting or at any speed. It may be varied by the resistance of the rotor, thus making it possible to start with maximum torque by increasing the rotor resistance. The reader may be trusted to draw many other instructive conclusions from the diagrams. Torque may be measured conveniently in "synchronous kw.," which is the kw. at angular velocity ω_1 corresponding to the torque at angular velocity, ω_2.

F. HIGHER HARMONICS IN THE FIELD BELT AND THEIR EFFECT UPON THE TORQUE

Examining Fig. 41 it is noticed that the field belt does not have the form of a sine wave. It consists, therefore, of a fundamental sine wave and of higher harmonics of this fundamental.

To study these effects, consider first a quarter-phase motor,

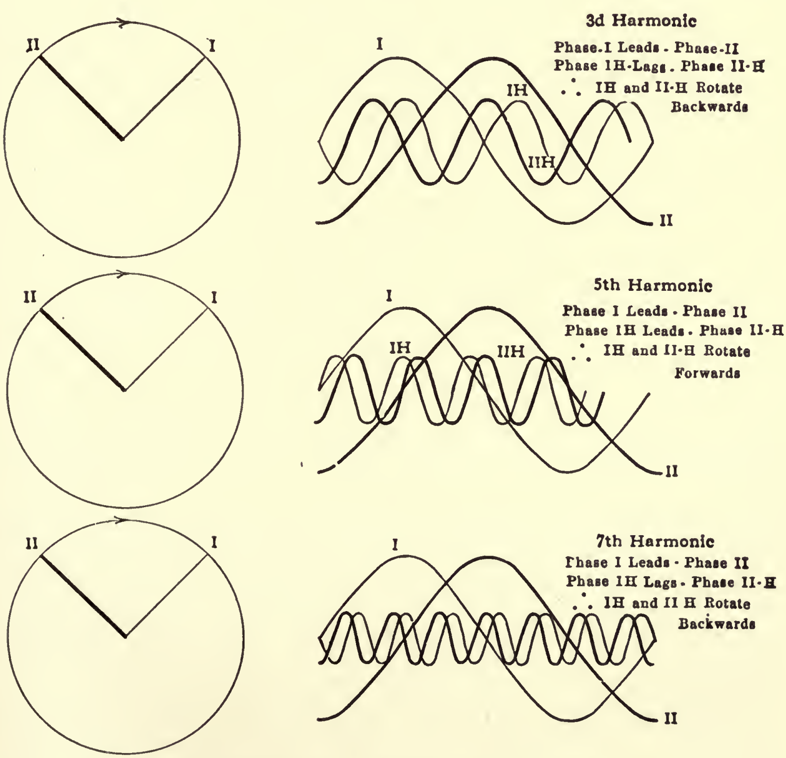


Fig. 58.—Harmonics in the field belt of a two-phase motor.

important application of which has recently been made to the U. S. Battleship New Mexico, as discussed fully in Chap. VIII. The third harmonic combines in the two phases in such a manner that, if Phase II Fundamental is behind Phase I Fundamental, then, Phase II 3d Harmonic is ahead of Phase I 3d Harmonic (Fig. 58). Hence, the third harmonic flux belt in a quarter-phase motor produces a backward torque.

It must be emphasized that we are talking about harmonics in the flux belt and not in the e.m.f. of the supply circuit. The

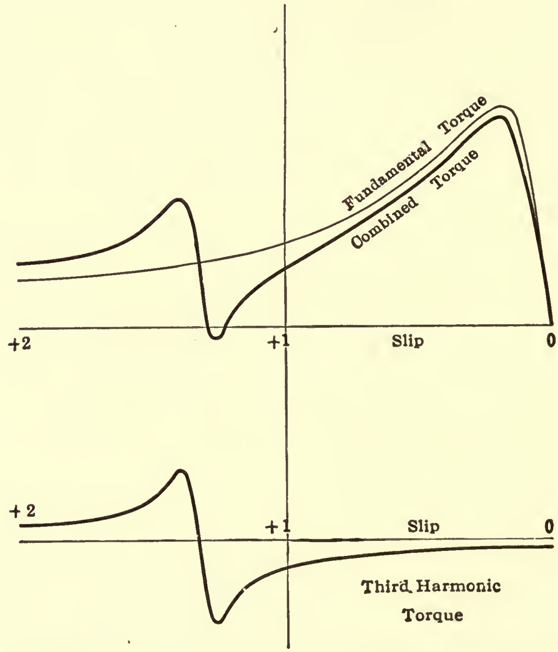


Fig. 59.—The effect of the third harmonic in the field belt on the torque of the two-phase motor.

effects of the latter may be neglected as a little thought indicates. The effects of the harmonics in the flux belt, however, consist in setting up rotating fields with angular velocity corresponding to the fundamental supply frequency, whose effect is therefore the same as the superimposition of 3, 5 or 7 times the number of poles would have upon the main fundamental flux belt.

We have already seen that in a two-phase motor the third harmonic acts as a brake (Fig. 59). In a two-pole motor fed from 25 cycles the synchronism of the fundamental is 1,500 r.p.m., while the synchronism of the third harmonic takes place at -500 r.p.m., but the field rotating backwards will produce a torque curve as indicated in Fig. 59. It is quite evident from Fig. 59, that such backward torque may be extremely serious as it diminishes the starting torque. If a squirrel-cage rotor is used, the motor may be unable even to start at all.

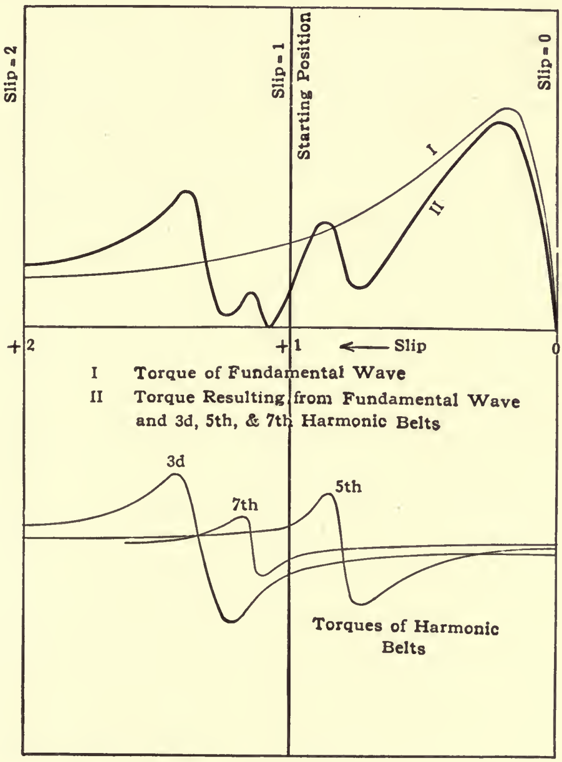


FIG. 60.—The effects of harmonics in the field belt on the torque of a two-phase motor.

Drawing the phases for the fifth harmonic, it will be seen that in the quarter-phase motor the fifth harmonic produces a forward torque whose synchronism occurs at $+300$ r.p.m. The seventh harmonic gives a backward torque whose synchronism occurs at -215 r.p.m. These three harmonics have been drawn into Fig. 60 showing the resultant torque. The dead points, which were so often observed in two-phase motors during the development stages, are particularly interesting.

An examination of the effects of the third harmonic in a three-phase system, if the sine waves are drawn, shows that the third harmonics of the three circuits are in phase with each other, therefore, they produce no rotating field. Their effect upon the torque is therefore that of a single-phase induction motor having three times the number of poles. Its torque is shown in Fig. 61

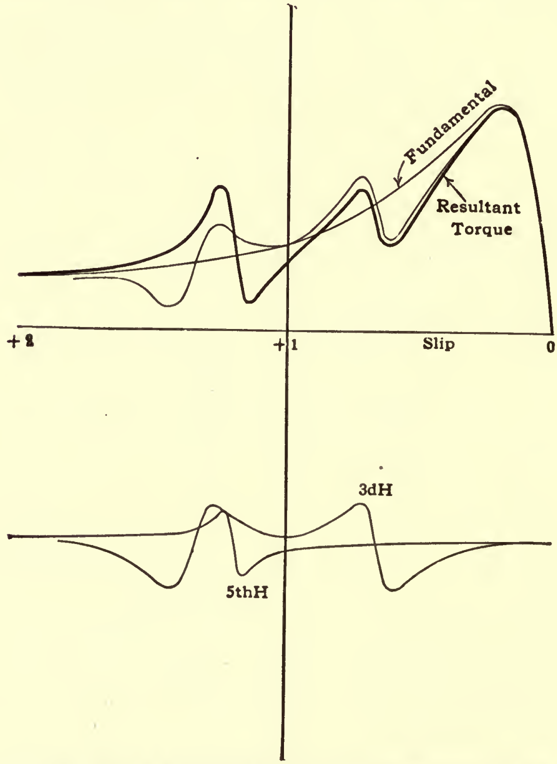


Fig. 61.—The effect of a third harmonic in the field belt on the torque of a three-phase motor. The torque for the 5th harmonic is shown but not added to the fundamental torque.

where also the torques of the fifth and seventh harmonics are indicated. The fifth harmonic gives backward torque, the seventh harmonic gives a forward torque. The torque curve of the single-phase induction motor will be treated at length inChap. XVII.[1]

[1] This entire subject of the effects of higher harmonics, both in the field belt and in the supply circuits, was treated brilliantly by André Blondel, as early as 1895, in his paper, "Quelques Propriétes Générales des Champs Magnétiques Tournants," *L'Eclairage Electrique*, Paris, to which classic and fundamental paper all readers should refer.

G. EXPERIMENTAL DATA

A vast amount of experimental material has been accumulated since the first publication of this theory 25 years ago. The experimental proof on induction motors of the circle characteristic was already given in my paper of Jan. 30, 1896, on one machine only. Literally, hundreds of thousands of motors, aggregating millions of horsepower have been tested since according to this method and the theory is now solidly established. In subsequent chapters numerous experimental data are given so that we shall not here refer to the subject any more.

H. COLLECTION OF DATA

The most important data in the design of the *three-phase induction* motor are here collected together:

Leakage Factor:

$$\sigma = v_1 v_2 - 1 \tag{14}$$

Maximum Power Factor:

$$\cos \psi_0 = \frac{1}{2\sigma + 1} \tag{19}$$

Magnetic Flux:

$$e_1 = 2.12 \sim \cdot z_1 \cdot F_1 \cdot 10^{-8} \text{ volts} \tag{49}$$

Magnetizing Current:

$$i_\mu = \frac{B \cdot 16 \cdot \Delta}{2n\sqrt{2}} \tag{45}$$

Slip:

$$s = \frac{\omega_1 - \omega_2}{\omega_1} = \frac{3i_2^2 r_2}{3i_2^2 r_2 + P} \tag{73}$$

Torque:

$$61.6\, D_{mkg} = \frac{p'}{\sim_1}\left[3i_2^2 r_2 + P_{watts}\right] \tag{71}$$

Input:

$$P_i = 3e_1 i_1 \cos \psi_1 \tag{74}$$

Output:

$$P_{watts} = 3e_1 i_1 \cos \psi_1 - 3i_1^2 r_1 - F - Q \tag{75}$$

where F = friction loss in watts, and Q the loss through hysteresis and eddy currents.

Efficiency:

$$\eta = \frac{P}{3e_1 i_1 \cos \psi_1} \tag{76}$$

CHAPTER VI

THE INDUCTION GENERATOR

A. THE THEORY OF TORQUE AND SLIP

If an induction motor is driven by an external torque applied to its rotor at a speed above synchronism, in the direction of its rotation as a motor, its slip becomes negative as ω_2 becomes greater than ω_1. As the supply circuit continues to impress upon the stator a rotating field through which the rotor conductors cut at angular velocity $\omega_1 - \omega_2$, it is apparent that electric energy will be impressed upon the supply circuit. The mechanical input required to turn the rotor is equal to the electrical output plus the losses in the rotor and stator, exactly as in the case of a synchronous generator. An induction machine operating above synchronism is therefore called an Induction Generator.

A careful consideration of all that has been said on the subject of the induction motor makes it clear that, on account of the negative slip, the secondary current will be generator current instead of motor current. Its vector will be "downwards" from F_2, instead of "upwards" as in the motor. If we neglect primary resistance the semi-circle below the abscissa is the locus of the primary current. If we take into account the primary resistance the complete circle is also swept out, but now divided differently into a motor range and a generator range.

The behavior of such an induction generator is indeed curious. Prof. M. I. Pupin[1] talks of "negative resistance" in this case. It is quite unnecessary to make a complicated subject any more complicated by mystery. It makes little difference how the field is supplied, whether from the external circuit or by some other contrivance. And herein seems to lie the difficulty of understanding the action of this machine. The effect of primary resistance in lowering the effective part of the impressed voltage is also worth pondering. As one never understands anything which one has not thought out for oneself, we shall leave these matters to the reader.

[1] *Transactions A. I. E. E.*, 1918, Part I., p. 685.

The *slip* is again expressed by the loss in the rotor divided by the electrical output plus the rotor loss. The scale of the slip, extended to the left, is therefore a measure of the negative slip of the induction generator (Fig. 62).

The *torque* is measured directly between the circle and the primary copper loss line. The results are also shown in Cartesian coordinates in Fig. 63.

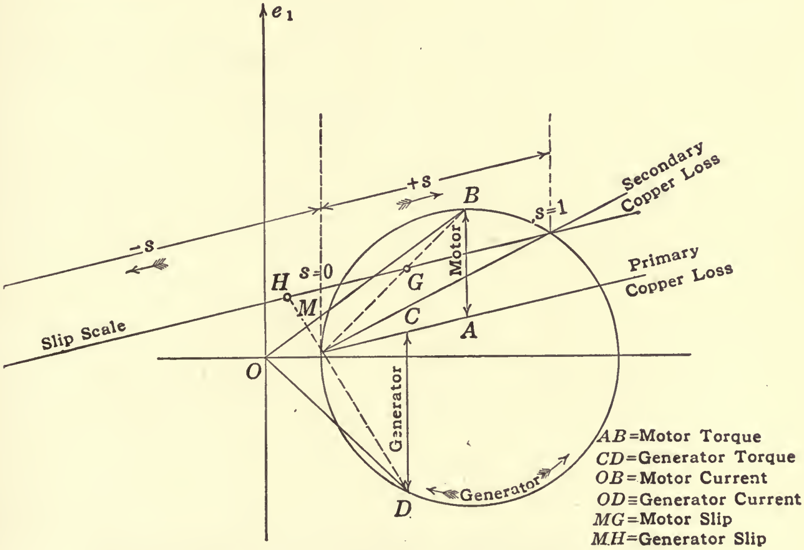


FIG. 62.—The torque and the slip in the induction generator.

B. STABILITY

If the torque which it is required to start by an induction *motor*, see Fig. 63, with short-circuited secondary, is *ab*, and if this torque increases according to the line *bA*, then the motor rapidly accelerates and operates at the point 1 in a stable condition. If the torque curve of the load follows *bB*, then if point *N* can be passed, a stable point will be reached at 2. If the torque curve follows the line *bC* then the motor will "stick" at *P*, and it will require the application of external torque to make it reach *Q*, from where the rotor will accelerate and operate stably, at least, theoretically, at point 3.

When operating as an induction *generator* a short-circuit of the line removes the excitation of the induction generator and its prime mover accelerates to the runaway point. It is also

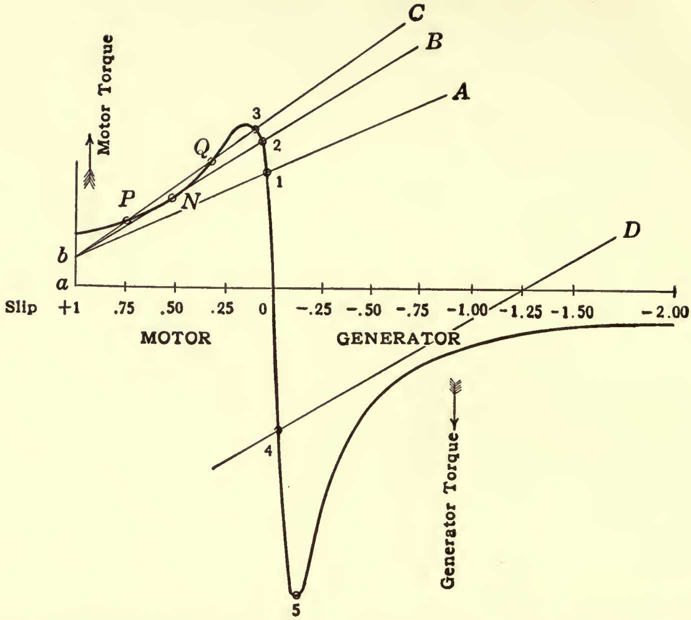


FIG. 63.—Stability in the induction motor and in the induction generator

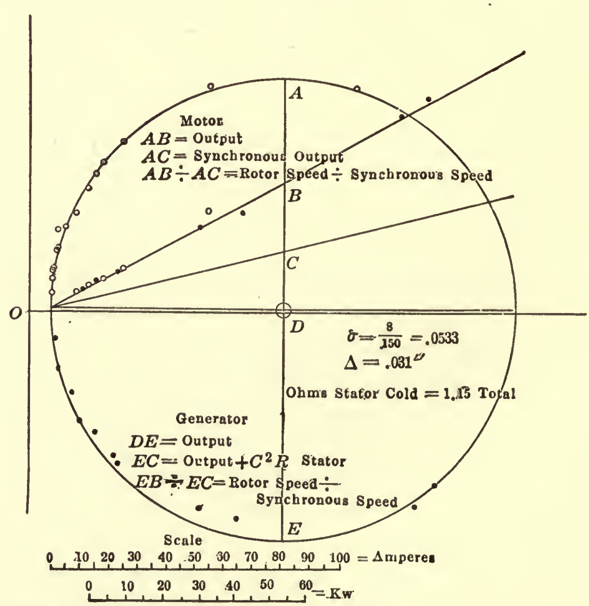


FIG. 64.—The experimental circle diagram for an induction machine acting as motor and generator.

essential that the characteristic torque of the prime mover shall not increase with increasing speed. Water wheels usually lose their torque at approximately twice the normal speed, therefore, such a torque characteristic of the prime mover as indicated by line *D*, would establish a stability point at 4. A sudden lowering of the supply voltage may, however, cause serious trouble, in view of the sensitiveness of the induction machine in regard to its primary impressed voltage, a drop of 10 per cent reducing the torque 20 per cent, and point 5 may be passed and the prime mover and generator run away.

C. EXPERIMENTAL DATA

Figure 64 shows a carefully obtained test on an induction machine over the entire range of operation, made in 1904, by the writer in his testing department in South Norwood, Ohio.

CHAPTER VII

THE SHORT-CIRCUIT CURRENT AND THE LEAKAGE FACTOR

A great aid in the design of poly-phase induction motors is afforded by the short-circuit characteristic. Attention was first called to this in 1893 by Dr. H. Behn-Eschenburg and Mr. Gisbert Kapp. In a motor having a squirrel-cage armature, the starting current under different voltages is identical with the short-circuit characteristic. If the resistances of the armature and of the field were known, the power factor of the supply current could be calculated; thus it would not even be necessary to use a wattmeter unless great accuracy was required. Theoretically, then, the short-circuit characteristic is sufficient for the determination of the leakage factor, if the magnetizing current is known. In practice, however, it is inadvisable, in the majority of cases, to depend upon the short-circuit curve, on account of the corrections which become necessary. If the total resistance of the motor is small, the lag of the current amounts to nearly a quarter of a period. The inductance of a motor at standstill should always be as small as possible, therefore, a very large current will go through the motor at full voltage. Now, the leakage factor is more or less dependent upon the intensity of the currents which cause the leakage, hence, the leakage factor at starting with only a small resistance in the armature, may be very different from—and as a rule it is smaller than—the leakage factor upon which the diagram is based. In fact, we do not work the motor in that quadrant of the circle which corresponds to the short-circuit characteristic. If, therefore, I shall not make much use of the short-circuit curve for the determination of the absolute value of the leakage factor, I shall all the more avail myself of it for comparative purposes, where it does not matter so much whether the absolute value is correct or not, as the relative value is chiefly of importance. Such questions as the influence of a closed or open slot, of the number of slots, of the air-gap, and of the pole-pitch upon the leakage factor, can all be answered by consulting the short-circuit characteristics. I shall proceed to discuss, point for point, the influence of these factors on the leakage coefficient.

A. THE SLOTS

The curves *A* and *B*, in Fig. 65, represent the short-circuit characteristics for a closed slot of the shape marked *A*, and of the open slots of the shape marked *B*. The slots of the rotor were closed in each case. Curve *C* shows the ideal short-circuit curve obtainable only if the leakage paths contain no iron at all. Curve *D* is the magnetizing current reduced from the measured value of 42.2 amperes at 1,900 volts to the various voltages of the

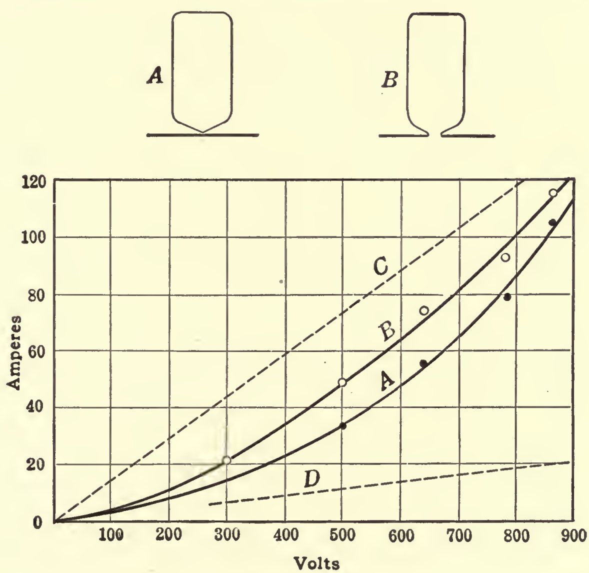


Fig. 65.—Short circuit characteristics for closed and open slots.

diagram. The increase of the magnetic reluctance of the main field through the opening of the slots proved too small to influence the magnetizing current in any perceptible manner.

We see that for voltages above 600 the curves *A*, *B*, and *C* converge; in other words, the short-circuit current at the full voltage of 1,900 will be almost the same whether the slots are open or closed. Hence, the maximum power input, which can be impressed upon the motor, is by no means so dependent upon the form of the slot as is usually assumed. The tendency of the closed slot is to change the fundamental diagram in the way shown by the full line curve in Fig. 65. From this curve it follows that, though the maximum power factor is slightly reduced, yet the maximum output of the motor is hardly affected as is indicated by the approximate equality of the maximum ordinates of the heavy line and dotted curves which measure the maximum power receptivity of the motor. If the iron bridges are kept very thin, excellent motors can be built with closed slots. The objection to closed slots is a commercial one, the high cost of labor of winding the coils within the machine instead of on formers outside the machine. During the development period of the induction motor, 25 to 30 years ago, labor cost was less of a telling factor and motors built in the pioneer factories of Oerlikon and of Brown, Boveri, and Company, utilizing the skilled Swiss labor then available, were ordinarily supplied with closed

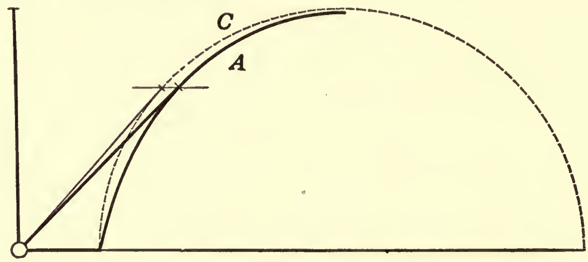


Fig. 66.—The effect of saturation of the leakage path upon the circle diagram. (From the 1st edition of 1900.)

slots and hand-wound coils. Needless to say, times have changed, skilled and inexpensive labor has departed, and cheap commercial methods—fool-proof methods which enable the manufacture of coils to be made by inexperienced hands—have supplanted the early notions. To such processes we apply the term of "progress," though it would perhaps be nearer the facts to say that evolution, far from meaning progress, is solely the adaptation to new conditions and environment. The survival of the fittest means, to be sure, the fittest in regard to its surroudings, the best adapted to these surroundings, not by any means, however, the "best" in any ethical sense. Huxley deplored that "fittest" had the connotation of "best" and was therefore a term unhappily chosen by Herbert Spencer. A design is a compromise, and the best design is that which combines the greatest number of advantages with the least number of drawbacks. As an old French proverb has it, our choice lies between the bad and the worse, and not between the bad and the good.

B. NUMBER OF SLOTS PER POLE

At the outset we wish to emphasize that we always consider one closed magnetic circuit consisting of a pair of poles. We trust the reader can apply the arguments readily to the particular multi-polar case, if he understands what goes on in one pair of poles.

Theoretically, we should have as many slots as possible, commercially, it is advisable to have as few as possible. Either extreme is impracticable.

In a general way, the influence of the number of slots upon the leakage can be seen. The more conductors we have in a slot the larger will be the leakage field surrounding the slot. With enough accuracy for our present consideration, the active field is the same whether we distribute the same number of conductors in a few slots or in many. The e.m.f. induced by this field may, therefore, be assumed constant independent of the number of slots. To fix ideas, let us take a specific case. If we have, for instance, 100 conductors arranged in five slots, there are 20 conductors in each slot. The leakage field per slot is produced by these 20 conductors, it is therefore proportional to their number. The e.m.f. induced by the leakage field per slot in the 20 conductors in each slot, is proportional to 20×20. Hence, the total leakage e.m.f. induced in the 100 conductors by the five leakage fields is proportional to $20 \times 20 \times 5 = 2{,}000$.

Now, let us arrange the 100 conductors in 10 slots. Each slot, then, contains 10 conductors, the leakage field per slot being proportional to 10. The e.m.f. induced by the leakage field in the 10 conductors in each slot is proportional to 10×10. Hence the total e.m.f. induced in all the conductors in the 10 slots is proportional to $10 \times 10 \times 10 = 1{,}000$. In other words, the e.m.f. of the leakage is twice as large in the case of five slots as in the case of 10 slots.

The above consideration assumes equal leakage reluctance in the two cases. The argument gives a general conception only.

C. CHARACTERISTICS OF ROTOR WINDINGS

To study the effect of the number of phases in the rotor and also the effect of the closeness of the rotor conductors to the air-gap, a series of tests was made as follows:

A. Rotor three-phase star-connected.
B. Rotor removed from stator. Leakage field closed through air.
C. Rotor squirrel-cage, two conductors in bottoms of slots.
D. Rotor squirrel-cage, two conductors close to top of slot.
E. Rotor squirrel-cage, one conductor close to top of slot.

The results are plotted in Fig. 67, showing the inferiority of the three-phase rotor to the squirrel-cage so far as leakage is

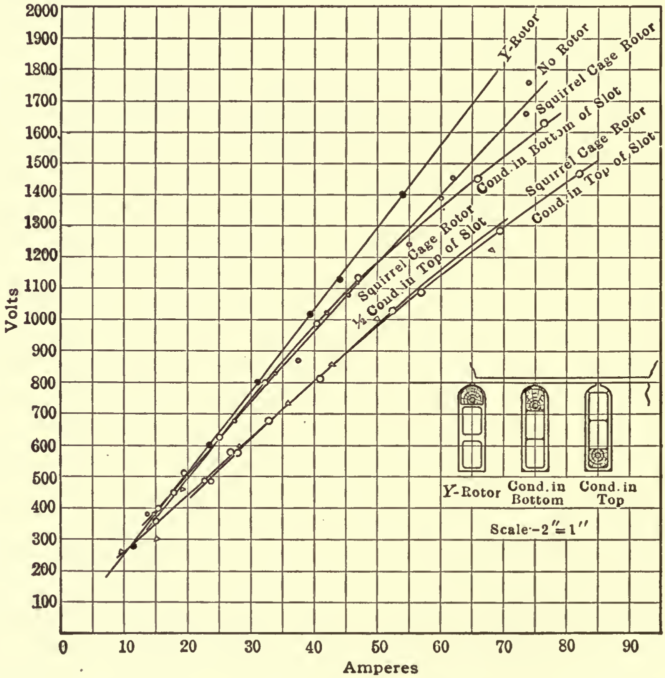


Fig. 67.—Short-circuit current of induction motor with (a) Y-connected rotor, (b) squirrel cage rotor, (c) rotor conductors near the gap, (d) rotor conductors at bottom of slot, (e) no rotor at all.

concerned and the advantage obtained by bringing the conductors close to the gap. Reducing the section of the conductors to one-half, Case "C," does not seem to constitute a noticeable gain.

It is interesting to note the relative independence of the magnitude of the leakage field in respect to the air-gap, as without a rotor inside the stator this leakage reluctance is only slightly smaller—considering both primary and secondary leakage reluctances in parallel—than with the rotor in position.

D. TEST DATA

To enable the reader to form for himself an opinion of the accuracy of the theory, I give the complete experimental data of a small three-phase current motor with short-circuited armature. I am taking an old motor designed 25 years ago.

The motor shall develop 20 hp., at a voltage of 380 between the lines, and a frequency of 47 p.p.s The slots in armature and field were closed, but the bridges were thin. The shape of the slots in armature and field is represented in Fig. 68. The motor had six poles, therefore its synchronous speed was 940 r.p.m.

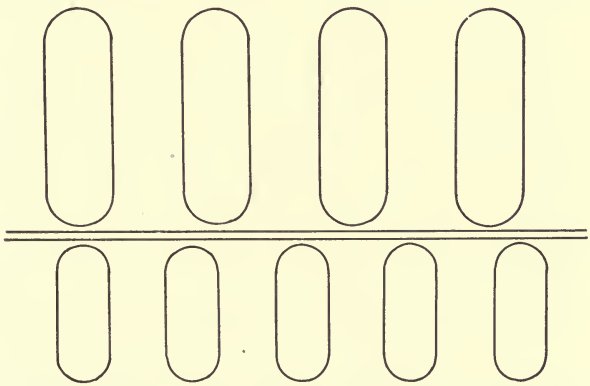


Fig. 68.—Slots of stator and rotor of 20 hp. six-pole motor, designed in 1896.

The following table shows the starting or short-circuit current as a function of the terminal volts measured between the lines:

Volts	Amperes	Frequency	Volts	Amperes	Frequency
82	11	46.7	200	56	46.7
108	20	....	233	70	....
135	30	....	273	90	....
162	40	....	314	112	....

These values are graphically represented in Fig. 69. From this curve we interpolate for 380 volts a current of 140 amperes. The energy dissipated into heat in the windings and in the iron amounted to 30 kw. This point lies upon the broken line curve in Fig. 71, deviating, therefore, not inconsiderably from the full line curve, representing the semi-circle of our diagram. The data determining this circle are given in the following table:

R.p.m. rotor	Frequency in r.p.m.	Volts	Amperes	Watts, input	Watts, output	Effi-ciency, per cent	Power factor, per cent	Apparent effi-ciency, per cent
930	932	380	5.5	2,100	1,300	62.0	58.0	36.0
931	940	382	9.0	5,400	4,050	75.0	90.7	68.0
926	930	380	14.5	9,000	7,300	81.0	94.4	76.5
910	918	380	20.5	12,900	10,800	84.0	95.5	80.2
912	925	380	26.0	15,600	13,200	84.5	91.5	77.3
894	912	380	31.0	18,600	15,600	84.0	91.0	76.5
892	922	380	42.0	25,100	20,600	82.0	91.0	74.5
880	922	380	57.0	32,400	24,300	75.0	86.5	65.0
846	896	382	64.5	35,400	25,500	72.0	83.0	59.7
860	940	391	74.5	40,100	26,100	65.0	79.5	51.7

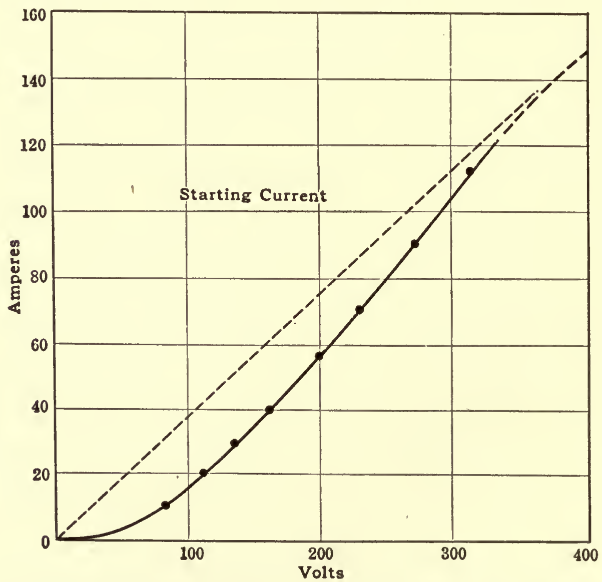


FIG. 69.—The short-circuit current of 20 hp. induction motor with closed slots. (Design of 1896.)

The curves corresponding to these data are graphically represented in Fig. 70. The maximum output of the motor is 35 hp. This point lies very near the maximum ordinate of the semi-circle in Fig. 71. The diameter of this circle is 122.5 amp. the magnetizing current 4.5 amp. hence

$$\sigma = \frac{4.5}{122.5} = 0.0367$$

The maximum power factor attainable is, equation (19),

$$\cos \phi_0 = \frac{1}{2\sigma + 1} = 0.93$$

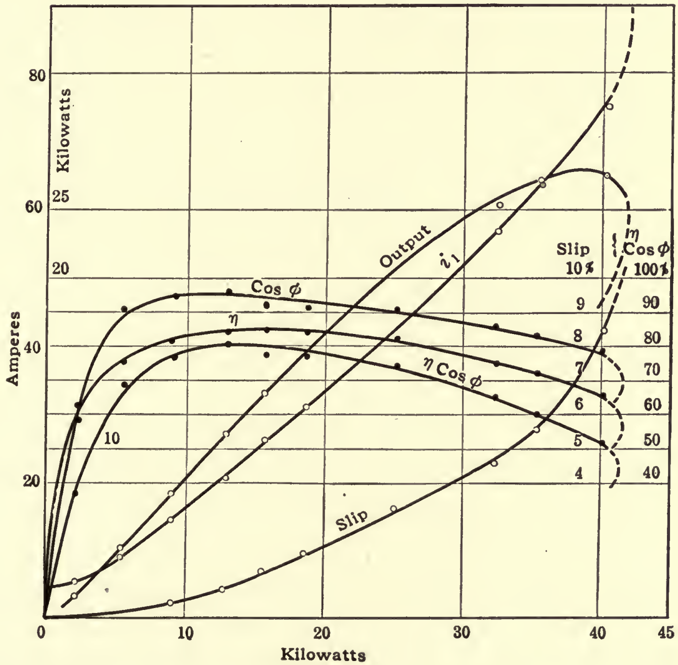


FIG. 70.—Characteristic data of 20 hp. induction motor from tests made in 1896.

Figure 71 and the data given above clearly show the inaccuracy which would arise if we were to use the short-circuit current as a means of determining the absolute value of the leakage factor.

I want to call attention to the load losses, which are always present in poly-phase motors, the causes of which are, however, still very obscure.

The maximum efficiency of this motor is 84.5 per cent. The losses are:

	Watts
Hysteresis, eddy currents and friction	800
Ohmic loss in primary	600
Ohmic loss in secondary	200
Total losses	1,600
Output	13,200
Input	14,800

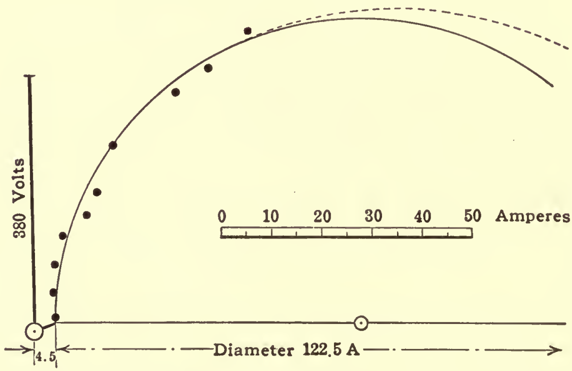


FIG. 71.—The circle diagram of 20 hp. induction motor, taken in 1896.

The energy which the motor actually consumed amounted to 15,600 watts, corresponding to an additional loss—the load loss—of 6 per cent of the output. The load loss increases rapidly with increasing load, until it becomes equal to all other losses taken together.

Opening the slots has a decided tendency to diminish the load loss considerably, therefore, it is probable that the seat of this waste of energy is in the bridges.

E. THE LEAKAGE FACTOR

There are two questions of the most vital interest which intrude themselves at every step upon the designer. The first is: "How does the maximum output a poly-phase motor is capable of yielding depend upon the length of the air-gap?" In other words, if the air-gap is increased, does this to any great extent decrease the output? And does a decreased air-gap increase the output of the motor? The second question is this: If a motor is wound for four poles, and we want to wind it for eight poles, provided the frequency and the induction in the air-gap remain the same, does the output decrease in the ratio of 4 ÷ 8, or, what relation exists between the maximum output of the motor and the number of poles? I shall now proceed to answer these questions.

F. THE INFLUENCE OF THE AIR-GAP UPON THE LEAKAGE FACTOR

In order to determine the interdependence between the magnetic reluctance of the main field and the leakage factor, the following experiment was made: The magnetic field—the stator—of a three-phase current motor was provided with two armatures, the diameters of which were so chosen as to create an air-gap of 0.5 mm. and one of 1.5 mm. The magnetizing currents were then measured as well as the short-circuit currents. The following tables contain the results of the experiment:

Magnetizing Currents at 50~

Δ = 0.5 mm.		Δ = 1.5 mm.	
Volts	Amperes	Volts	Amperes
37.0	1.00	16.5	1.05
56.5	1.50	34.7	2.25
78.0	2.10	55.5	3.40
95.5	2.55	75.0	4.70
116.0	3.20	77.0	4.80
....	....	77.5	5.00
....	....	99.0	6.40
....	....	113.0	7.30
....	....	114.5	7.40

Short-circuit Currents at 50~

Δ = 0.5 mm.		Δ = 1.5 mm.	
Volts	Amperes	Volts	Amperes
17.4	5.1	16.2	5.40
36.0	12.5	36.0	14.85
36.5	12.7	37.6	14.00
58.0	20.3	55.5	22.50
77.0	29.5	79.0	33.50

Resistance of primary: Each phase, 0.05 ohm.
Resistance of secondary: Each phase, 0.50 ohm.

The data of these tables are graphically represented in Figs. 72 and 73. The most remarkable fact illustrated by these curves is that the short-circuit current remained almost unaltered for the two different air-gaps. We interpolate from these curves for 110 volts in each phase the respective short-circuit and magnetizing currents, and thus get the following values:

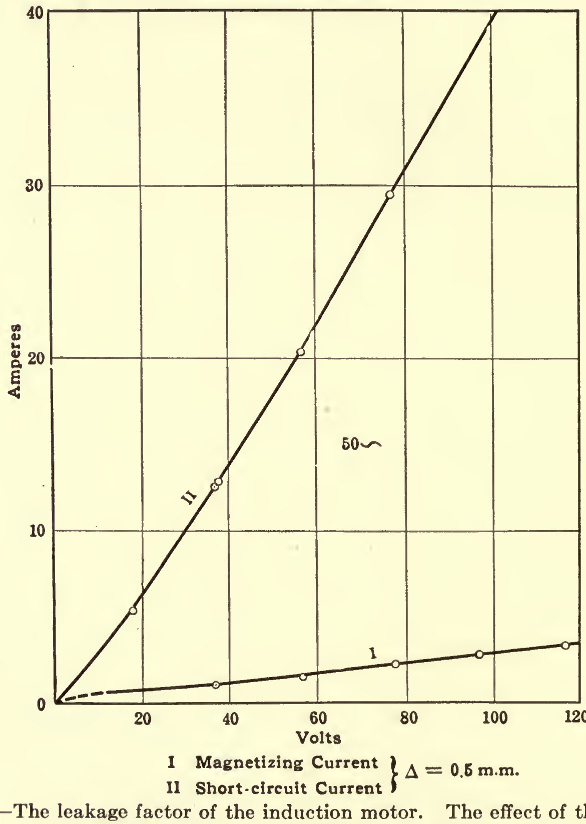


FIG. 72.—The leakage factor of the induction motor. The effect of the air-gap on the leakage factor. Air-gap Δ = .50 mm.

Δ = 0.5 mm.
Magnetizing current: 3 amp.
Short-circuit current: 42 amp.
Watts consumed: 8,550 watts.
Δ = 1.5 mm.
Magnetizing current: 7 amp.
Short-circuit current: 47.5 amp.
Watts consumed: 9,100 watts.

With these data the semi-circles in Fig. 74 are drawn. We thus find for the leakage factor the following values:

$\Delta = 0.5$ mm. $\sigma = 0.058$

$\Delta = 1.5$ mm. $\sigma = 0.128$

The leakage factor is, according to these experiments, directly proportional to the magnetizing current, or, in other words, to

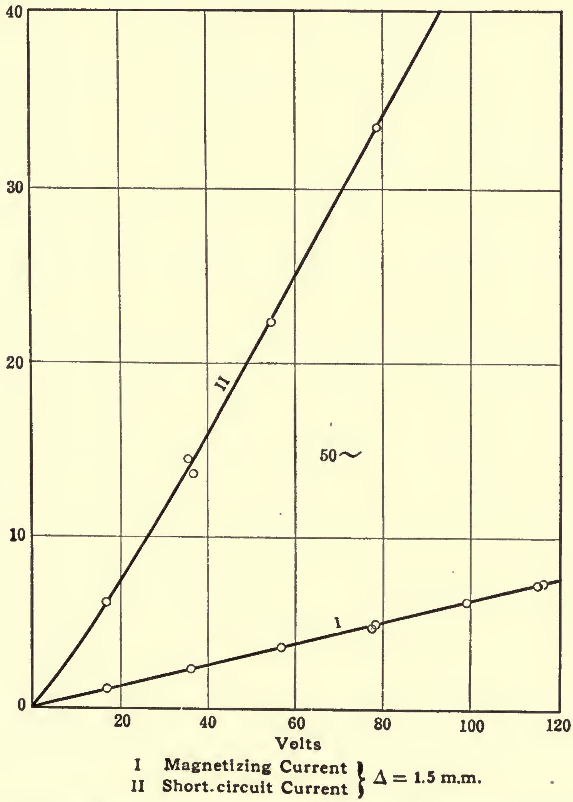


Fig. 73.—The leakage factor of the induction motor. The effect of the air-gap on the leakage factor. Air-gap $\Delta = 1.50$ mm.

the magnetic reluctance of the main field. In our case the reluctance of the iron path is not negligible, as the air-gap of 0.5 mm. is very small, otherwise the leakage factor would have been proportional to the air-gap.

Now, this result is highly interesting. As a glance at Fig. 74

shows, the energy which the motor is capable of taking in at the voltage of 110 is the same whether the air-gap is small or large. Hence, *the overload that a motor is able to stand is independent of the air-gap.*

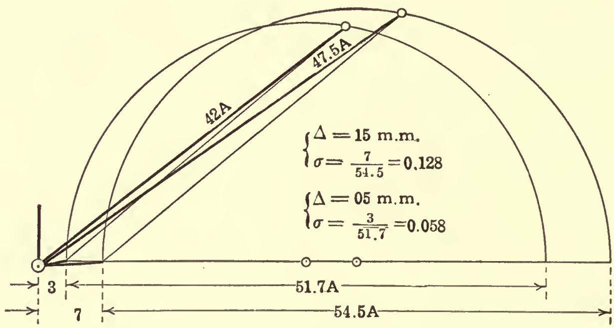


FIG 74.—The leakage factor of the induction motor. The circle diagram for different air-gaps.

This, of course, holds good only of small air-gaps.

The air-gap influences merely the strength of the magnetizing current, but not the output.

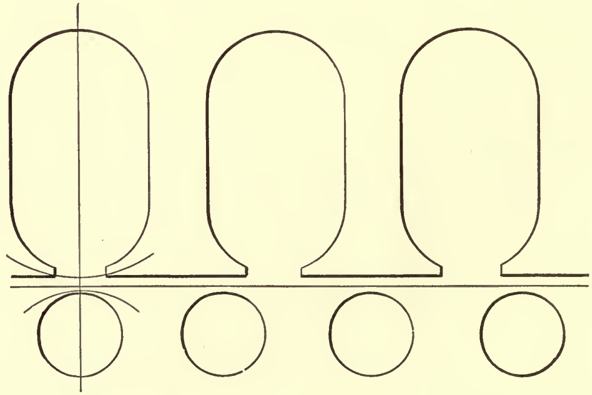


FIG. 75.—The leakage factor of the induction motor. The shape of the slots in the experimental motor. (1896.)

The curves of the short-circuit currents in Figs. 72 and 73 are almost straight lines owing to the open slots in the field of our motor.

There remains to be answered the second question: How is the leakage factor dependent upon the pitch of the poles?

G. THE INFLUENCE OF THE POLE-PITCH UPON THE LEAKAGE FACTOR

Before entering upon the experiments made to clear up this point, I shall attempt to show deductively how the leakage factor may be expected to vary with the pole-pitch.

Figure 75 gives a view of the slots in a poly-phase motor. The broken lines mark the leakage flux threading each slot. Now, let us assume that we have a motor with 48 slots in the field, which we want to wind as a two-pole, four-pole, or eight-pole motor, the armature being provided with a squirrel-cage winding, thus being suitable for any number of poles.

If the number of ampere-conductors per slot remains the same for any number of poles, the leakage flux per slot also remains constant.

The total number of ampere-turns spread over the circumference of the field is then constant whether the motor has two, four, or eight poles.

But the number of ampere-turns per pole is inversely proportional to the number of poles; hence, the induction produced by these ampere-turns in the air-gap is also inversely proportional to the number of poles. In other words, the magnetic field per pole, being proportional to the product of the induction in the air-gap into the pole-pitch, varies inversely with the square of the pole-pitch.

The leakage field, as we have seen, is constant for each slot. Hence, the total amount of leakage—the sum of all the leakage fields pertaining to each slot—is also constant. The number of leakage lines per pole is proportional to the number of slots per pole.

The ratio of leakage field ÷ main field is therefore inversely proportional to the pole-pitch for the same number of ampere-turns per slot.

This result is verified by the following series of tests:

A.—Three-phase current motor for 36 hp., 380 volts between the lines, six poles, 42 $\sim$.

$$\text{Air-gap } \Delta = 0.62 \text{ mm.}$$
$$\text{Pole-pitch } t = 30.50 \text{ cm.}$$

Volts between the lines	Amperes, field	Amperes, armature	Frequency
81	36.0	95	42.0
120	74.0	180	....
150	106.0	260	....
170	135.0	320	41.0
—	—	—	—
383	8.5	0	43.2

If we draw the short-circuit curve, we can interpolate for 380 volts the short-circuit current of 380 amp. We thus get for the leakage factor the value

$$\sigma_1 = \frac{8.5}{380} = 0.0224$$

B. The same magnetic frame was wound for 24 hp. 190 volts between the lines, 10 poles, 50 $\sim$.

$$\text{Air-gap } \Delta = 1.1 \text{ mm.}$$
$$\text{Pole-pitch } t = 18.3 \text{ cm.}$$

Volts between the lines	Amperes, field	Frequency
20.0	20.0	51
25.0	31.5	
33.0	50.5	
43.5	75.0	51
66.0	139.0	
83.0	185.0	
95.0	220.0	

Magnetizing current: 31.2 amp. at 190 volts. The short-circuit current at 190 volts amounts to 470 amp. Hence,

$$\sigma_{II} = \frac{31.2}{470} = 0.0664$$

The following table shows the results of the tests:

Air-gap cm.	Pole-pitch cm.	σ
0.062	30.5	0.0224
0.110	18.3	0.0664

For equal air-gaps we have

$$\sigma_{I} = 0.0224 \cdot \frac{0.11}{0.062} = 0.0396 \tag{78}$$

$$\sigma_{II} = 0.0664$$

Hence,

$$\frac{\sigma_{II}}{\sigma_{I}} = \frac{0.0664}{0.0396} = 1.68, \tag{79}$$

or, in other words,

$$\frac{\sigma_{II}}{\sigma_{I}} = \frac{t_{I}}{t_{II}}. \tag{80}$$

The leakage factor is inversely proportional to the pole-pitch, or directly proportional to the number of poles.

By the above experiments it has been demonstrated that the leakage factor is directly proportional to the air-gap, and inversely proportional to the pole-pitch. We may, therefore, write the formula for the leakage factor,

$$\sigma = C \cdot \frac{\Delta}{t} \tag{81}$$

in which equation C is a factor dependent upon the shape and size of the slots, and upon a great many other conditions of which we are still rather ignorant. For practical purposes, however, C can be determined with satisfactory accuracy, though it will still be left to the designer to estimate the value of C between certain limits. For slots, as shown in Fig. 75, C varies between 10 and 15.

Since this formula (81) was first developed by the author in 1896 and 1897, it has been constantly tested and it has been the subject of considerable discussion. Other formulas, also almost altogether empirical, have been suggested by Mr. H. M. Hobart, Dr. Behn-Eschenburg, and others.

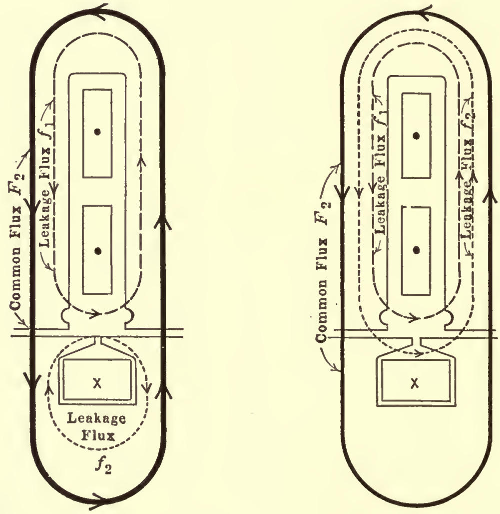
H. THE DIFFERENT LEAKAGE PATHS

There are at least three pronounced leakage paths in an induction motor.

A. The leakage of the end windings. The reluctivity of this leakage field is probably approximately proportional to the length of the projecting windings, *i.e.*, proportional to the pole-pitch. At least it is a fair guess that the longer the pole-pitch the greater is this flux. However, the length of the leakage path may also increase with the pole-pitch, so that after all proportionality may not exist. To approach the subject mathematically requires the equipment of a Maxwell or a Heaviside.

B. The so-called zig-zag leakage, by which is probably meant the peculiarity that in different positions of the rotor some of the primary (or secondary) flux does not reach around the secondary (or primary) conductors. Whether this should be called leakage at all is problematical.

C. The so-called slot leakage. This leakage is supposed to close around the slots without embracing the induced member. To attempt to obtain a physical picture of this leakage is also very difficult. The leakage lines are frequently shown by writers to intersect the main field which is an assumption contrary to



Conventional and incorrect picture of leakage flux diagram.

Correct picture of leakage flux diagram.

FIG. 76.

our conception of Faraday's lines or tubes of force. The leakage lines are also often shown as indicated in Fig. 76, which is physically an error, as there is no m.m.f. for the closed magnetic circuit in which the lines are shown as though they surrounded an m.m.f.

The entire subject seems somewhat vague and problematical, greatly confused by elaborate speculations based upon flimsy and insufficient premises contradictory of our fundamental conceptions of the magnetic field.

It is possible, however, to obtain a somewhat vague, physically plausible, picture, at least in rough outline, which luckily harmonizes sufficiently well with observed data.

We have seen that

$$\sigma = v_1 v_2 - 1 \tag{14}$$

$$\Phi_1 v_1 = \frac{X_1}{\rho} + \frac{X_1}{\rho_1} \tag{82}$$

$$\Phi_2 v_2 = \frac{X_2}{\rho} + \frac{X_2}{\rho_2} \tag{83}$$

where Φ_1 and Φ_2 are the fictitious primary and secondary fluxes, X_1 and X_2 the primary and secondary m.m.fs., ρ the reluctance of the common magnetic circuit, ρ_1 and ρ_2 the reluctances of the primary and secondary leakage circuits.

From (82) and (83)

$$v_1 = \frac{X_1}{\Phi_1}\left[\frac{1}{\rho} + \frac{1}{\rho_1}\right]$$

$$v_2 = \frac{X_2}{\Phi_2}\left[\frac{1}{\rho} + \frac{1}{\rho_2}\right]$$

But

$$\frac{X_1}{\Phi_1} = \rho \tag{84}$$

$$\frac{X_2}{\Phi_2} = \rho \tag{85}$$

Therefore

$$v_1 = 1 + \frac{\rho}{\rho_1} \tag{86}$$

$$v_2 = 1 + \frac{\rho}{\rho_2}$$

$$\sigma = \left(1 + \frac{\rho}{\rho_1}\right)\left(1 + \frac{\rho}{\rho_2}\right) - 1 \tag{87}$$

$$= \frac{\rho}{\rho_1} + \frac{\rho}{\rho_2} + \frac{\rho}{\rho_1 \rho_2} \tag{88}$$

$$= \rho\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) \text{ approximately} \tag{89}$$

Now,

$$\rho = K\frac{\Delta}{bt} \tag{90}$$

where b is the width of core of the motor.

$$\frac{1}{\rho_1'} = K_1 \cdot t \text{ for the end connections} \tag{91}$$

$$\frac{1}{\rho_1'} = K_2 \cdot \frac{d}{w} \cdot b \text{ for the slots} \tag{92}$$

where d is depth of slot, and w width of slot. Hence, adding

$$\frac{1}{\rho} = K_1 t + K_2 \cdot \frac{d}{w} \cdot b \tag{93}$$

$$= KK_1 \frac{\Delta}{b} + KK_2 \frac{d}{w} \cdot \frac{\Delta}{t} \tag{94}$$

$$\sigma = \frac{\Delta}{t}\left(KK_1 \cdot \frac{t}{b} + KK_2 \cdot \frac{d}{w}\right) \tag{95}$$

$$\sigma = \frac{\Delta}{t} \cdot C \tag{96}$$

where C is a factor which varies in a number of ways depending on features of design. It can be guessed at successfully by the experienced designer, but it is not amenable to sound scientific calculation. It varies between the limits of 6 and 15.

I. FURTHER EXPERIMENTAL DATA

From an exhaustive investigation, whose results are given in the table below and plotted in Fig. 77 the following formula has been derived:

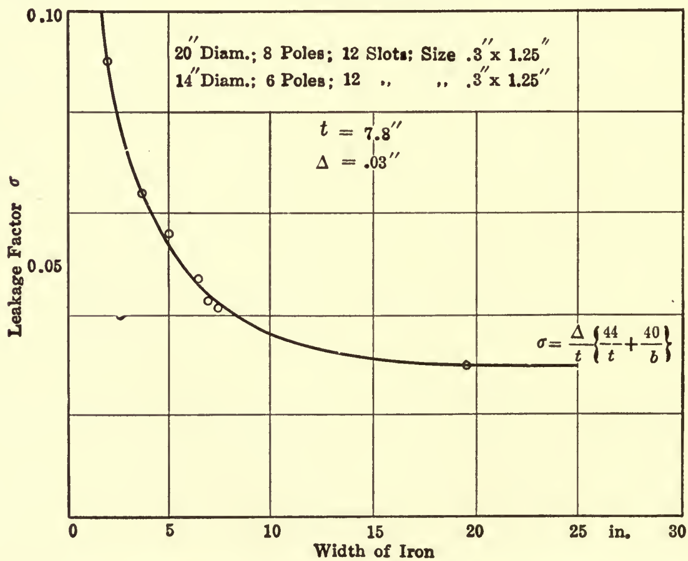


Fig. 77.—The leakage factor σ and its dependence upon the width of the motor.

$$\sigma = \frac{\Delta}{t}\left(5.1\frac{t}{b} + 5.65\right) \tag{97}$$

in which Δ, t, and b are to be substituted in inches.

60-Cycle Induction Motors Stators 12 slots per pole Slot 0.3 in. by 1.125 in. $t = 7.8$ in. $\Delta = 0.03$ on one side	
b Inches	σ
2.00	0.0900
3.75	0.0640
3.75	0.0540
5.00	0.0560
6.50	0.0465
7.00	0.0430
7.50	0.0413
19.50	0.0300

It is well to keep in mind that σ is approximately equal to the magnetic reluctance of the common magnetic field divided by the reluctance of the parallel leakage fields

$$\frac{1}{\rho_0} = \frac{1}{\rho_1} + \frac{1}{\rho_2} \tag{98}$$

$$\rho = \frac{1}{\dfrac{1}{\rho_1} + \dfrac{1}{\rho_2}} \tag{99}$$

$$\sigma = \frac{\rho}{\rho_0} \tag{100}$$

K. WINDING THE SAME MOTOR FOR DIFFERENT SPEEDS

Formula (96) permits us to determine the change in the output, power factor, and so forth, of a motor wound for a different number of poles, for instance, for eight, four, or two poles. If the field has 48 or 72 slots, it can easily be wound so as to satisfy this demand. We will assume the induction in the air-gap, or, which is the same, in the teeth, to remain constant for all three cases. We will further assume that the motors are to be wound for the same voltage. Then it is clear, according to equation (49), that the total number of active conductors must be proportional to the number of poles; in other words, if the eight-pole motor has, for instance, 720 conductors, or 10 conductors in each of 72 slots, then the four-pole motor must have 360, and the two-pole motor 180, in order to get the same induction in the air-gap. To calculate the relative value of the magnetizing current we need only know the number of active conductors n per pole, see equation (45). We have for n in the eight-pole motor $\frac{720}{8} = 90$; in the four-pole motor $\frac{360}{4} = 90$; and in the two-pole motor $\frac{180}{2} = 90$. Hence, as B and n are the same in each of the three cases, it follows that the magnetizing current also remains the same.

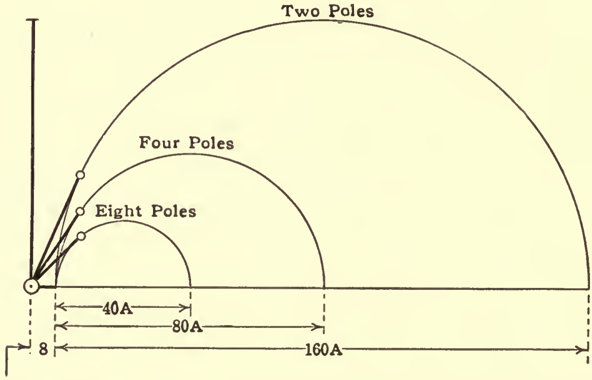


FIG. 78.—The primary current locus of the induction motor The leakage factor and the circle diagram for different numbers of poles on the same frame.

As the shape and size of the slots are the same in all three cases, the factor in equation (96) for the leakage coefficient also remains the same. Hence, as the leakage factor is proportional to the quotient of the air-gap divided by the pole-pitch, we find the short-circuit current to be inversely proportional to the number of poles. This is graphically represented in Fig. 78. A glance at the diagram teaches us that the maximum energy that can be impressed upon the motor, and, therefore, also very nearly the output, vary in proportion to the pole-pitch. According to the diagram we find the leakage factor for the two-pole motor equal to $\frac{8}{160} = 0.05$; for the four-pole motor equal to $\frac{8}{80} = 0.10$; and for the eight-pole motor equal to $\frac{8}{40} = 0.20$. The maximum power factor in each case can now be calculated with the help of formula (19). This is done in the following table:

Number of poles	Leakage factor	Maximum power factor	Relative output	Revolutions per minute
2	0.05	0.910	80	3,000
4	0.10	0.835	40	1,500
8	0.20	0.715	20	750

The output is therefore proportional to the number of r.p.m. It may not be amiss here to remark that if the motor is ordinarily wound for four poles, the induction in the iron above the slots may for the same frame become too high in the two-pole motor, thus increasing the magnetizing current, and possibly creating undue heating.

L. DRAWBACKS OF A HIGH FREQUENCY

If the circumferential speed of the armature is limited, and this is generally the case, then the pole-pitch is also limited for a given number of r.p.m. The air-gap cannot indefinitely be diminished, hence, a high frequency necessitates a large leakage factor according to formula (96).[1] We labor here under the same difficulties that we have met with in the design of alternators for high frequencies. It is doubtless possible to build motors for frequencies between 60 and 100, but the higher the frequency the lower will be the power factor, and the larger will be the lagging currents. It has also to be borne in mind that motors for high frequencies, if they are to be as good as those for low frequencies, must be made not inconsiderably larger.

Allowing again the induction in the air-gap to be the same for different frequencies, which is a more or less challengeable proposition, it follows from formula (49) that the total number of active conductors around the circumference of the field must also be the same, for the pole-pitch is inversely proportional to the frequency, hence, the product of the frequency into the number of lines of induction per pole remains the same if the induction in the air-gap is the same.

The magnetizing current, however, being proportional to the ratio of the induction B divided by the number of active

[1] This is the reason why Mr. Tesla and the Westinghouse Company failed to design a successful motor between 1888 and 1890 as a frequency of 135 was then commonly used. See B. G. Lamme, "The Story of the Induction Motor," *A. I. E. E. Journal*, March, 1921.

conductors per pole, is thus inversely proportional to the frequency. The leakage factor is, according to formula (96), directly proportional to the pole-pitch, or inversely proportional to the frequency—because the pole-pitch is, in the case under consideration, inversely proportional to the frequency—hence, it follows that, as the magnetizing current has been shown to be proportional to the frequency, the diameter of the semi-circle remains constant for all frequencies.

Figure 79 shows the polar diagram for the same motor, but for different frequencies. The maximum energy that the motor is capable of taking in, and, therefore, also the maximum output, is the same for 100∼, 50∼, or 25∼. But the maximum power factor is considerably smaller for the high frequencies, as a

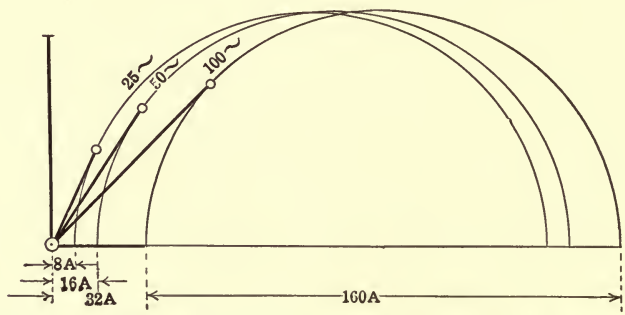


Fig. 79.—The leakage factor of the induction motor. The circle diagram for different frequencies.

glance at the diagram shows. The following table shows the leakage factor and the power factor in relation to the frequency:

Frequency	Leakage factor	Maximum power factor
25	0.05	0.910
50	0.10	0.830
100	0.20	0.715

I wish to call attention to the fact that the motor for the higher frequencies is here represented less unfavorably than it really is, because the induction in the air-gap has to be reduced if the motor is to be wound for a higher frequency. The immense lagging currents invariably bound up with the higher frequency are very clearly shown in the diagram.

It is to be remembered that the current in the armature is dependent upon the leakage factor, since the transformation factor v_1 forms part of the leakage factor. (See Fig. 16.) The transformation factors v_1 and v_2 are connected with σ through equation (14),

$$\sigma = v_1 v_2 - 1 \tag{14}$$

Hence, it follows, as $i_2 = \overline{AD} \cdot \frac{n_1}{n_2} \cdot v_1$, see Fig. 16, that the current in the armature is larger for the motor running at a high frequency than for that running at only 25∼. In our case, setting $v_1 = v_2$, we get for v at 25∼, 0.978, and at 100∼, 0.912, therefore the current in the armature of the motor for 100∼ is, for the same $\overline{AD}$, 1.07 times larger than for 25∼. This corresponds to an increased armature loss of about 14 per cent. But as the primary current is also larger for 100∼ than for 25∼, the armature loss is still greater than here calculated. Thus to the drawback of large lagging currents, there has to be added the further drawback of considerably larger losses.

The foregoing experiments and considerations are, within my knowledge, the first attempt to deal in a rational, systematic manner with the conditions underlying the leakage in polyphase motors. I am far from claiming for this treatment completeness of conclusiveness; on the contrary, I deem it a necessity to revise it by the light of forthcoming experience. I am tolerably confident that the main propositions will be proved true, while minor points may need some qualification.

Considering the immense complexity of the phenomena in poly-phase motors, the greater or less arbitrariness which hangs about most of our assumptions which have to be made in order to be able to calculate at all, I cannot forbear from wondering that so approximate a solution can be attained at all. It may be that there are errors inherent in our fundamental assumptions which all so counteract one another as to cause the result of calculation to deviate but little from experiment and observation. This view will commend itself to those who are familiar with some branch of physiology, for instance, physiological optics; here we have the testimony of Helmholtz that the eye, having "every possible defect that can be found in an optical instrument," yet gives us a fairly accurate image of the outer world because these various defects balance one another almost completely.

The above remarks will be distasteful to those who have accustomed themselves to look upon only one side of a question, and who try to shut their eyes to the inevitable uncertainties that beset us in all intellectual problems. I was once taken to task by a critic for having adduced experimental evidence qualifying my theory, and narrowing the limits of its application, and I was told that these experiments invalidated my argument, while my intention—to lay stress upon the incompleteness and the shortcomings of the theory—was obviously not even thought of by my critic. Politicians and propagandists may have to hide the weak sides and spots of their arguments, but men of science are bound to point them out and to expose them.

M. HISTORICAL AND CRITICAL DISCUSSION OF THE LEAKAGE FACTOR

In *The Electrical Engineer*, London, Dec. 11, 1903, Mr. H. M. Hobart discussed the equation (96) in his usual thoughtful manner. He points out that too much weight has been placed upon the pole-pitch and that the formula might lead to motors of too large a diameter. He says: "In the following article the writer wishes, in the first place, to emphasize the importance of that part of the total inductance which is due to the end connections, and, in the second place, to develop a simple and practical method by which the best dimensions may be decided. On p. 36 of Behrend's excellent treatise on induction motors, the following formula for calculating σ is given:

$$\sigma = C\frac{\Delta}{t}$$

where C is a figure which is dependent on the slot dimensions and other conditions, Δ is the length of air-gap, and t the pole-pitch. Behrend estimates that C varies between 10 and 15 for half-closed slots. This formula is extremely useful on account of its simplicity, especially if experimental results are available by which to decide the value of C."

Mr. Hobart then proceeds to give in a table the ratio of $\frac{b}{t}$ and determines C accordingly. He closes with the statement: "It is, however, preferable to keep the formula as simple as possible, so that the writer thinks it better to retain Behrend's original formula, together with the values of C got from Table I and Fig. 1."

The Institution of Electrical Engineers, London, held a meeting Jan. 14, 1904, at which Prof. Silvanus P. Thompson presented a paper prepared by Dr. H. Behn-Eschenburg, "On the Magnetic Dispersion in Induction Motors, and its Influence on the Design of these Machines." Dr. Behn-Eschenburg starts with a slightly different definition of the coefficient of leakage from that used by us.

$$\sigma_0 = 1 - \frac{M_1 M_2}{L_1 L_2} \tag{101}$$

$$= 1 - \frac{1}{v_1 v_2} \tag{102}$$

$$\therefore \sigma_0 = \frac{\sigma}{\sigma + 1} \tag{103}$$

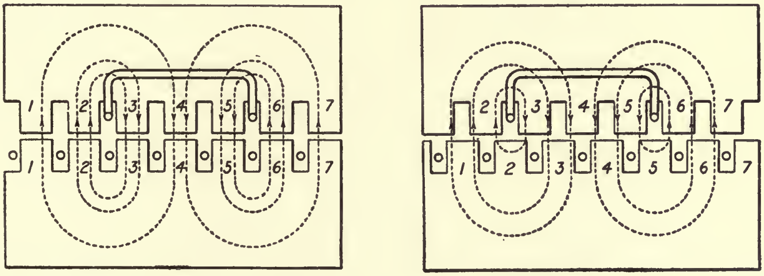


FIG. 80. FIG. 81.

FIGS. 80 and 81.—Dispersion of the flux. (After Behn-Eschenburg).

This σ_0 is the ratio of the magnetizing current to the short-circuit current while our δ is the ratio of the magnetizing current to the diameter of the circle. The following six figures which are instructive are taken from the Behn-Eschenburg paper. The investigation results in the formula

$$\sigma_0 = \frac{3}{n^2} + \frac{\Delta}{\xi \cdot n \cdot t} + \frac{6\Delta}{b} \tag{104}$$

in which n is the mean number of slots for stator and rotor in the pole-pitch; ξ the opening in centimeters of the half-closed slot. The remaining notation is identical with ours.

A comparison of Dr. Behn-Eschenburg's formula shows that it is practically identical with ours equation (96).

In the discussion Mr. H. M. Hobart said:

"The data contributed in Dr. Behn-Eschenburg's paper are of great value to those engaged in induction motor design. The exhaustive series of comparative tests which are described in the paper, throw more light on the subject than any contributions since the publication of Behrend's investigations. Dr. Thompson has mentioned Behrend's formula for the determination of σ. This formula of Behrend's is in refreshing contrast with many that have been proposed, and it is satisfactory to note that Dr. Behn-Eschenburg's formula is also of fairly moderate length." Mr. Hobart then proceeds to recommend his for-

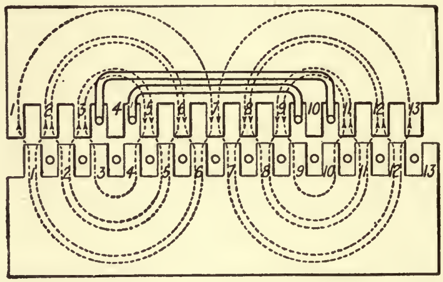


Fig. 82.

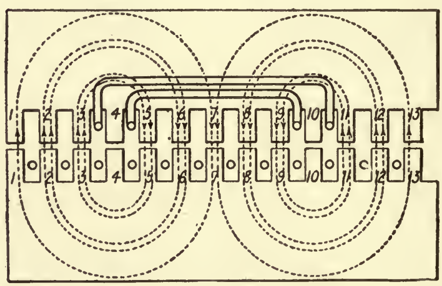


Fig. 83.

Figs. 82 and 83.—Dispersion of the flux. (After Behn-Eschenburg).

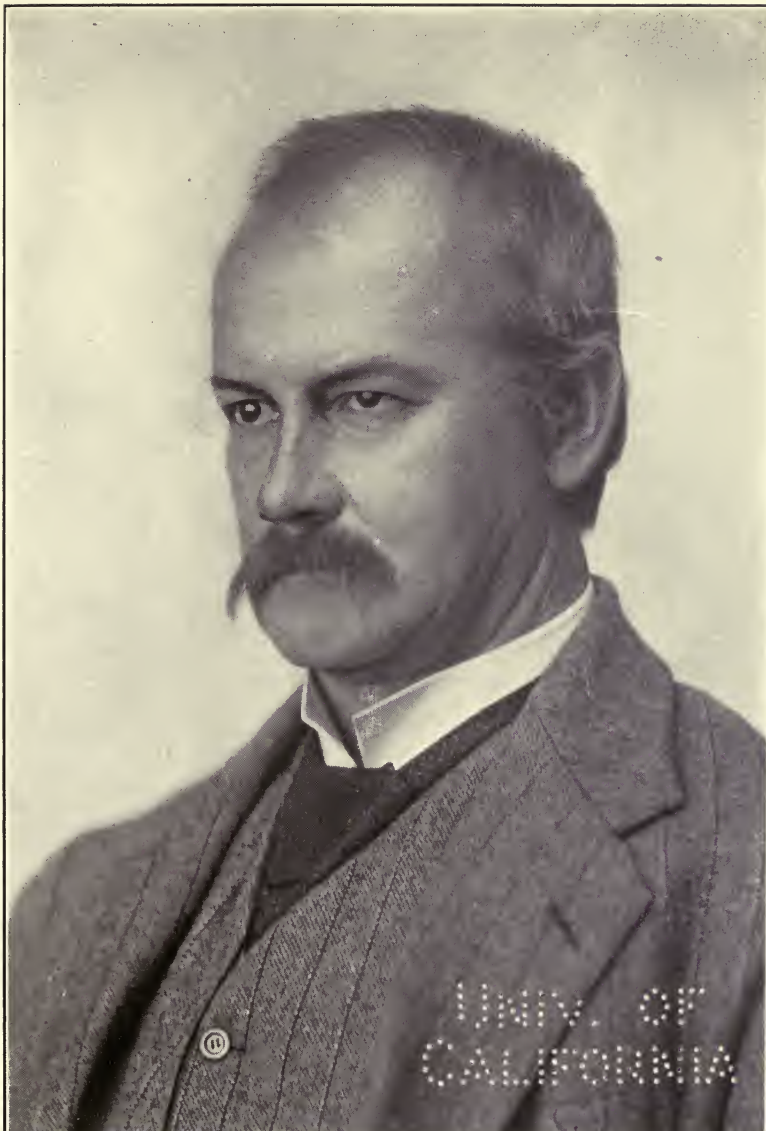
mula already referred to. "It appears fairly certain that the use of Behrend's formula in its present form, namely:

$$\sigma = CC^1 \frac{\Delta}{t} \tag{105}$$

the constants C and C^1 being determined respectively from the curves of Figs. 1 and 2, will, for practically all commercial types of induction motors yield very good results."

Prof. Silvanus P. Thompson stated that, "What has been done by Kolben, Behrend, and others has been to state certain very simple empirical rules, of which Behrend's is perhaps the best."

This formula for σ was developed and used by me in 1896 and 1897 in the testing department of the Oerlikon Company on the



In alter Zuneigung u. Hochschätzung
Ihr ergebener
Hs. Behn-Eschenburg

(Facing page 112)

basis of my circle diagram, with which engineers were then quite unfamiliar. It was published by me in January, 1900, in lectures delivered at the University of Wisconsin, republished in the *Elec-*

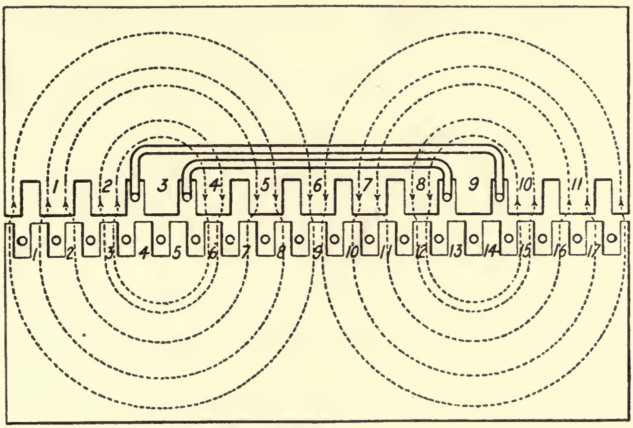


Fig. 84.

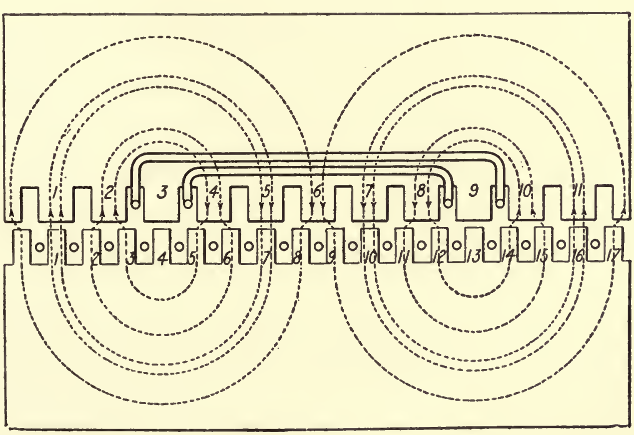


Fig. 85.

Figs. 84 and 85.—Dispersion of the flux. (After Behn-Eschenburg).

trical World, and afterwards in book form. Dr. Behn-Eschenburg published a slightly different formula in the pamphlet *Sur le Calcul des Machines électriques*, Oerlikon, June, 1900.

8

N. BIBLIOGRAPHY

R. E. HELLMUND: Reference should be made also to the papers of. "Graphical Treatment of the Rotating Field," *Trans.* Am. Inst. El. Engrs., June–July, 1908.

"Graphical Treatment of the Zigzag and Slot Leakage in Induction Motors," London, Institution of Electrical Engineers, June, 1909 and May, 1910.

PROF. C. A. ADAMS: "The Leakage Reactance of Induction Motors," International Electrical Congress, St. Louis, September, 1904.

"A Study in the Design of Induction Motors," *Trans.* Am. Inst. El. Engrs., June, 1905.

R. GOLDSCHMIDT: "The Leakage of Induction Motors," London: "The Electrician," Printing & Pub. Co., Ltd. No date.

DR. A. S. MCALLISTER: "Alternating Current Motors," New York: McGraw-Hill Book Co., Inc., 1909.

E. ARNOLD ET J.-L. LA COUR: "Les Machines d'Induction," Paris, Librairie Ch. Delagrave, 1912.

ALEXANDER GRAY: "Electrical Machine Design," New York: McGraw-Hill Book Co., 1913.

CHAPTER VIII

THE DOUBLE SQUIRREL-CAGE INDUCTION MOTOR

An ingenious suggestion was made in the early nineties by M. Dolivo-Dobrowolsky,[1] which has recently found an important application in the quarter-phase induction motors propelling the battleship U. S. S. New Mexico.[2] It consists in the use of two squirrel-cage windings on the same rotor. The winding near the surface has a high resistance and high leakage reluctance, while another winding placed beneath this winding has a low resistance and a low leakage reluctance. The high resistance of the outer winding is obtained by the use of an alloy of approximately 18 per cent german silver, while the low leakage reluctance of the second winding is obtained by a separate leakage path.

The ostensible idea of the arrangement is to utilize the outer high resistance winding in starting, and the inner low resistance winding for running. It appears plausible that, in view of the high frequency of the secondary currents when starting, and the low secondary frequency when running, these conditions may be satisfactorily fulfilled. This condition can be determined only by a careful quantitative analysis of the arrangement. Suffice it to say that, in spite of the fact that the arrangement has been known for over 25 years, it has found no application until recently.

Figure 86 represents the arrangement of the windings in the slots, and Fig. 87 represents the distribution of the magnetic flux in the different circuits. It is clear that the Low Resistance Winding III, whose m.m.f. is X_3, is closed through the leakage path ρ_3, whose leakage flux is f_3. The flux produced by X_3, if acting alone, through the reluctance ρ is Φ_3, not shown in the figure. The m.m.f. X_2 of the High Resistance Winding II acts in series with the m.m.f. X_3 so that its flux Φ_2, not shown in the figure would, be $\Phi_2 = X_2 \div \rho$, if acting alone, and therefore if only X_2 and X_3 were acting upon the circuits, the flux $\Phi_2 + \Phi_3 = (X_2 + X_3) \div \rho$, would exist in the air-gap. These fluxes are prevented from becoming established through the existence of the m.m.f. X_1.

[1] U. S. Patent No. 427,978, May 13, 1890.

[2] "General Characteristics of Electric Ship Propulsion Equipments." By E. F. W. Alexanderson. *General Electric Review*, April, 1919.

To fix ideas we also represent in Fig. 88 the electric circuits to which this type of motor is equivalent, assuming a ratio of transformation of one to one. As both rotor windings have

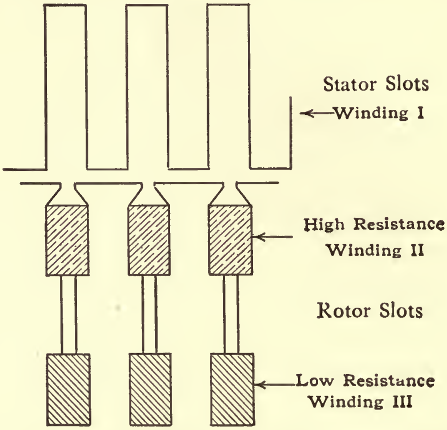


Fig. 86.—Arrangement of slots of double-squirrel cage motor.

the same slip, the variation in speed corresponds to a transformer with variable resistances, as indicated in the Fig. 88, *viz.*,

$$r_2 \div s \text{ and } r_3 \div s \tag{106}$$

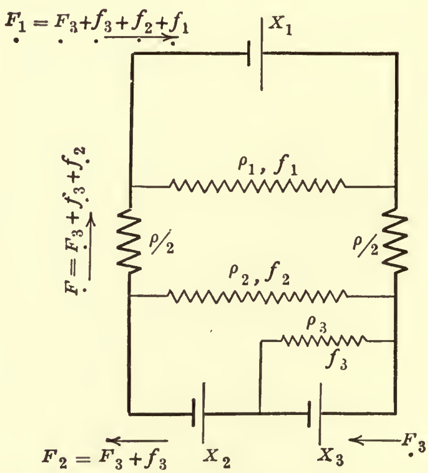


Fig. 87.—The leakage paths of the magnetic circuit of the double-squirrel cage motor.

We begin with Winding III. Its resultant flux is F_3. This flux sets up the e.m.f. which sends current through the ohmic resistance of the Low Resistance Winding III. This e.m.f. is equal to

$$e_3 = 2.12(\sim_1 - \sim_2)z_3F_3 10^{-8} \text{ volts} \tag{107}$$

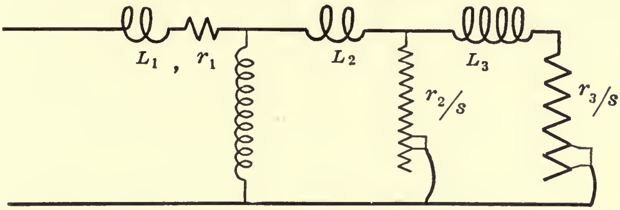


FIG. 88.—Equivalent transformer circuits for double-squirrel cage induction motor.

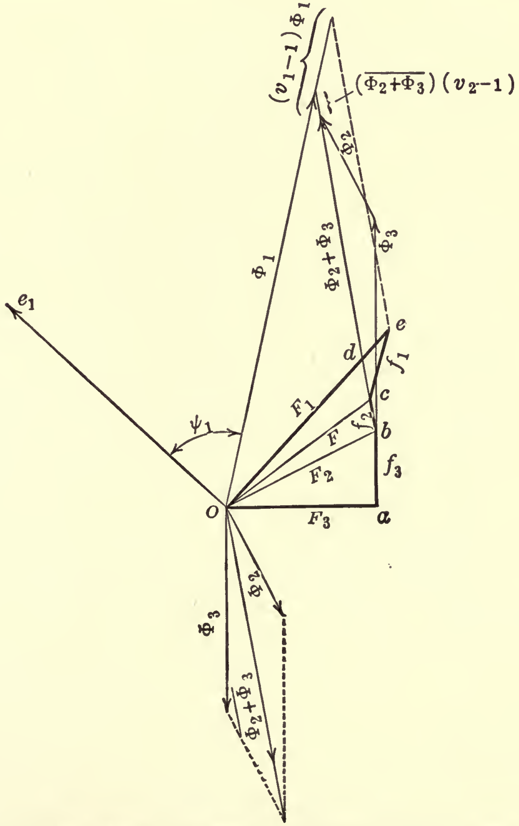


FIG. 89.—The diagram of fluxes of the double-squirrel cage motor.

The current produced by this e.m.f. is equal to $i_3 = e_3 \div r_3$.

The leakage field f_3 through the path of reluctance ρ_3 is in time-phase with and proportional to i_3 and it is to be estimated in the usual manner. In Fig. 89 it is represented by $ab = f_3$. OB in Fig. 89 is the flux F_2, as also indicated in Fig. 87.

If X_3 created by i_3 were acting alone, it would circulate a flux Φ_3, Fig. 89. Likewise, the e.m.f. e_2 induced by F_2,

$$e_2 = 2.12(\sim_1 - \sim_2)z_2F_2 10^{-8} \quad \text{volts} \tag{108}$$

produces a current $i_2 = e_2 \div r_2$, and an m.m.f. X_2 which, acting alone, would circulate a flux Φ_2 in quadrature with F_2, as shown in Fig. 89. Combining Φ_3 and Φ_2 vectorially, gives $\dot{\Phi}_2 + \dot{\Phi}_3$, represented in the diagram in line with bc.

As in the general flux theory of the induction motor, so here,

$$v_3\Phi_3 = \dot{\Phi}_3 + f_3 \tag{109}$$

$$\therefore (v_3 - 1)\Phi_3 = f_3 \tag{110}$$

$$v_2(\dot{\Phi}_2 + \dot{\Phi}_3) = (\dot{\Phi}_2 + \dot{\Phi}_3) + f_2 \tag{111}$$

$$\therefore (v_2 - 1)\dot{\Phi}_2 + \dot{\Phi}_3) = f_2 \tag{112}$$

$$f_3 = ab \tag{113}$$

$$f_2 = bc \tag{114}$$

$$f_1 = ce \tag{115}$$

As before in the theory of the induction motor, it follows readily

$$\left(1 - \frac{1}{v_1}\right)\left(\Phi_2 + \Phi_2\right) = cd \tag{116}$$

The diagram of Fig. 89 shows clearly and significantly the composition of the fluxes F_3, f_3, f_2, and f_1 into the primary resultant flux F_1 which induces the counter e.m.f., which balances the primary impressed e.m.f. As before, the primary resistance r_1 is neglected. It can easily be taken into account as in Chap. III.

An inspection of the diagram shows the influence of the low reluctance of the Leakage Path III. To show the effect of this leakage, a complete performance of a motor has been worked out for a range of slip from zero to infinity for given motor characteristics, as follows:

$$r_3 = 0.06$$
$$r_2 = 0.6$$
$$v_3 = 1.3$$
$$v_2 = 1.1$$
$$v_1 = 1.1$$
$$\Phi_3 = \frac{F_3 \cdot s}{r_3}$$
$$\Phi_2 = \frac{F_2 \cdot s}{r_2}$$
$$F_1 = 33.5$$

These characteristics correspond closely to a large slow speed motor with the exception that the leakage is assumed somewhat larger than it would be in reality, as well as the reluctance of the main magnetic circuit. The real motor, therefore, would have a higher power factor.

The following table is obtained from corresponding points carefully worked out:

Slip	Cosine ψ_1	Prim. current	Torque		
∞	0.0000	...	0	Current and Cos ψ_1 do not correspond to these points.	
50.0	0.0375	192.0	66		
10.0	0.2250	180.0	400		
4.0	0.3300	143.0	470		
3.0	0.3350	129.0	430		
2.0	0.3150	116.0	370	256	
1.5	0.2950	109.0	312	350	
1.0	0.2500	105.0	263	500	132
0.9	0.2300	104.0	244	546	150
0.8	0.2370	104.0	244	600	165
0.7	0.2230	102.0	225	643	180
0.6	0.2350	100.0	232	710	205
0.5	0.2380	98.5	230	760	230
0.4	0.2400	96.0	235	793	280
0.3	0.2900	91.0	270	776	320
0.2	0.3700	87.0	320	635	345
0.1	0.4800	69.0	337	370	275
0.06	0.5200	54.0	276	210	190
0.0	0.0000	33.5	0	0	0

These results are represented in the polar diagram, Fig. 90. An analysis of this figure yields the following results:

First, the locus of the primary current is no longer a circle.

Secondly, if the Low Resistance Winding III did not exist, the locus of the primary current would be the circle about O' as center, with a diameter

$$ab = \frac{Oa}{\sigma} \tag{117}$$

$$\begin{aligned} \text{where } \sigma &= v_1 v_2 - 1 \\ &= 1.1 \times 1.1 - 1 \\ &= 0.21 \end{aligned}$$

This circle is shown in the figure.

Thirdly, if the High Resistance Winding II did not exist, the locus of the primary current would be the circle about O'' as center, with a diameter

$$ac = \frac{Oa}{\sigma}$$

$$\begin{aligned} \text{where } \sigma &= v_1v_2 - 1 \\ &= 1.1 \times 1.3 - 1 \\ &= 0.43 \end{aligned}$$

This circle is also shown in the figure.

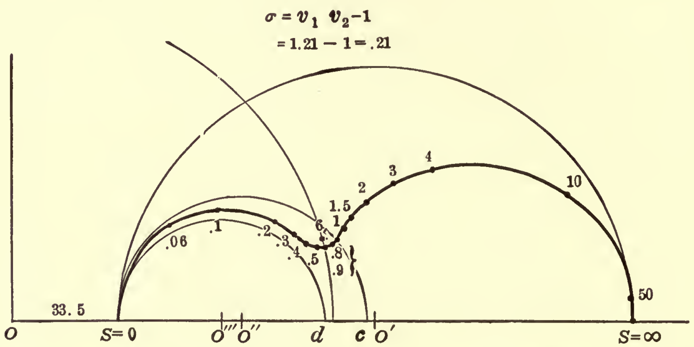


FIG. 90.—The polar diagram of the locus of the primary current of the double squirrel cage motor.

Fourthly, it appears that the actual performance of the motor is even less satisfactory than that which would correspond to a motor equipped only with the Low Resistance Winding III, as is natural enough, as there exists besides the leakage path II.

Fifthly, adding the two leakage paths II and III we obtain

$$\begin{aligned} v_2 &= 1.40 \\ \sigma &= 1.1 \times 1.4 - 1 \\ &= 0.54 \end{aligned}$$

The circle corresponding to such a motor is shown over ad with O''' as center.

Sixthly, the torque of the motor in synchronous watts for any given speed or slip is represented, as in the ordinary induction motor, by the watt component of the primary current. An analysis of the relations between torque and slip shows the interesting fact that, though the maximum torque obtainable from a certain frame is greatly reduced by the Low Resistance Winding

III, the slip of the motor while running near synchronism is considerably less than it would have been had a single-rotor winding of high enough resistance been used to obtain the same starting torque with the same starting current as in the case of the double squirrel-cage motor.

A single high resistance winding in the position of winding II would, on the other hand, give a performance circle *ab*. Its resistance could be so chosen that its slip at normal load were equal to that of the double squirrel-cage motor. However, in this case, the starting torque and the starting current would be approximately twice those of the double squirrel-cage motor

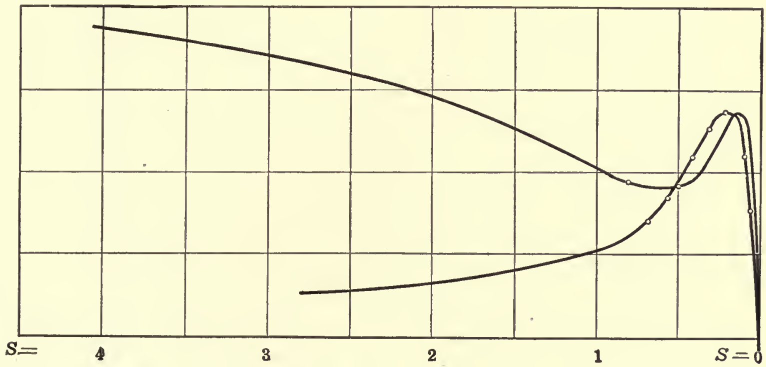


FIG. 91.—Upper curve: Torque of a double-squirrel cage motor. Lower Curve: Torque of a single-squirrel cage motor.

and it would be necessary to reduce the voltage 40 per cent at starting in order to obtain equivalent conditions for the single winding motor with the double squirrel cage. As this requires the use of auto-transformers, it may be less desirable under certain conditions than the double squirrel-cage arrangement.

A torque curve is shown in Figs. 91 and 92 which brings out the interesting and singular fact that the double squirrel-cage motor cannot accelerate its starting torque as the dip in the torque curve shows that the torque at 60 per cent slip is 13 per cent less than at 100 per cent slip.

For regular commercial use this type of motor would seem to be unsuitable in view of the great reduction of maximum output as a result of the great leakage of the low-resistance winding.

For special conditions, like the electric propulsion of ships, it may have the advantage claimed for it, *viz.*, the elimination of a large control rheostat, but in a sense it embodies in its own frame a variable reactance which takes the place of a starting auto-transformer installed outside the motor. Nor is it altogether clear whether smaller and lighter motor frames with a single high resistance winding of low leakage would not yield equally good results in the individual case in which one or two

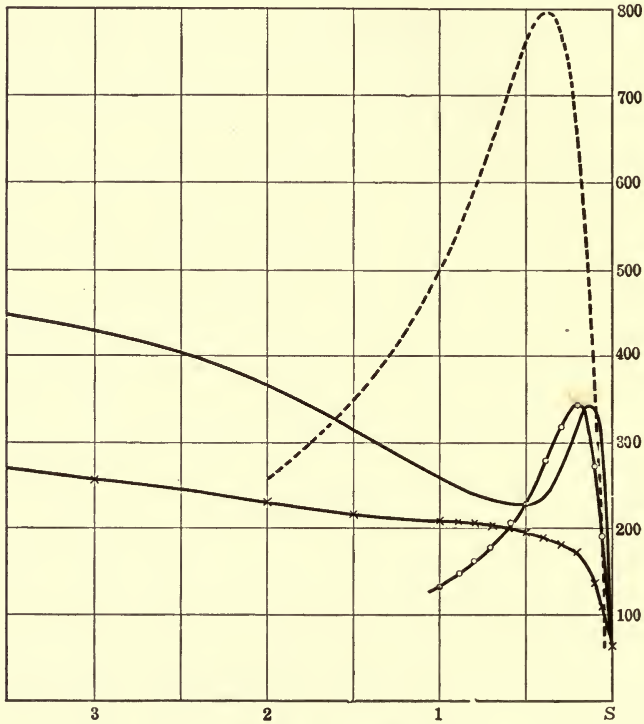


FIG. 92.—Torque and current of double-squirrel cage induction motor. 1. Lower curve is the current. 2. Upper curve in heavy lines the torque of the double-squirrel cage motor. 3. Dotted curve the torque of the same motor without the inner cage. 4. Short solid curve torque of the same motor without the outer cage.

motors may be operated from one generator as a unit, if it is desired to dispense with an external rheostat in the rotor circuit.

To sum up, the performance of a double squirrel-cage rotor as to power factor and overload torque approaches very closely the performance of an induction motor with a single-rotor winding, the constants of which are the same as those of the high-leakage winding. Thus power factor and overload torque are greatly reduced, in comparison with a normal low-leakage motor. The advantage consists in lowering the slip near synchronism, and thus raising the efficiency of the motor, on the assumption that the starting torques of both types are approximately the same, as well as their starting currents. While, if the slip is the same at normal load, the starting current of the double squirrel-cage motor is increased over that of the low-resistance, high-leakage type of motor. All this is clearly shown in our diagrams.

CHAPTER IX

POLY-PHASE COMMUTATOR MOTORS
PROPERTIES OF COMMUTATORS

A. THE ACTION OF THE COMMUTATOR

In this chapter we shall study the characteristics of an armature wound like a direct-current machine, rotating in a stator without windings. The commutator connected to this armature carries a set of poly-phase brushes stationary in space and poly-phase current of a given frequency is supplied to the armature through these brushes.

Neglecting for the present the phenomena of commutation, we see at a glance that the action of commutator and stationary

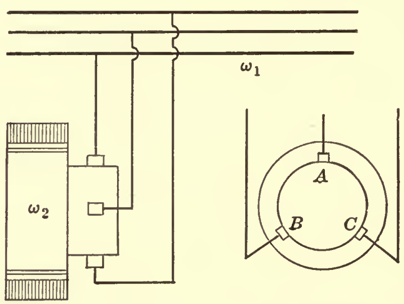


FIG. 93.—Poly-phase commutator motor. The action of the commutator.

brushes consists in creating stationary *groups* of coils, Fig. 93, as though these coils or *groups* of coils belonged to the stationary part of an induction motor. Supposing then that the supply circuit has a given constant frequency $\sim_1$ and that the commutator-rotor revolves with angular velocity ω_2. The supply frequency being $\sim_1$, a rotating field of angular velocity $\omega_1 = 2\pi\sim_1$ is set up in our two-pole model and there is a slip $s = \frac{\omega_1 - \omega_2}{\omega_1}$ set up between the rotor coils and the rotating magnetic field F. Between brushes $A - B$, $B - C$, and $C - A$, there is therefore going to be induced by rotation an e.m.f., Eq. (47).

$$e = 1.84(\sim_1 - \sim_2)z_2F10^{-8} \text{ volts} \tag{118}$$

where, as usual, z_2 is the number of active rotor conductors between A and B, B and C, and C and A.

Equation (118) may also be written

$$e = 1.84 \sim_1 s \cdot z_2 \cdot F 10^{-8} \text{ volts} \tag{119}$$

where s is the slip.

If in Fig. 94 F is the flux of the rotating field it is obvious that at standstill, neglecting losses, the e.m.f. to be impressed upon the rotor through the stationary brushes must lead by a quarter-time phase, as the magnetizing current i must be a watt-less current and therefore lagging by a quarter-time phase.

However, with increasing speed of the rotor the slip approaches zero at synchronism, while above synchronism the phase of e reverses. Thus we are led to the brilliant discovery of Leblanc later utilized by M. Marius Latour and A. Scherbius that such a commutator armature, if rotated at a negative slip in its own field, acts like a condenser, or expressing it more rigorously, it takes lagging currents from the supply circuit or it generates leading currents through its rotation in its own field.

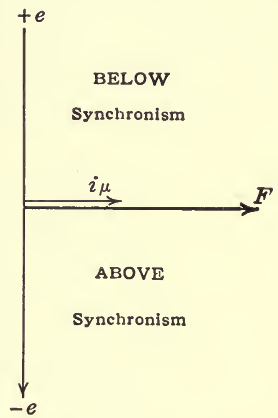


Fig. 94.—Time phase diagram showing e.m.f. in commutator supplied with poly-phase currents above and below synchronism.

The reasoning pursued above is based on the assumption that the magneto-motive force belts of the winding generate solely a rotating field and that there are no single-phase fields remaining over as leakage fields whose effect is prominent at constant frequency instead of slip frequency. The apparent reactance per phase X_2^0 of the rotor in regard to the rotating field F is equal to

$$X_2^0 = \frac{e}{i_\mu} = \frac{1.84 \cdot \sim_1 \cdot s \cdot z_2 \cdot F 10^{-8}}{k \dfrac{F}{z_2}}$$

$$X_2^0 = k_2^0 \cdot s \tag{120}$$

In other words, X_2^0 is proportional to the slip and becomes negative at negative slips.

Supposing, however, that, failing to obtain a perfect rotating field, there remain local fields entirely independent of the slip.

These fields would set up an e.m.f. proportional to the impressed frequency and current and therefore the reactance X_2'' corresponding to the effect of these fields is constant.

$$X_2'' = k_2'' = \text{constant} \tag{121}$$

Considering now the effect of the short-circuit currents under the brush as produced by the rotating field F, they effect the generation of an e.m.f. proportional to the product of the slip and the field divided by the impedance of the short-circuited coil. Assuming this impedance to be primarily a resistance, it is not unreasonable to assume it to be constant. Hence, the

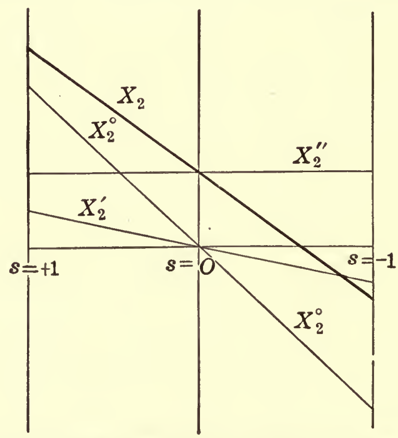


FIG. 95.—The local leakage reactance of the poly-phase commutator rotor as composed of its various parts showing the effect of the variable and constant portions. (Leblanc-Latour).

effect of the short-circuited turns under the brushes corresponds to a reactance X_2' which is proportional to the slip,

$$X_2' = k_2's \tag{122}$$

Therefore, the total reactance of the rotating commutator armature is equal to

$$X_2 = X_2^0 + X_2' + X_2''$$
$$X_2 = k_2^0 \cdot s + k_2' \cdot s + K_2'' \tag{123}$$

In rectangular coordinates, these relations may be represented as shown in Fig. 95, from which it appears that the reactance existing at $s = O$, *i.e.*, at synchronism, is equal to the constant part of the total reactance of the commutator rotor.

B. PROPERTIES OF PHASE LAG OR LEAD OF THE POLY-PHASE COMMUTATOR

The use to which Leblanc, Latour, Scherbius, and the Brown, Boveri Company have put these rotors is now apparent. Operated by a small motor at high speed with a large negative slip, either with a mechanically-stationary magnetic circuit, or with conductors embedded in the revolving iron ring,[1] they are connected in series with the rotor winding of a slip ring type of

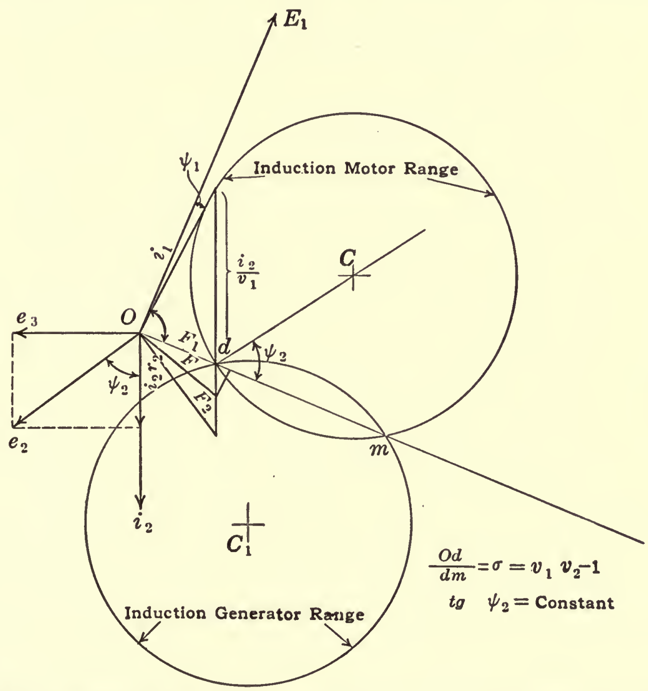


FIG. 96.—The effect of leading currents in the rotor of a slip-ring type of induction motor upon the primary currents.

induction motor. Thus they are excited by low frequency currents and an e.m.f. proportional to the rotor current but in lagging quadrature therewith is induced in the rotor. The polar diagram is drawn in Fig. 96, which already has been discussed in Chap. III, C. It shows the locus of the primary current for both induction motor and induction generator ranges so long as the current which flows into the Leblanc commutator-

[1] A. SCHERBIUS, "The Electrician," London, 1912, p. 582.

rotor leads the impressed e.m.f. at its brushes. This condition must be fulfilled in order to obtain the benefit of the commutator action.

Saturation of the magnetic circuit of the Leblanc rotor has been proposed by Scherbius in order to obtain as rapidly as possible a high-power factor at small loads without low-leading power factors at greater loads, see Fig. 160, which is self-explanatory.

C. COMPARISON BETWEEN INDUCTION MOTORS WITH ROTORS SHORT-CIRCUITED THROUGH RINGS OR OF THE SQUIRREL-CAGE TYPE; AND ROTORS SHORT-CIRCUITED THROUGH SYMMETRICAL POLY-PHASE BRUSHES

The e.m.f. induced through rotation in a rotor of the slip-ring type is proportional to the slip and it is of slip-ring frequency.

The e.m.f. induced through rotation in a rotor of the commutator type is proportional to the slip but it is of primary frequency.

At standstill and at synchronism it appears evident that both types of motor must operate identically if we consider for example two three-phase rotors, or a rotor with slip-rings on one side and a commutator on the other, alternately short-circuited.

Consider for a moment a rotor without leakage reactance of any kind. The mechanical brush shift in the direction of rotation of the magnetic field leads to an induced e.m.f. which lags behind the e.m.f. induced if the brushes are in a position in which the primary and secondary entries are opposite each other. The maximum ampere turns on the rotor occurring thus *later in time-phase*, the rotor position being *advanced in space-phase*, these two conditions automatically compensate each other and thus it appears that it is immaterial in which position the brushes are placed, the same argument holding if the brushes are shifted in a direction opposite to that of the rotation of the magnetic field.[1] The method of proof here given is, of course, not limited to a rotor without leakage, as the phase lag of the current remains unaltered if the position of the rotor relative to the stator does not affect the leakage reactance of the rotor.

But in a rotor without leakage reactance, perfect reflection of the rotor m.m.f. into the stator is the same in both the slip-ring type and the commutator type. However, with secondary

[1] For this simple method of exposition I am indebted to my friend, Prof. V. Karapetoff. I am also indebted to him for the happy term "perfect reflection."

leakage reactance it appears that in the slip-ring type this reactance is proportional to the slip on account of the fact that the secondary currents are of slip frequency, whereas in the commutator type these currents are of primary frequency. It is, therefore, to be assumed at the outset that the two types will not act the same if there is secondary leakage reactance.

In the slip-ring type, both the effect of the rotating leakage field $\Phi_2(v_2 - 1)$ and the effect of single-phase leakage fields are obviously of the same nature. This is not so in the commutator

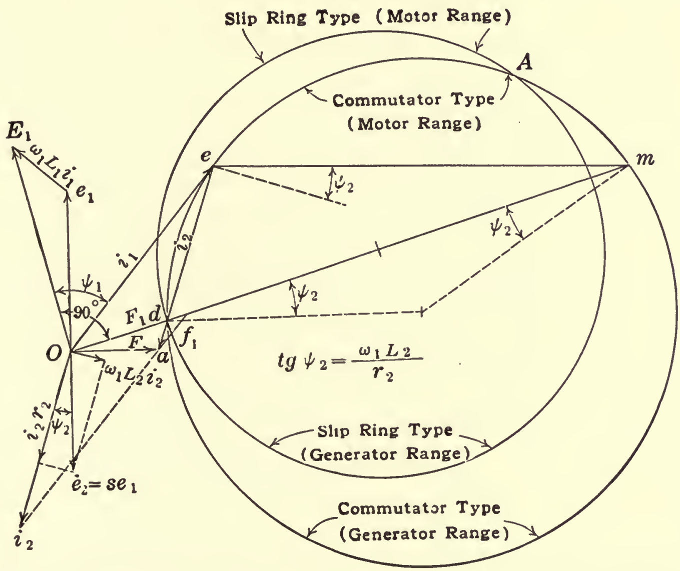


FIG. 97.—Comparison of commutator type of induction motor with the slip ring or squirrel-cage type.

type as here the effect of a rotating leakage field $\Phi_2(v_2 - 1)$ and the effect of single-phase leakage are altogether different.

The assumption of constant secondary reactance leads to a very simple diagram, but this assumption is not warranted. However, to bring home the subject, let us for a moment consider the logical consequences of such an assumption. In Fig. 97 F is the common resulting flux in primary and secondary; $e_2 = s.e_1$ is the e.m.f. induced by rotation in a group of coils between brushes. ψ_2 is the secondary lag

$$tg\psi_2 = \frac{\omega_1 L_2}{r_2} \tag{124}$$

where L_2 is the secondary leakage inductance, and $\omega_1 L_2$ is the constant secondary leakage reactance. The remainder of the diagram is familiar to the reader. Now, as ψ_2 is constant by our assumption of constant secondary leakage reactance, the angles at "a" and "e" are equal to $\frac{\pi}{2} + \psi_2$ and therefore e moves on the arc of a circle so that angle *dem* is always equal to $\frac{\pi}{2} + \psi_2$.

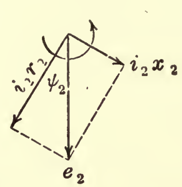
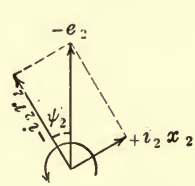
It is interesting to note that here the generator range is greatly benefitted by the use of the commutator, while the motor range has been affected detrimentally. Both types have the point "A" in common at starting, for which the secondary current and phase relations are identical.

We commend to the reader a careful study of the diagram and especially the curious effects above synchronism, to which attention has been drawn by Rüdenberg[1] and Altes, and which are so important that a brief reference to them should be made here.

D. THE REFLECTION INTO THE PRIMARY CIRCUIT OF THE M. M. F. OF THE SECONDARY WITH SLIP-RING AND COMMUTATOR ROTORS

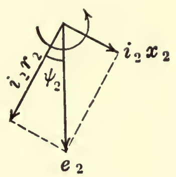
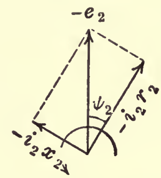
At positive and negative slips, the angle of current phase appears reflected differently into the primary if the slip is positive or negative, and if the rotor is of the slip-ring type or of the commutator type. This is illustrated in the following table:

Slip-ring Type

$+s$	$-s$
$+e_2$ $+i_2$ $+i_2x_2$ $+i_2r_2$	$-e_1$ $-i_2$ $+i_2x_2$ $-i_2r_2$
 	

[1] R. Rüdenberg, *E. T. Z.*, 1910, p. 1087. Also W. C. K. Altes, *A. I. E. E. Trans.*, 1918, p. 309.

Commutator Type

$+e_2$ $+i_2$ $+i_2x_2$ $+i_2r_2$	$-e_2$ $-i_2$ $-i_2x_2$ $-i_2r_2$
	

For a negative slip, which corresponds in the short-circuited types to generator action, we see therefore that in the slip-ring type the reflected m.m.f. of the secondary, though its current lags behind the induced secondary e.m.f., appears in the primary vector diagram as a leading current.

In the commutator type, however, the reflected m.m.f. of the secondary appears with a lag due to the fact that induced e.m.f. and induced current reverse, while the reactance component is constant, *viz.*, $tg\psi_2$ = constant.

E. VARIABLE AND CONSTANT SECONDARY REACTANCE OF THE COMMUTATOR MOTOR

The next step now is to consider, in addition to the constant secondary reactance, that part which varies with the slip. We assume, at standstill, point A, the total secondary leakage composed of the rotating field leakage Bb and the single-phase leakage aB, so that angle

$$\sphericalangle BOa = tg\psi_2' = \text{const.} \tag{125}$$

Then it follows from Chap. III, B, that we obtain the circle O'' for the locus of the primary current of a commutator type of induction motor with short-circuited brushes, while the circle O' is the locus for a squirrel-cage or slip-ring type of motor.

The circle is larger over the generator range due to the fact that the part of the secondary leakage reactance which is constant appears in the primary as would a leading current in the slip-ring type. It is worth while to follow this matter up with a physical illustration.

Let Fig. 99 be a Leblanc commutator. A rotating field F is produced by the currents flowing through the brushes, its

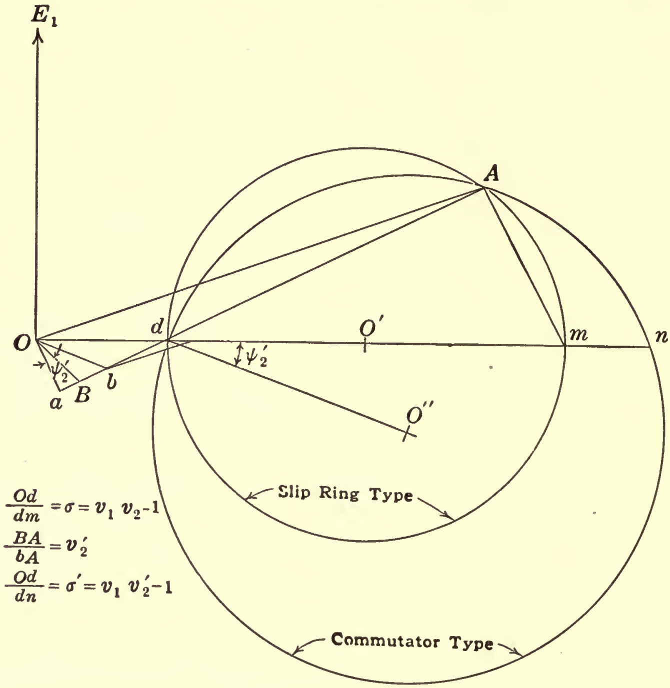


Fig. 98.—Variable and constant secondary reactance of the commutator induction motor.

angular velocity being ω_1. If $\omega_2 > \omega_1$ then we have shown that the current leads the impressed e.m.f. Now, it makes no differ-

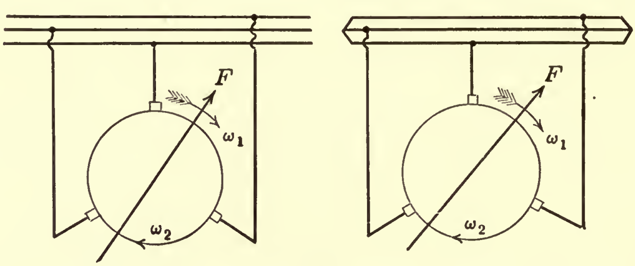


Fig. 99.—The operation of the Leblanc commutator.

ence whether F is produced by the currents from the line, or whether it is set up by other means, if the brushes are short-circuited, the short-circuit corresponding to the negligible impedance of the line, as was so brilliantly pointed out by M. Latour. Thus the two diagrams of Fig. 99 are physically the same, and the effect of operating a commutator motor above synchronism consists in creating leading currents in it, as viewed from the stator, and thus the constant part of the secondary leakage reactance above synchronism acts like a condensance. The reader is cautioned

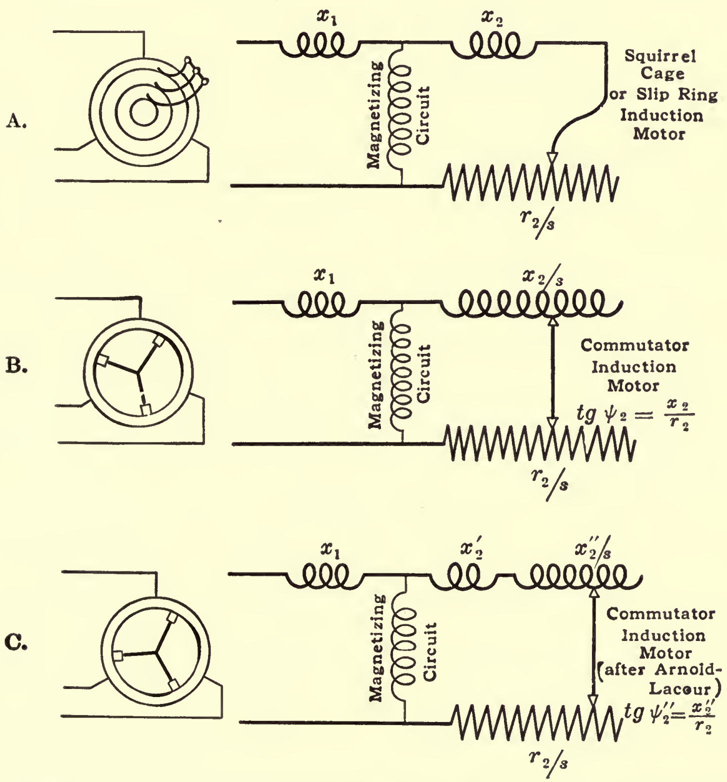


FIG. 100.—Commutator and slip ring, or squirrel cage, types of induction motor, and their equivalent circuits.

to distinguish between the total reactance and the leakage reactance, which is a prolific source of confusion.

Equivalent circuits can be drawn for the different types of induction motors as is indicated in Fig. 100, where "*A*" is the equivalent circuit of the slip-ring type, "*B*" the equivalent circuit of a commutator type assuming constant secondary reactance, and "*C*" is the equivalent circuit for a commutator type in which both constant secondary reactance and secondary reactance varying with the slip are indicated.

Tests have been made to check the performance of these motors by E. Arnold and la Cour[1] and by L. Dreyfus and F.

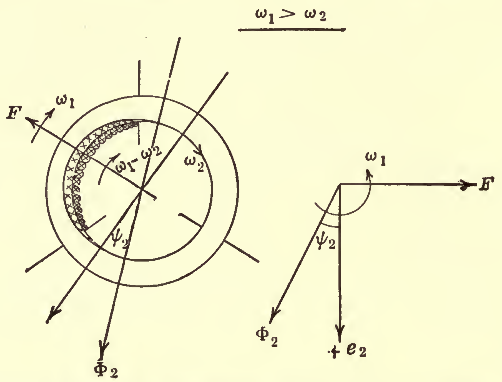


Fig. 101.—Slip-ring type of induction motor. Space diagram *below* synchronism.

Hillebrand.[2] The latter equipped a rotor with slip-rings on one side and a commutator on the other and thus recorded the standard circle diagram and the displaced circle for the same type of motor.

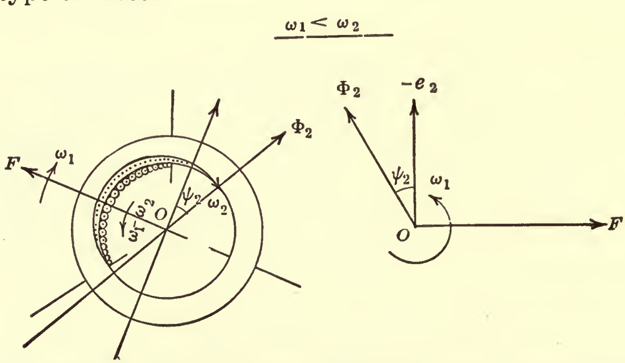


Fig. 102.—Slip-ring type of induction motor. Space diagram *above* synchronism.

In Figs. 101, 102, 103, and 104 there are traced out the m.m.f. belts produced by the rotation of the resultant field F with relative angular

[1] E. Arnold and J. L. la Cour, Vol. V, 2, p. 221.

[2] L. Dreyfus and F. Hillebrand, "Zur Theorie des Drehstromkollector-Nebenschlussmotors." *Elektrotechnik und Maschinenbau*, 1910, p. 886.

velocity $\omega_1 - \omega_2$ towards the rotor. For $\omega_1 > \omega_2$ the machine is a motor, while for $\omega_1 < \omega_2$ it is a generator. The m.m.f. belts indicate that, in the *Slip-ring Type,* viewed from the primary, the m.m.f. of the secondary, or its fictitious flux Φ_2, lags behind F by $\frac{\pi}{2} + \psi_2$

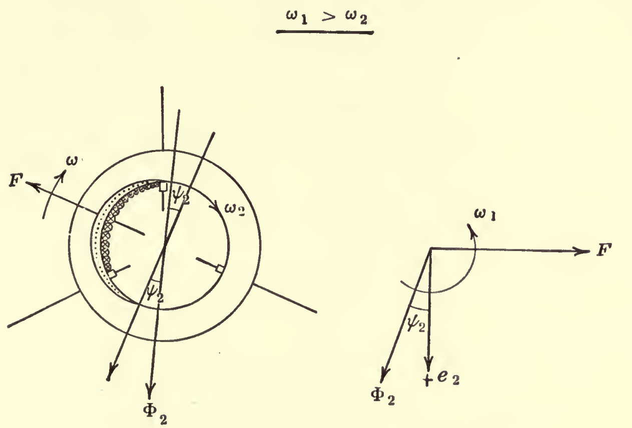


FIG. 103.—Commutator-type of induction motor. Space diagram *below* synchronism.

electrical degrees, because the secondary currents, being of slip frequency $\omega_1 - \omega_2$, set up a rotating magnetic field which is carried around by the rotor with angular velocity ω_2 in the

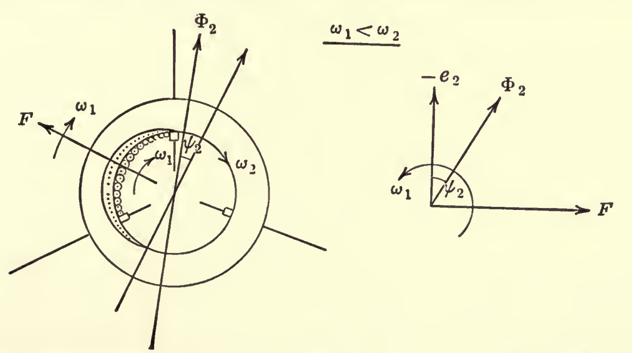


FIG. 104.—Commutator-type of induction generator. Space diagram *above* synchronism.

direction of mechanical rotation. Therefore, the electric phase lag of the secondary current appears also as a lag in the combined space and time diagram.

For the generator action of the slip-ring type of motor, it is to be noted that the secondary currents set up a magnetic field rotating in opposition to the mechanical rotation of the machine, thus the lagging current in the secondary appears in the primary and in the combined space and time diagram leading the flux F by a time or space angle of $\frac{\pi}{2} + \psi_2$.

These relations are different in the *Commutator Type,* in which the secondary currents are of line frequency and the coil groups between brushes are stationary. Therefore, the secondary current reflected into the primary appears as a lagging current relative to the induced e.m.f. Therefore, this consideration leads again to the curious result that, in the *Commutator Type of Induction Generator* running above synchronism, constant secondary reactance strengthens the resultant field of the machine and it therefore acts as capacity does in the range below synchronism.

F. THE SLIP-RING COMMUTATOR TYPE AS FREQUENCY CHANGER

The first to suggest the use of an armature provided with a commutator on one side, on which poly-phase brushes are placed, and slip-rings on the other, appears to be Mr. B. G. Lamme,[1] who noticed that, in a rotary converter without field excitation, running below synchronism, a current of low frequency appeared at the brushes on the commutator. This low frequency disappeared at synchronous speed.

If poly-phase currents of frequency $\sim_1$ are sent through the brushes upon the slip-rings, then a magnetic field is set up rotating with angular velocity ω_1 relative to the armature. If the armature revolves with angular velocity ω_2 against the direction of rotation of the field, then in the groups between the stationary poly-phase brushes upon the commutator, there will be induced an e.m.f. of the frequency $\sim_1 - \sim_2$. Hence, if the commutator of such a device were connected to the rotor of a slip-ring type of induction motor, it would receive currents of slip frequency $\sim_1 - \sim_2$, and on its slip-rings it would deliver currents of the frequency $\sim_1$. An application of this interesting phenomenon is described in Chap. XIII, *C.*

[1] B. G. Lamme, United States Patent No. 682,943. Sept. 17, 1901. Application filed July 24, 1897.

CHAPTER X

THE SERIES POLY-PHASE COMMUTATOR MOTOR

A. THE THEORY FOR CONSTANT CURRENT AND CONSTANT POTENTIAL IN THE IDEAL MOTOR

The properties of a commutator, as discussed in Chap. IX, are now to be applied to the Series Poly-phase Commutator Motor, first described in 1888 by Wilson in the British Patent No. 18,525 and by H. Goerges in the German Patent No. 61,951 of Jan. 21, 1891. Mr. H. Goerges also described the Shunt Poly-phase Commutator Motor and outlined the theory of these motors in the *E. T. Z.*, 1891, p. 699.

Figure 105 shows diagrammatically the connections of the motor. The stator windings, which are indicated here in Y-connection, are in series with the delta-connected armature which is rotating counter-clockwise with the angular velocity ω_2. The rotating field resulting from the action of the poly-phase currents is assumed to have a counter-clockwise rotation ω_1, and the brushes are shown shifted forward by an angle α in the direction of rotation. The neutral position of the brushes, or the datum from which we count the brush-shift, is defined as that in which the current passing in series through stator and rotor produces two fictitious magnetic fields, which cancel each other, neglecting leakage. As usual we assume a two-pole magnetic structure and distributed windings in rotor and stator. The brushes slide on the commutator here assumed to be the surface of the rotor wnidings.

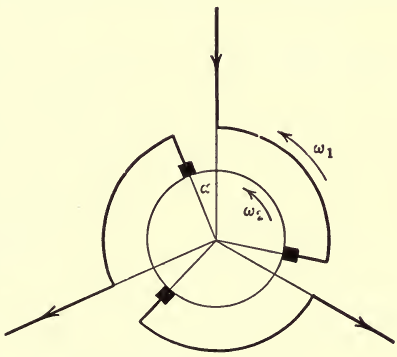


Fig. 105.—Three-phase series poly-phase commutator induction motor (H. Goerges). Brush shift angle α, no transformer.

Great attention has to be paid to the conception of time and space phases. To simplify the analysis, we assume identical windings on stator and rotor, the series connection being obtained through a series transformer whose magnetizing current and leakage we neglect for the present, Fig. 106.

The rotor and stator currents are assumed equal and in time-phase, so that, with brush-shift angle $\alpha = 0$, the two fictitious fields of rotor and stator would obliterate each other.

If the stator windings acting alone produce a counter-clockwise or positively rotating magnetic field Φ_1, then the same current—equal in magnitude and time-phase—acting alone in the rotor windings, produces a magnetic field Φ_2, whose *position in space* at a given moment of time is represented by the vector Φ_2 in Fig.

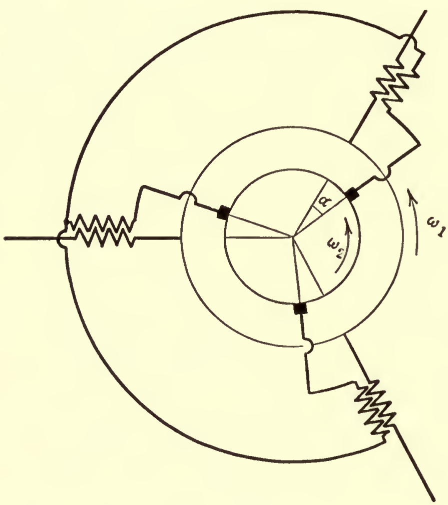


FIG. 106.

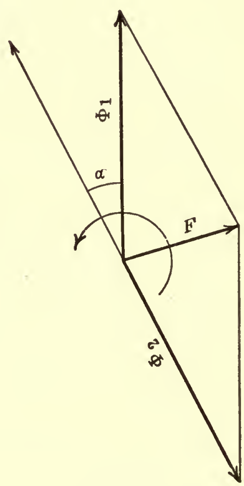


FIG. 107.

FIG. 106.—Three-phase series A. C. commutator motor with series transformer. Brush shift angle α, Δ-Connection in stator and rotor.

FIG. 107.—Space diagram of fluxes in series motor with brush shift angle α. Heavy lines indicate space vectors.

107. At *the same* moment of time the same current produces a magnetic field Φ_1, whose position in *space* is indicated by the vector Φ_1. The vector difference of Φ_1 and Φ_2 is the *resultant* really existing rotating magnetic field F.

With $\alpha = 0$, there is no torque; with brush shift clockwise or negative, we obtain clockwise rotation; with brush shift α counter-clockwise or positive, we obtain counter-clockwise rotation of the armature, the direction of rotation reversing with the shift of the brushes. *The torque is exerted in the direction of the* *brush shift from the defined datum* $\alpha = 0$, *as is indicated by a simple consideration of the magnetic fluxes* Φ_1 and Φ_2 *and their mutual attraction.*[1]

Still assuming no leakage, we know that F induces in stator and rotor windings e.m.fs. which are in time quadrature with the resultant flux which embraces the windings. As the e.m.f. induced in the rotor windings must appear earlier in time-phase than that induced in the stator windings, with α negative, or later with α positive, it follows that the e.m.fs. induced in the stator and rotor differ in *time-phase* by the same time angle α as do the *space* fields Φ_1 and Φ_2 by the same space angle α.

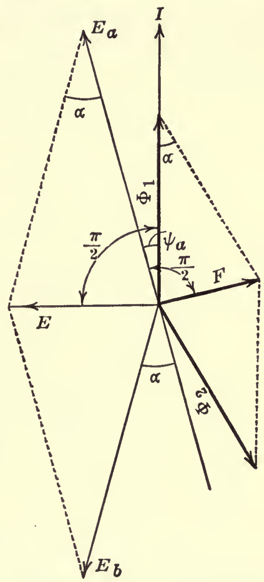


Fig. 108. — Combined space and time diagram of fluxes and e.m.f.'s in series poly-phase commutator motor with brush shift angle α.

Assume the resultant flux F to be projected on a vertical time axis for reference (Fig. 108). Assume it to be zero at a certain time. Then the voltage E_a induced in the stator winding of the series motor is, barring leakage and resistance, in quadrature with the resultant field F and, as the current I is in time-phase with the fictitious flux Φ_1, we now have the essential elements for the determination of the complete vector diagram of the poly-phase series motor.

Assume the rotor and stator resistances zero, and the rotor standing still. The fictitious fluxes Φ_1 and Φ_2 differing in space phase by the angle α, produce the resultant real field F.

I is in time-phase with Φ_1, and E_a is in time quadrature with F. Thus the time lag ψ_a of the current I behind E_a is determined. E_a and E_b are in time-phase opposition if the brush-shift angle $\alpha = 0$. They differ by the angle α in time-phase. We thus obtain E as the resultant voltage impressed upon the motor, in time quadrature with I, in the case of a resistance-less imaginary motor. This relation follows from the similar m.m.f., or flux, and e.m.f. triangles.

Assuming I constant and the motor beginning to turn in the

[1] See V. Karapetoff, "The Secomor," *Trans. A. I. E. E.*, Feb. 16, 1918.

direction of its rotating field, then the counter e.m.f. of rotation induced in the rotor diminishes proportionally to the slip. It disappears at synchronism.

An examination of the diagram Fig. 108 shows that, *below synchronism* the stator takes energy from the line, while the rotor delivers energy back to it. *At synchronism* the stator alone takes energy from the line; *above synchronism,* it is seen that both stator and rotor take energy from the line, the sum being transformed into mechanical energy. Such a motor is therefore described as "doubly-fed," a term widely used in the great work of the joint authorship of E. Arnold, J. L. la Cour, and A. Fraenckel.

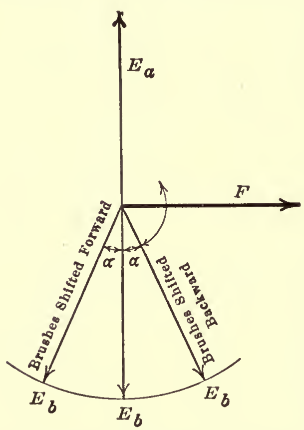


Fig. 109.—Brush shift and field rotation affect the time diagram.

We have already seen that the time-phase of E_b depends upon the brush shift α which determines the space angle of the m.m.fs. To emphasize this important point once more, for $\alpha = 0$, E_a and E_b are in time-phase opposition, Fig. 109. Shift the brushes clockwise, or backward in the direction of the rotation of the magnetic field, and the maximum of E_b will occur earlier in time-phase by the angle α, which is now a time angle; shift the brushes counter-clockwise, or forward in the direction of rotation of the magnetic field, and the maximum of E_b will occur later in time-phase by the angle α. We shall consider only this latter case in which the rotor and the magnetic field rotate in the same direction.

Still neglecting leakage and resistance, we re-draw (Fig. 110) the time diagram of the e.m.fs. and of the current and we see at a glance that for constant current the e.m.f. triangle is OBC, in which OB is the e.m.f. induced in the primary or stator winding, BC is the e.m.f. induced in the rotor at slip s, and it is therefore under our previous assumption of equal numbers of turns equal to E_a. The point B, therefore, corresponds to synchronous rotation and the range BA corresponds to speed above synchronism.

The angle α at B remains constant for a full speed range, and so does the angle $\alpha - \psi_a$ at A. If, therefore, we were to keep $OC = E$ constant, as well as α, allowing the current to vary, it is obvious that A lies on the periphery of a circle described about OC as chord. The current I is always proportional to OA, therefore, I may be measured by the chord drawn from O to the periphery of the circle. (Fig. 111.)

The Torque.—The current at starting is A_{sh}, while the torque at starting is proportional to the product of the space quadrature component of the secondary m.m.f. into the magnetic flux F, or

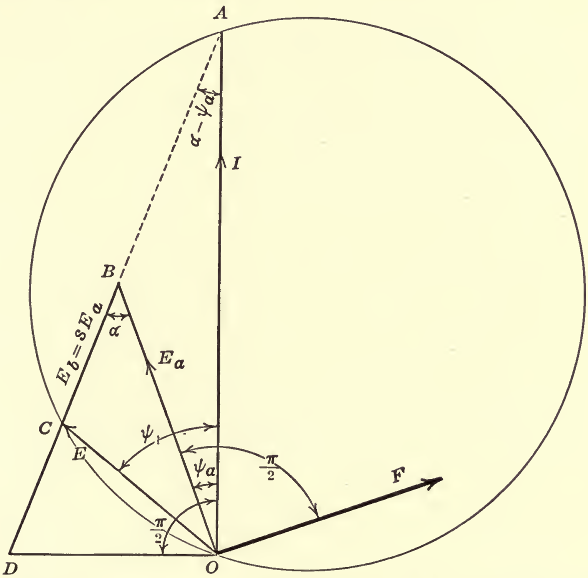


FIG. 110.—"D," Starting; "B," Synchronism; "A," Double Synchronism. Time-phase diagram for the locus of the primary current in the series poly-phase commutator induction motor for constant potential. Currents measured from O to periphery of circle.

$$\text{Torque} = K \cdot I \cdot F \cdot \sin \frac{\alpha}{2} \tag{126}$$

But in our diagram F is proportional to and in time quadrature with E_a, and E_a is also proportional to I. Therefore,

$$\text{Torque} = K_1 I^2 \sin \frac{\alpha}{2} \tag{127}$$

The torque, therefore, may be represented by the square of the current and, as we have seen on previous occasions, the square of the chords in a circle may be measured graphically perpendicular to the diameter of the circle as shown in Fig. 112 by T.

The Slip.—Drop a perpendicular from A_{sy}, the current locus at synchronism, upon the diameter of the circle OA_0. (Fig. 113.) The point of intersection a between OA and $A_{sy}S$, cuts

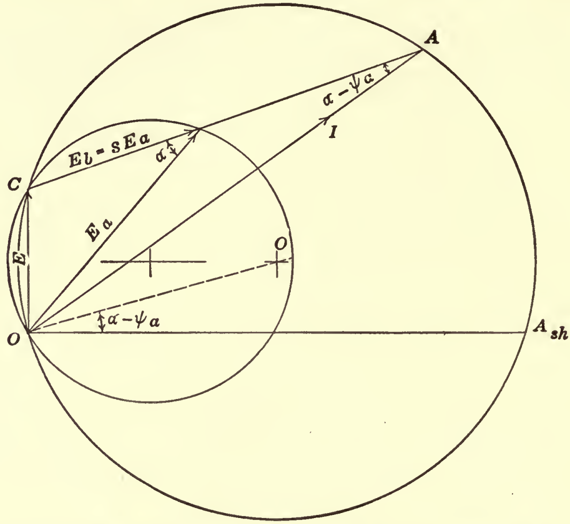


FIG. 111.—Circle $OA_{sh}A$ is locus of current I for constant $E = OC$. The current and the e.m.f.'s in the ideal series motor for constant potential.

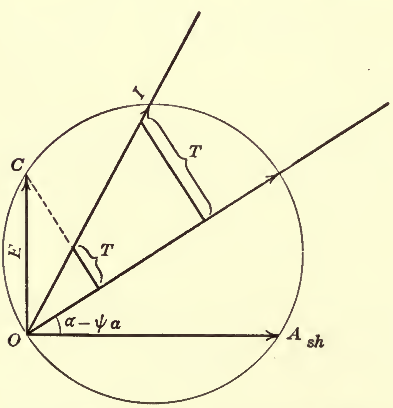


FIG. 112.—The torque in series poly-phase commutator motor. $OC = E$ = primary impressed voltage. For $\psi_1 = O, OC = I$ and T is corresponding torque.

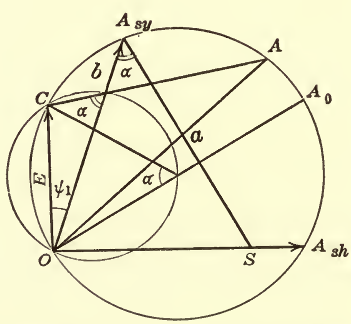


FIG. 113.—The slip in the series poly-phase commutator motor for constant potential.

off $A_{sy}a$, which is a measure of the slip. Point S corresponds to 100 per cent slip, point A_{sy} to a slip 0 per cent.

Proof.—Triangles ObC and $OA_{sy}a$ are similar. The slip is the ratio of $Cb : Ob$, therefore,

$$Cb : Ob :: aA_{sy} : OA_{sy}$$

$$\therefore s = \frac{aA_{sy}}{OA_{sy}} \tag{128}$$

At double synchronism, as is shown by inspection of the diagram, the power factor becomes unity.

B. THE THEORY FOR CONSTANT CURRENT AND CONSTANT POTENTIAL IN THE REAL MOTOR

The diagram of m.m.fs., or fluxes, remains the same if we take the leakage into consideration by using the e.m.f. induced by the

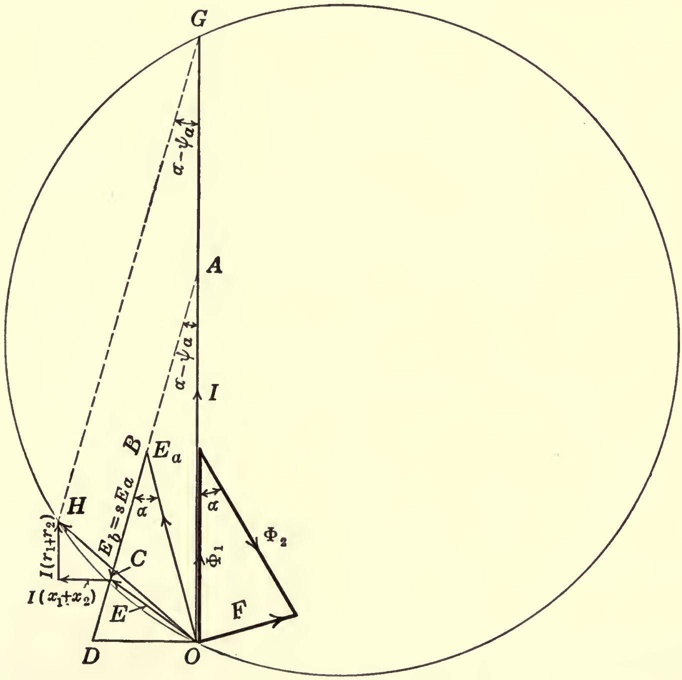


FIG. 114.—The time-phase diagram of the series poly-phase commutator induction motor. Including resistance and leakage.

leakage field which is the simplest method in this case as we must combine e.m.fs. affected by the speed. In the theory of the slip-ring or squirrel-cage induction motor, we found it simpler to employ leakage fields instead of the voltages induced by them.

The leakage reactance of the stator windings is constant at all speeds. The leakage reactance of the rotor windings, as shown in Chap. IX, is composed of a part constant at all speeds and of another dependent upon the speed of rotation of the armature. This latter is positive at speeds below synchronism, zero at synchronism, and negative at speeds above synchronism. Without a serious error we may permit ourselves the license of viewing the total leakage reactance of the motor as constant, and in

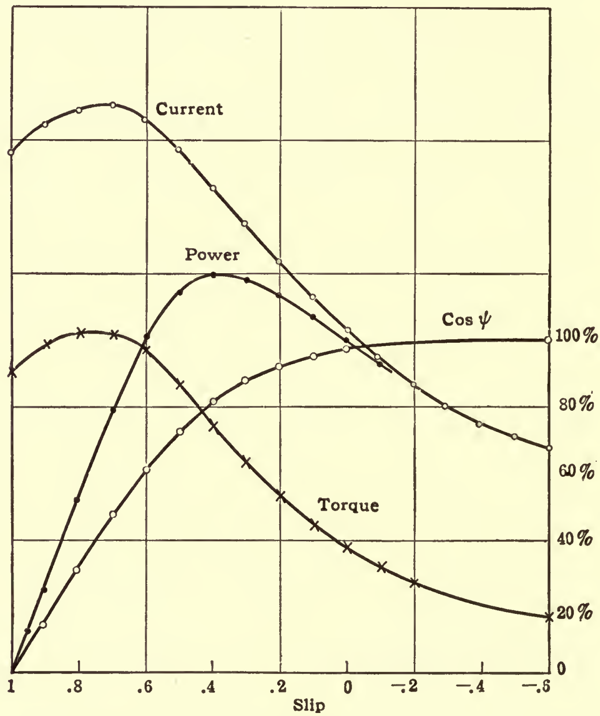


Fig. 115.—The torque, current, power factor and slip in the series poly-phase commutator induction motor.

quadrature time-phase with the current, Fig. 114. Adding vectorially to the e.m.f. thus obtained $I(r_1 + r_2)$, a point G is derived at the intersection of OA and the line OH through H. The angle $(\alpha - \psi_a)$ at G is again constant.

If we assume again constant primary voltage OH for the entire operating range, a circle on whose periphery lie the points O, G, and H becomes the locus of a radius vector from O, like OG, which is a measure of the current I.

All other relations follow from this diagram as before. We have plotted in Fig. 115 in rectangular coordinates the current, the power factor, the torque, and the power as a function of the rotor slip. At speeds above synchronism the power factor approaches unity. It may be suggested to the reader to draw similar diagrams for different brush-shifts α.

C. THE NECESSITY OF SATURATION FOR STABILITY

The series poly-phase motor is self-exciting as a generator. If it possesses remnant magnetism, it may generate direct current as the supply circuit forms virtually a short circuit for such currents. Saturation of the motor frame or of the series transformer may prevent these effects to a great extent. On this subject the reader may consult the following:

U. S. Patent 1,164,223, Dec. 14, 1915, A. Scherbius: Stabilized Commutator Machine.

E. Arnold, J. L. la Cour and A. Fraenckel, Die Wechselstromkommutatormaschinen, 1912, p. 59.

V. Karapetoff, "The Secomor." *Trans. A.I.E.E.*, 1918, Vol. XXXVIII, Part 1, p. 347.

W. C. K. Altes, "The Poly-phase Shunt Motor." *Trans. A. I. E. E.*, 1918, Vol. XXXVII, Part 1, p. 385.

CHAPTER XI

THE SHUNT POLY-PHASE A. C. COMMUTATOR MOTOR

A. HISTORICAL INTRODUCTION

The shunt poly-phase commutator motor as well as the series type appear to be the invention of H. Goerges who described them in the *E. T. Z.*, 1891, p. 699. The 10 years which succeeded its invention were devoted to the practical development of the Tesla induction motor and thus the commutator type did not receive much attention. Exactly 10 years after Georges' publication, A. Heyland described again (in the *E. T. Z.*, 1901, No. 32), the Goerges motor, showing that it can be used to compensate the watt-less component of the primary by proper brush-shift. Although Prof. Blondel contends[1] that this was a matter of course, it would have been an interesting contribution had not Mr. Heyland claimed with great emphasis that his motor was entirely different in principle from the shunt motor of Goerges. This has been disproved with great precision and clarity by Prof. Blondel in the papers cited. Mr. Heyland had suggested the use of stationary sliding contacts on the squirrel-cage rings, thus introducing an external e.m.f. at points equally spaced on the commutator, but the low resistance of these end rings acts as a powerful shunt and this arrangement proved ineffective. The author tried two independent windings with somewhat better success, and tests on a similar motor are reported by Prof. C. A. Adams.[2] It is now no longer open to doubt that Mr. Heyland's suggestion covers solely a shunt poly-phase motor with additional shunts placed between the commutator bars. No practical application seems to have been made of this modification.

Great activity in devising modifications and improvements of poly-phase commutator motors followed the general enthusiasm created by Mr. B. G. Lamme's single-phase railway motors. The use of these motors in order to obtain speed regulation without undue loss in efficiency, and finally their application to the speed regu-

[1] André Blondel, Théorie des Alternomoteurs Poly-phasés à Collecteur. *L'Eclairage Electrique*, 1903, pp. 121 to 495.

[2] C. A. Adams, *Trans. A. I. E. E.*, 1903.

lation of large induction motors with which they are concatenated, has secured for the Goerges motor a wide and interesting field.

In order to obtain the appropriate voltage on the rotor of a Goerges shunt poly-phase A. C. commutator motor, it is necessary either to use a separate transformer, or to utilize the stator winding by means of taps, or to employ a regulating winding. These methods are treated in Chap. XIII where the work of Osnos, La Cour, and Schrage is given consideration.

B. THE THEORY OF THE SHUNT POLY-PHASE A. C. COMMUTATOR MOTOR FOR CONSTANT POTENTIAL

Assume a stator like that of a standard induction motor in which a rotor is mounted wound like a direct-current armature equipped with a commutator. Let poly-phase current be supplied to both the rotor and the stator from the same supply circuit.

We thus obtain a "doubly-fed" type of poly-phase motor, which is called a shunt poly-phase A. C. commutator motor.

The two e.m.fs. in the stator and rotor, being derived from the same supply circuit, are of the same frequency and in time-phase.

If the rotor-brush position is such that the entries on both primary and secondary are opposite each other, then the m.m.f. belt of the secondary in relation to that of the primary depends solely upon the time-phase of the two circuits.

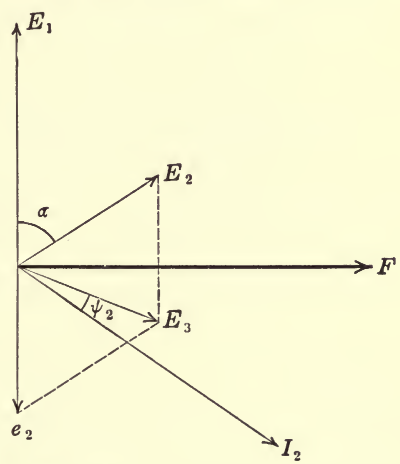


Fig. 116.—Composition of e.m.f's. in the rotor of the shunt poly-phase commutator motor.

E_1 = e.m.f. impressed on stator.
E_2 = e.m.f. impressed on rotor.
e_2 = e.m.f. induced in rotor.

If the rotor brushes are shifted the brush-shift angle displaces the impressed e.m.f. of the rotor relative to the stator by the amount of the angle of shift. Thus, in Fig. 116, let E_1 be the impressed e.m.f. on the stator, then, if α is the brush shift, E_2, the impressed e.m.f. on the rotor appears displaced by the angle α relative to the impressed e.m.f. E_1 so far as space relations are concerned. That is to say, the m.m.f. belts due to the currents which are produced by E_1 and E_2, must be determined as though, in a stationary transformer, the two e.m.fs. were displaced in time-phase by the angle α.

If we assume for the moment no secondary leakage, then the resultant flux F induces an e.m.f. e_2 through the relative angular velocity $\omega_1 - \omega_2$ of the rotor in respect to F, and the vector sum of E_2 and e_2 results in E_3, which produces a current which would be in phase with E_3 if there were no leakage in the secondary. If there is leakage, I_2 would lag behind E_3 by a time angle ψ_2.

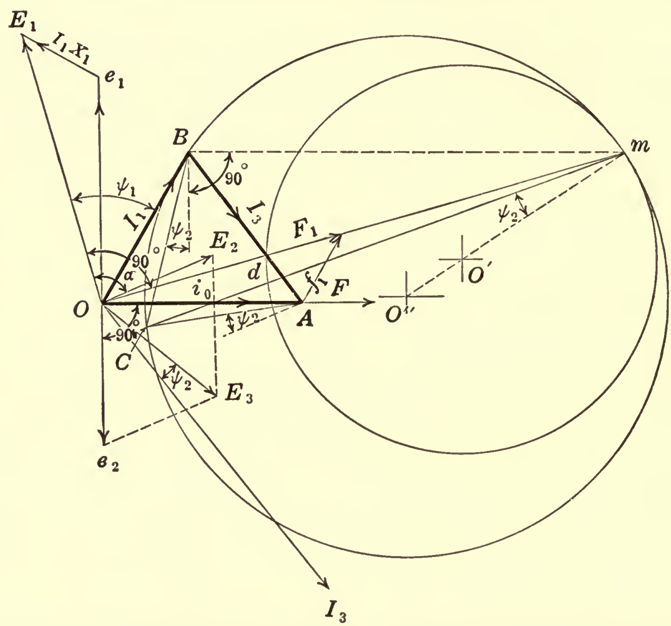


FIG. 117.—Time-phase vector diagram of the shunt poly-phase commutator induction motor. Secondary current I_3 is composed of currents BC due to e_2 and AC due to E_2. $I_1 = OB$, $I_3 = BA$, $i_0 = OA$.

The phase relation of E_1 and E_2 depends solely upon the brush position, if they are derived from the same supply circuit. It is difficult but important to bear in mind that we are tracing rotating fields in space and that time and space-phases are utilized in the same diagram so that we can show the mutual effect of rotor and stator m.m.fs. in one diagram instead of in two. This may be confusing, but the other treatment loses in physical reality.

The m.m.f. of the rotor due to I_3 and the m.m.f. of the stator due to I_1 result in the magnetizing m.m.f. equivalent to i_0 in the stator windings (Fig. 117). Assuming constant secondary reactance and neglecting as a good approximation that part of the secondary reactance which is proportional to the slip, I_3 is composed of BC due to e_2, and of AC due to E_2. Point C, therefore, is a fixed point as long as the brush-shift angle α and E_2 remain constant. Point m also is a fixed point, $Od : dm$ being equal to $v_1 - 1$, as is readily seen. Angle CBm is equal to $\frac{\pi}{2} + \psi_2$ and constant so that point B moves on the arc of a circle described over mC. The circle described about O' as center is the primary locus of the commutator-induction motor with short-circuited brushes. The circle described about O'' as center is the locus of the stator current of the shunt poly-phase motor, to which has to be added, or from which has to be subtracted, vectorially, the current in the primary of the transformer feeding the rotor. (Fig. 118.)

C. DETERMINATION OF THE TOTAL PRIMARY CURRENT

With the limiting assumption of a constant secondary lag ψ_2 between the secondary total e.m.f. E_3 and the total secondary current I_3 we may now proceed to determine the total current taken from the line supplying both stator and rotor. (Fig. 118.)

The stator current is OB.

The rotor current is $Bd \cdot v_1$. This current being fed into the rotor at the voltage E_2 which is smaller than E_1, if the ratio of transformation is n, then the current to be added to the stator current on account of the current I_3 fed into the rotor from E_2 is $\frac{I_3}{n}$, to be added to I_1 in such a manner that, as outside the motor E_2 and E_1 are in time-phase, the phase lag between I_3 and E_1 must be the same as that between I_3 and E_2. Thus results a simple graphical method[1] which sets off $Bh = \frac{I_3}{n}$ at the constant brush-shift angle α, triangle dBh for all secondary currents being similar, angle dBb always being α and angle Bdh being ϵ, also constant. Thus draw dO'', make triangle $dO''O'''$ similar to

[1] A. Blondel, *L'Eclairage Electrique*, 1903, p. 178.

triangle dBh, and O''' is the center of the new circle for the total current.

This total current, as was to be expected in view of the double feeding of this motor through its primary and secondary, is smaller than the stator current over the motor range of the shunt machine and it may be seen at a glance also that the rotor returns energy to the supply circuit. The whole arrangement

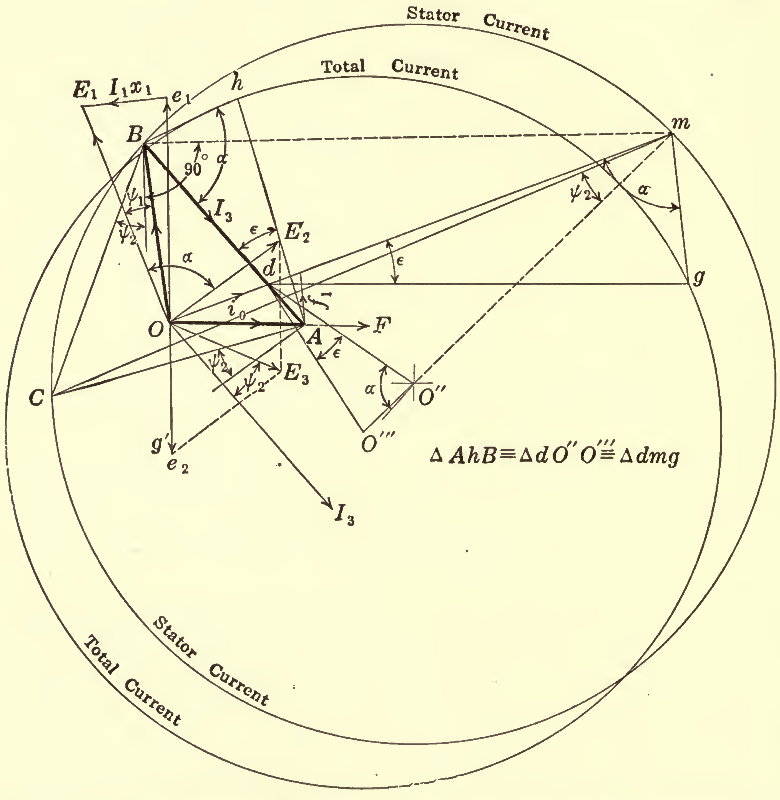


FIG. 118.—Time-phase vector diagram of the shunt poly-phase commutator induction motor. Circle loci of the stator current and of the total current.

may be simulated by a sort of equivalent arrangement of e.m.fs., resistances, and reactances, as is shown in Fig. 119, where E_1 and $\frac{E_2}{s}$ are mechanically coupled together, spaced apart by a space angle α. In order to obtain similarity of current and e.m.f. relations, it is necessary to divide E_2 by the slip s.

D. SPEED REGULATION AND THE SLIP

It is evident that, since BC, Fig. 117, represents the secondary current due to e_2 only, and as e_2 is proportional to the product of the slip into F, $tgCmB$ is a measure of the slip. By changing E_2 and α any slip may be obtained and thus the poly-phase shunt motor acts in this respect entirely differently from the induction motor, the shunt motor having three degrees of freedom, while the induction motor has only one.[1] A counter e.m.f. may be injected into the rotor of such magnitude and phase that the

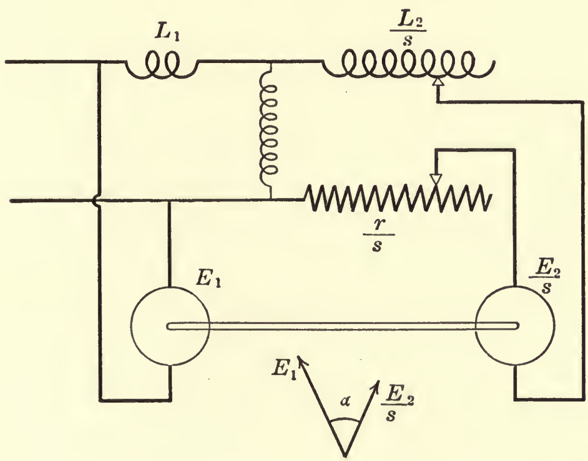


FIG. 119.—Equivalent electrical and mechanical combination simulating the action of the shunt poly-phase commutator induction motor.

motor will run, say, at 20 per cent slip, of which say 4 per cent is ohmic drop, while the remaining 16 per cent is due to the injected e.m.f. which may be made in phase with the ohmic drop. However, incidentally, the phase of the injected e.m.f. may be shifted so as to magnetize the motor and thus to supply through the secondary the magnetizing current ordinarily supplied through the primary. As this can be done with much less K. V. A. due to the low voltage of the rotor caused by the high slip, it appears obvious that such a motor may have a very high power factor.

[1] This likeness to problems in dynamics is due to PROF. V. KARAPETOFF.

E. BIBLIOGRAPHY

Before leaving the subject of these interesting motors, a short list of papers may be given chronologically.

The motor was described in *E. T. Z.*, 1891, p. 699, by H. GOERGES, by whom it was also patented Jan. 21, 1891, in the German patent No. 61,951.

It was brought back to light 10 years later largely through the sensational paper by A. Heyland, *E. T. Z.*, 1901, No. 32, in which the author described the same doubly-fed motor, calling it, however, an induction motor excited from the secondary. Through shifting the brushes any primary-phase angle may be obtained. No tests were made or described.

A fundamental advance was made in the theory of these motors by PROF. A. BLONDEL, in *L'Eclairage Electrique*, Apr. 25, 1903 *et seq.*, where, however, a curious assumption was made. PROF. BLONDEL assumes that to the secondary impressed voltage E_2 the rotor offers resistance only. He thus composes $\frac{E_2}{r_2}$ and i_2 into I_3 and then applies to this current the flux theory of the induction motor assuming a secondary leakage field to be produced by this m.m.f. His results are thus marred by this assumption, which also implies that the secondary leakage lag diminishes with the slip of the motor, which is at best only partially true and which leads to the establishment of a specious equivalence between the squirrel-cage or slip-ring motor and the commutator-induction motor short-circuited across its brushes, a result which the tests do not seem to bear out. BLONDEL is thus led to semi-circular loci where we have arrived at arcs. The value of BLONDEL's methods is fortunately in no way impaired by these assumptions.

Three months after the appearance of the BLONDEL circle diagrams of this motor in Apr. 25, 1903, MR. HEYLAND published in the *E. T. Z.* No. 30, July 23, 1903, a diagram identical with BLONDEL's diagram in spite of the curious assumption made by BLONDEL. No reference whatever appears to have been made by MR. HEYLAND to BLONDEL's papers here referred to.

An exhaustive study of the shunt motor and the derivation of a correct diagram appeared in the *E. T. Z.*, 1903, p. 368 *et seq.*, by PROF. O. S. BRAGSTAD.

M. EDOUARD ROTH, of Belfort, France, published a masterly thesis in *L'Eclairage Electrique*, April to June, 1909.

The work of DR. E. KITTLER and DR. W. PETERSEN, Stuttgart, F. ENKE, devotes a great deal of space to these motors.

Vol. V, Part 2, of E. ARNOLD, LA COUR and FRAENCKEL, Berlin, J. SPRINGER, 1912, is a mine of valuable information on alternating-current commutator machines in general.

DR. F. EICHBERG'S "Gesammelte Elektrotechnische Arbeiten," 1897–1912, Berlin; J. SPRINGER, 1914, may also be consulted.

The papers by L. DREYFUS and F. HILLEBRAND in *Elektrotechnik & Maschinenbau*, 1910, pp. 367 *et seq.*, may be consulted with profit.

The latest contributions are the papers by N. SHUTTLEWORTH, "Polyphase Commutator Machines and their Application," *The Journal of the Institution of Electrical Engineers*, Mar., 1915, and the paper by W. C. K. ALTES, "The Polyphase Shunt Motor," *Trans. A. I. E. E.*, 1918.

CHAPTER XII

METHODS OF SPEED CONTROL

A. CONCATENATION

The similarity in theory between an induction motor and a transformer is due to the fact that, as the secondary frequency in the rotor of the induction motor varies from full primary frequency at standstill to zero at synchronism, the constant resistance of the rotor is in effect equivalent to a variable resistance $r_2 \div s$ in the secondary of a transformer, where r_2 is the secondary resistance of the rotor and s the slip, *viz.*, the difference between primary and secondary frequencies divided by the primary frequency. Thus we obtain the conception of the equivalent circuits which simulate the physical phenomena of magnetizing current, leakage fields, etc.

If the induced e.m.f. in the secondary of one motor at the frequency of its slip is impressed upon a second motor (we will assume it to be impressed upon the stator with a winding having a number of conductors equal to that of the rotor of the first motor), then the second motor will not operate like a standard induction motor of constant impressed voltage and constant frequency, but both its impressed voltage and frequency will vary in a peculiar manner. If the rotors of both motors are mounted rigidly on the same shaft, then they have a common mechanical angular velocity ω_2. We thus obtain the following relations assuming the same number of poles in both motors; and designating the angular velocity of the primary field of Motor I by ω_1, its slip by s_1, and the slip of Motor II by s_{II}.

$$s_{I} = \frac{\omega_1 - \omega_2}{\omega_1} \tag{129}$$

But

$$\omega_2 = \omega_1(1 - s_I) \tag{130}$$

The relative angular velocity at which the rotor conductors of Motor II cut through the field impressed by the e.m.f. of the secondary of Motor I is $\omega_1 - \omega_2 - \omega_2 = \omega_1 - 2\omega_2$.
Therefore, the slip of Motor II,

$$s_{II} = \frac{\omega_1 - 2\omega_2}{\omega_1 - \omega_2} \tag{131}$$

And, because of equation (130),

$$s_{\mathrm{II}} = \frac{\omega_1 - 2\omega_1(1 - s_{\mathrm{I}})}{\omega_1 - \omega_1(1 - s_{\mathrm{I}})} \tag{132}$$

$$\therefore s_{\mathrm{II}} = \frac{s_{\mathrm{I}}}{2s_{\mathrm{I}} - 1} \tag{133}$$

$$s_{\mathrm{I}} = \frac{2 - s_{\mathrm{II}}}{1} \tag{134}$$

$$s_{\mathrm{I}}s_{\mathrm{II}} = 2s_{\mathrm{I}} - 1 \tag{135}$$

These relations, which are here obtained only for equal numbers of poles in both motors, are of fundamental importance in understanding the operation of concatenation or cascade operation of induction motors.

Beginning the examination from the secondary of Motor II, the relative angular velocity between its rotor conductors and its resultant rotating field is $\omega_1 - 2\omega_2$. Therefore, the e.m.f., induced in this winding, can be written as follows:

$$e_4 = 2.12(\sim_1 - 2\sim_2)z_4F_4 10^{-8} \text{ volts} \tag{136}$$

$$e_4 = 2.12\left(\frac{\sim_1 - 2\sim_2}{\sim_1 - \sim_2}\right)\left(\frac{\sim_1 - \sim_2}{\sim_1}\right)\sim_1 z_4F_4 10^{-8} \text{ volts} \tag{137}$$

$$e_4 = 2.12(s_{\mathrm{II}}s_{\mathrm{I}})\sim_1 z_4F_4 10^{-8} \text{ volts} \tag{138}$$

In the equivalent circuits with constant frequency $\sim_1$ the resistance which is to be the equivalent of r_4, the rotor resistance of Motor II, in which the frequency is equal to $(\sim_1 - 2\sim_2)$, is therefore obtained by dividing r_4 by $(s_{\mathrm{I}}s_{\mathrm{II}})$.

The effect of the leakage fields remains of course unaffected by the different frequencies in the different parts of the concatenated circuits.

The resistances of the primary of Motor II and the secondary of Motor I are situated in circuits of variable frequency $(\sim_1 - \sim_2)$, therefore, the e.m.fs. to overcome these are:

$$e_3 = 2.12(\sim_1 - \sim_2)z_3F_3 10^{-8} \text{ volts} \tag{139}$$

$$e_2 = 2.12(\sim_1 - \sim_2)z_2F_2 10^{-8} \text{ volts} \tag{140}$$

These equations may be written:

$$e_3 = 2.12\left(\frac{\sim_1 - \sim_2}{\sim_1}\right)\sim_1 z_3F_3 10^{-8} \text{ volts}$$

$$e_2 = 2.12\left(\frac{\sim_1 - \sim_2}{\sim_1}\right)\sim_1 z_2F_2 10^{-8} \text{ volts}$$

or
$$e_3 = 2.12(s_{\text{I}}) \sim_1 z_3 F_3 10^{-8} \text{ volts} \tag{141}$$
$$e_2 = 2.12(s_{\text{I}}) \sim_1 z_2 F_2 10^{-8} \text{ volts} \tag{142}$$

In other words, in the equivalent electric circuits, simulating the circuits of concatenation, the e.m.fs. impressed upon the resistances of the primary of Motor II and of the secondary of Motor I, are obtained by dividing r_2 and r_3 respectively by s_{I}.

A diagram of equivalent circuits is shown in Fig. 120. At low frequency a large current may pass in spite of the large leakage field which it produces, but this current, in order to pass through the ohmic resistance of the primary of Motor II and the secondary of Motor I, requires a magnetic field in the rotor of Motor I whose magnitude is $\sim_1 \div s_{\text{I}}$ times that which would be required for the same current if the frequency were $\sim_1$. The important fact thus becomes outstanding that, as the impressed frequency on Motor II diminishes, the effect of its primary resistance is

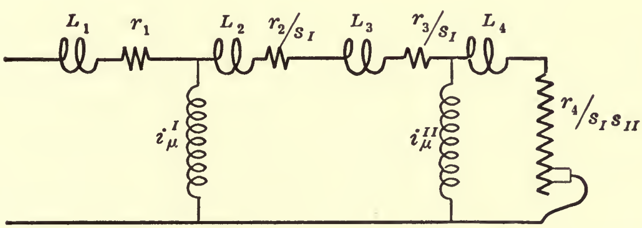


Fig. 120.—Concatenation: Equivalent circuits, leakage and resistance.

enhanced to the point that, at synchronism of Motor I, its effect is equivalent to infinite resistance or an open circuit of the primary of Motor II.

An inspection of this method of connection of the two motors shows that, exactly at half synchronous speed, the relative angular velocity of the primary field of Motor II in respect to its rotor is zero. Therefore, there are no currents induced in Rotor II, and the magnetizing current i_μ^{II} is the load current of Rotor I. Motor II, therefore, runs idle, as it were, at half the frequency impressed upon Motor I, and the torque of Motor I, since its rotor current is in quadrature with its rotor field, must vanish. The magnetizing current of the two motors, supplied through the stator of Motor I, is therefore approximately twice the magnetizing current of Motor I by itself, and equal to twice that of the group at $s_{\text{I}} = 0$.

Now, consider the relation betwen $s_I s_{II}$ and s_I, between which there exists the equation

$$s_I s_{II} = 2s_I - 1 \tag{143}$$

For

$$s_I = +1$$
$$s_I s_{II} = +1$$

For

$$s_I = +\tfrac{1}{2}$$
$$s_I s_{II} = 0$$

For

$$s_I = 0$$
$$s_I s_{II} = -1$$

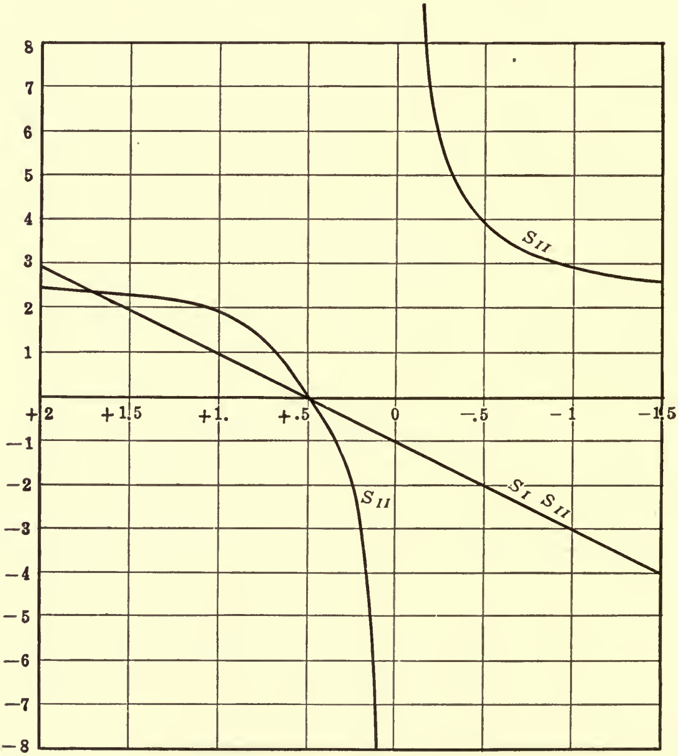


FIG. 121.—The slip of the second motor as a function of the slip of the first motor in concatenation.

$$s_I s_{II} = 2s_I - 1$$

These relations are shown in diagram Fig. 121, which also shows the slip s_{II} as a function of s_I. It is noticed that $s_{II} = \infty$ for $s_I = 0$.

We are now prepared to examine more closely into the relation of the fields, currents, and e.m.fs. existing in the concatenated circuits. Let F_4, Fig. 122, be the resultant magnetic field in the secondary of Motor II. It induces an e.m.f. e_4 according to equation (138).

$$e_4 = 2.12(s_{\text{I}}s_{\text{II}})\sim_1 z_4 F_4 10^{-8} \text{ volts} \tag{138}$$

This e.m.f. produces a current

$$\dot{i}_4 = e_4 \div r_4 \tag{144}$$

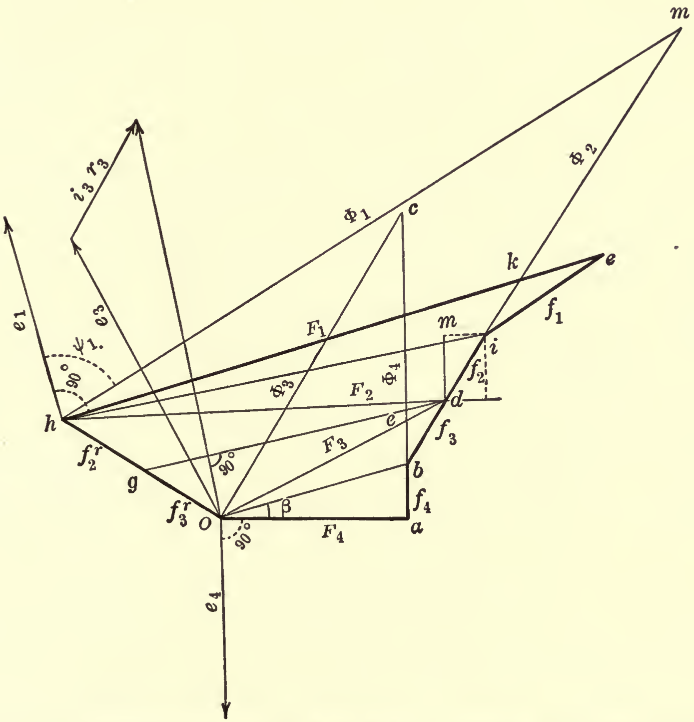


FIG. 122.—Concatenation: The flux diagram. Leakage and resistance taken into account.

whose m.m.f. is in quadrature with F_4. The leakage field, $f_4 = ab$, is in phase with this m.m.f. The flux $\Phi_4 = bc$ completes the triangle of the magnetic fluxes in which $Oc = \Phi_3$, the flux which is proportional to the primary m.m.f. of Motor II.

The primary resultant and actually existing flux of Motor II is $Od = F_3$.

In order to take account of the primary resistance of Motor II, we represent it by that magnetic field which, at the same frequency as F_3, would induce the same ohmic drop i_3r_3. As the frequency of F_3 is $(\sim_1 - \sim_2)$ we have

$$e_3 = 2.12(s_{\mathrm{I}})\sim_1 z_3 F_3 10^{-8} \tag{145}$$

Therefore

$$i_3r_3 : e_3 :: f_3^r : F_3$$

$$\therefore\ f_3^r = \frac{i_3r_3}{e_3} \cdot F_3$$

$$= \frac{i_3r_3}{2.12(s_{\mathrm{I}})\sim_1 z_3 10^{-8}}$$

$$= \frac{i_3r_3}{s_{\mathrm{I}}} \cdot K \tag{146}$$

where $$K = 1 \div 2.12\sim_1 z_3 10^{-8} \tag{147}$$

Hence, f_3^r may be represented by a vector proportional to the ohmic drop divided by the slip s_{I}. It is now clear that with $s_{\mathrm{I}} = O$, f_3^r becomes infinite, which is the equivalent of a completely open circuit of the primary of Motor II.

The impressed e.m.f. of Motor II is generated by the secondary of Motor I. The relation of its phase to its current is determined by the consideration that its current is the same as the current in the primary of Motor II and its phase is the same as that of the voltage impressed on the primary of Motor II relative to its current. The current in circuits II and III being the same, the leakage fields and resistance drop fields must also be the same. Hence

$$Og = gh = f_3^r = f_2^r \quad \text{and}$$
$$bd = di = f_3 = f_2$$

The actual resultant magnetic field in the secondary of Motor I is now represented by $hi = F_2$.

The fictitious flux Φ_2 corresponding to the secondary m.m.f. of Motor I is represented by $Oc = im$, while $hm = \Phi_1$, the primary fictitious flux proportional to i_1. The primary leakage flux is $ei = f_1$, and $hl = F_1$ is the primary resultant flux generating a counter e.m.f.

$$e_1 = 2.12\sim_1 z_1 F_1 10^{-8} \text{ volts} \tag{49}$$

whose phase is in quadrature with i_1 and Φ_1 or hm.

An examination of the diagram shows at a glance the vectorial composition of the secondary flux F_4 with the leakage fluxes f_4, f_3, f_2 and f_1, and the resistance drop fluxes f_3^r and f_2^r, into the resultant primary flux F_1. It is most interesting and instructive to note that this composition can take place actually in the same motor in the case of internal concatenation, which can most readily be realized by two windings with numbers of poles in the ratio of 2 : 1, as in this case the windings are mutually independent in respect of mutual induction. A similar case of great theoretical interest is the case of a poly-phase motor with a single-phase secondary which will be discussed in Chap. XII *B*.

The diagram of the composition of fluxes, Fig. 122, neglects as usual the primary resistance of Motor I, which can be taken into consideration by the simple graphical correction given in Chap. III, if such correction should prove desirable.

It is not necessary, in order to determine a number of points for different speeds to develop more than the flux polygon *O-a-b-d-i-e-h*. The leakage flux $ab = f_4$ is proportional to the secondary current of Motor II. The ohmic drop i_4r_4 is proportional to the flux F_4 multiplied by the secondary frequency of Motor II, *viz.*, $(\sim_1 - 2\sim_2)$. Therefore, equation (138)

$$e_4 = i_4r_4 = 2.12(s_{\mathrm{I}}s_{\mathrm{II}})\sim_1 z_4F_4 10^{-8} \text{ volts} \tag{148}$$

$$i_4 = K_4\Phi_4 \tag{149}$$

$$f_4 = (v_4 - 1)\Phi_4$$

$$\therefore i_4 = K_4\frac{f_4}{v_4 - 1} \tag{150}$$

From (148) follows:

$$(s_{\mathrm{I}}s_{\mathrm{II}}) = \frac{i_4r_4}{2.12\sim_1 z_4F_4 10^{-8}} \tag{151}$$

$$= \frac{K_4f_4}{(v_4 - 1)2.12\sim_1 z_4 10^{-8}}$$

$$= K_4'\left(\frac{f_4}{F_4}\right) \tag{152}$$

$$(s_{\mathrm{I}}s_{\mathrm{II}}) = K_4' tg\beta$$

Thus $s_{\mathrm{I}}s_{\mathrm{II}}$ being known from (152), we obtain

$$s_{\mathrm{I}} = \frac{1}{2 - s_{\mathrm{II}}} \tag{153}$$

The procedure is now as follows: Assume F_4 and s_{I}; calculate $s_{\mathrm{I}}s_{\mathrm{II}}$ from which we obtain f_4; determine F_3 in the usual manner, *eb* being equal to $(1 - \frac{1}{v_3})\Phi_4$. The direction and magnitude of the primary current of Motor II thus being known from f_3, determine the resistance drop fields $Og = f_3'$ and $gh = f_2'$, thus obtaining F_2. The leakage field $f_2 = di$, and the point k are then determined as before, $ik = (1 - \frac{1}{v_1})\Phi_2$, and $hl \div hk = v_1$, obtaining $F_1 = hl$, in quadrature with which we find e_1. The angle of lag is ψ_1, between Φ_1 and e_1. The construction of this diagram has been carried out for a number of values of the slip s_I and the results have been plotted in Figs. 123, 124, 125, 126, and 127.

The torque of Motor II is proportional to the vector product of i_4 and F_4, it is therefore represented by the vector product of

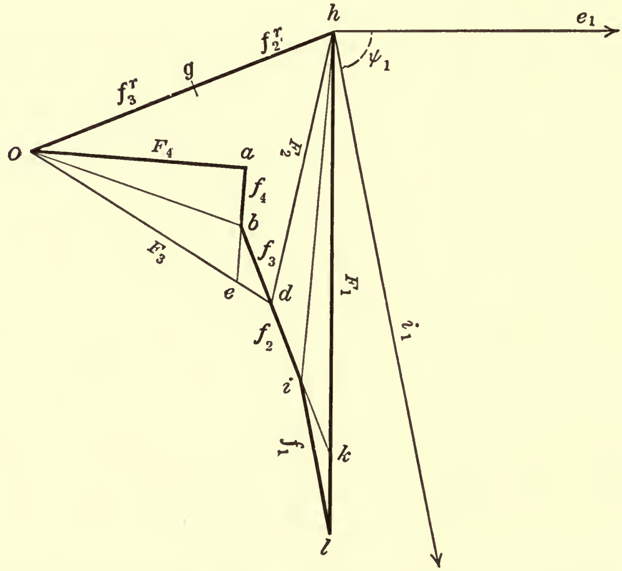


Fig. 123.—Concatenation: Flux diagram motor II as generator. Leakage and resistance taken into account.

F_4 and f_4. The torque of Motor I is proportional to the product of the quadrature component of i_2, *viz.*, the component *md* of the leakage flux f_2 into the field F_2. This calculation has been carried out and the values of the two torques have been plotted in Figs. 125 and 126.

It has been customary, in popular theories of concatenation,[1] to assume that the torque of Motor I follows, through the entire range of slip from $s_I = I$ to $s_I = 0$, the torque curve of an ordi-

[1] See F. Eichberg, *Zeitschrift für Elektrotechnik*, Vienna, December, 1898. Also B. G. Lamme, *Electric Journal*, Pittsburgh, September, 1915.

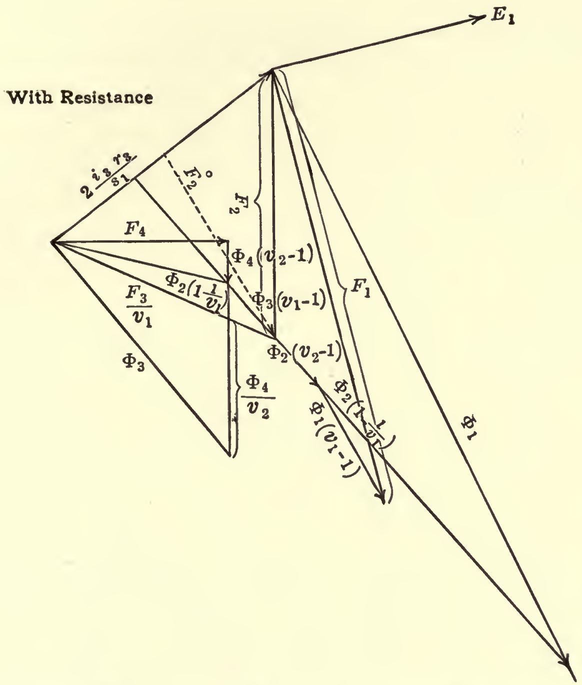


FIG. 124.—The fluxes of concatenated motors. Leakage and resistance taken into account motor II acting as generator.

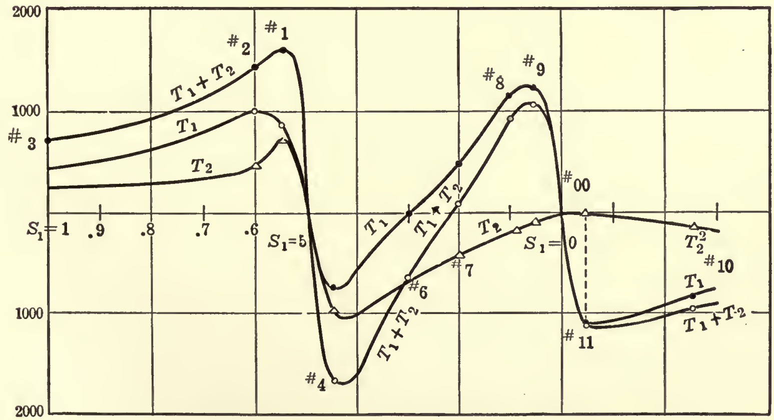


FIG. 125.—The torque curves of two concatenated motors. T_1 is the torque of motor I. T_2 is the torque of motor II. $T_1 + T_2$ is the resultant torque.

nary induction motor at constant potential, while the torque curve of Motor II is assumed to be that of a constant potential motor whose synchronism is reached at half the synchronous

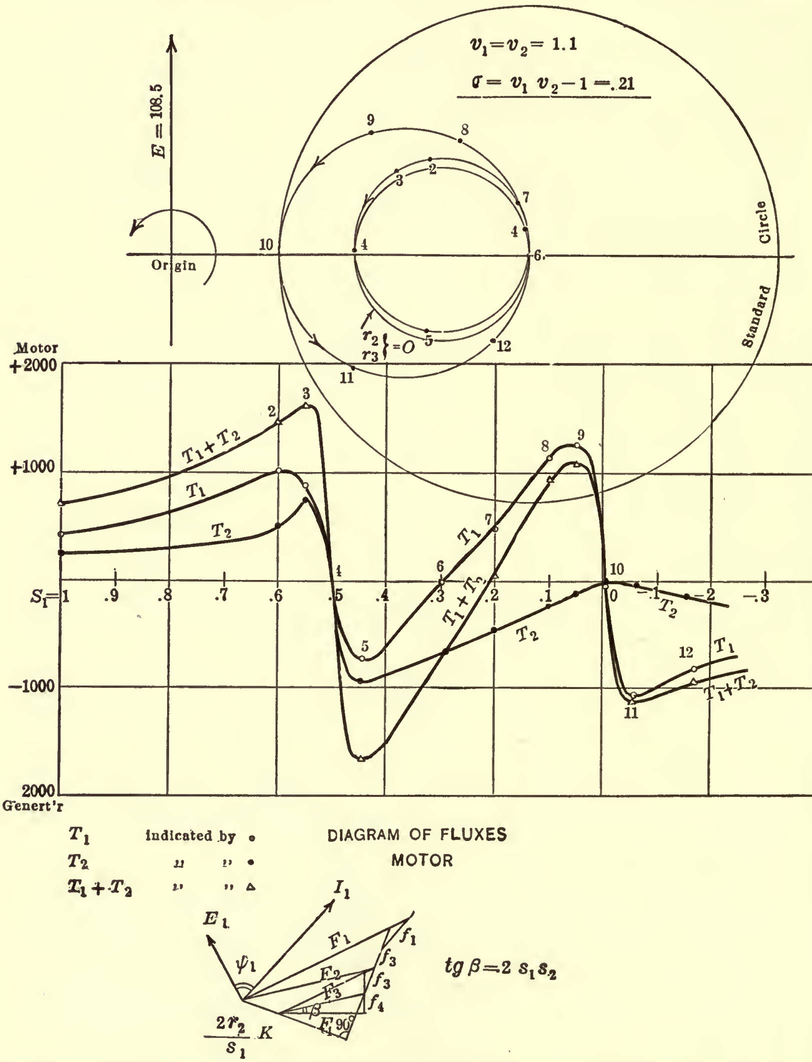


FIG. 126.—Concatenation: The torques and the loci of the primary current. Leakage and resistance taken into account.

speed of Motor I operating by itself, in Fig. 128. It is clear from our analysis and also from the consideration of the fact that the torque of Motor I for $s_1 = 0.5$ must be zero, that this plausible explanation is incorrect, and misleading in regard to the physical aspects of this problem.

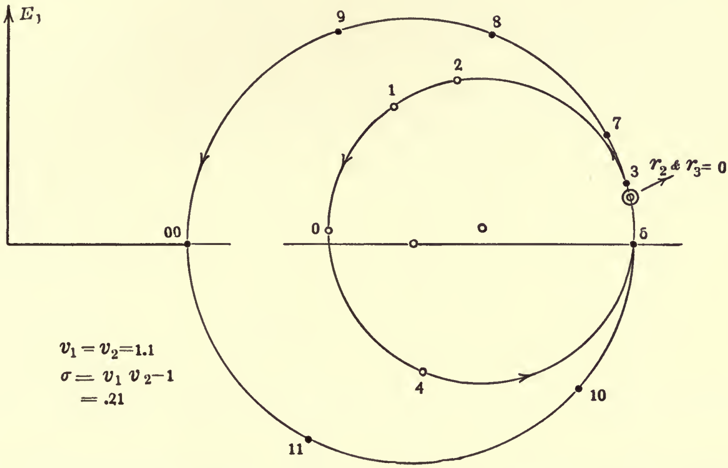


FIG. 127.—Concatenation: The primary current locus of two concatenated motors with leakage and resistance taken into account.

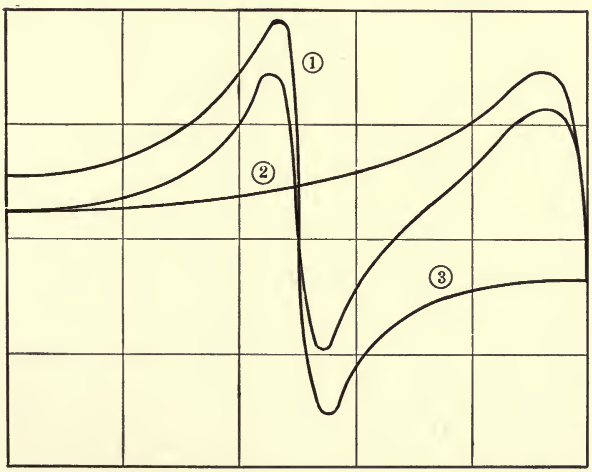


FIG. 128.—Convential but incorrect method of representing the torque of two concatenated motors. (3) Torque of one motor operating at double slip. (2) Torque of other motor operating at normal slip. (1) Torque of the concatenated group.

It is necessary to consider carefully the effect of the second motor on the characteristics of the group. Above half synchronous speed, $s_{\mathrm{I}} = 0.5$, Motor II runs above its synchronous speed relative to its supply frequency $\sim_1 - \sim_2$, and therefore acts throughout its range to $s_{\mathrm{I}} = 0$ as an induction generator. Yet the effect of the resistances r_2 and r_3 consists in changing over the torque of Motor I between $s_{\mathrm{I}} = 0.3$ and $s_{\mathrm{I}} = 0$, from a generator torque into a motor torque. This point is so important that we give in Fig. 123 and 124 a complete polar diagram for this condition, which should now be self-explanatory.

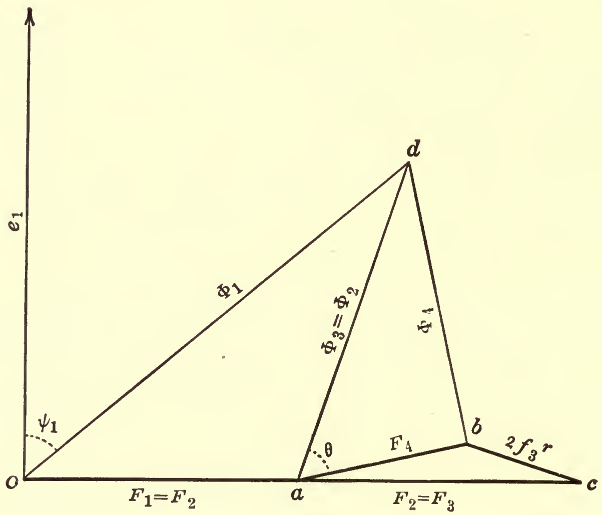


Fig. 129.—Concatenation: Resistance only and no leakage. Motor II acting as motor.

From Fig. 125 it is evident that the torques of both motors are motor torques up to $s_{\mathrm{I}} = 0.5$, while from $s_{\mathrm{I}} = 0.5$ to $s_{\mathrm{I}} = 0.3$ both torques are generator torques. Between $s_{\mathrm{I}} = 0.3$ and $s_{\mathrm{I}} = 0$ Motor I is a motor, while Motor II remains a generator. At negative slips the group acts individually and collectively as a generating unit. Figure 127 shows the polar diagram of the primary current of Motor I. The locus of this curent is a curve of the fourth power. The slip s_{I} is marked everywhere and it is interesting to note how the current from standstill gradually diminishes to half-synchronous speed, as in a single ordinary induction motor. The magnetizing current at half-synchronous speed is almost double that of a single motor. Above this speed the group acts like an induction generator until, as a result of the effect of the secondary resistance of Motor I and the primary resistance of Motor II above $s_{\mathrm{I}} = 0.21$ the group becomes a motor, and the primary current, while at first increasing, is gradually choked off by these resistances until at $s_{\mathrm{I}} = 0$ their

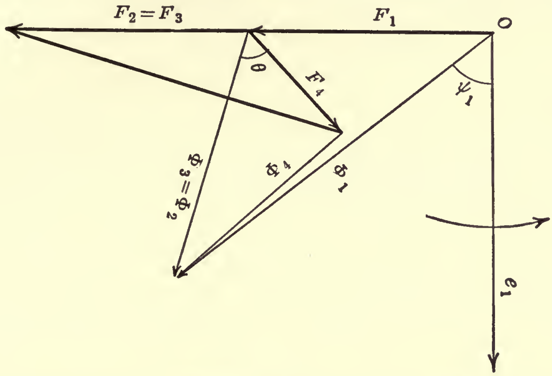


FIG. 130.—Concatenation: Resistance only and no leakage. Motor II acting as generator.

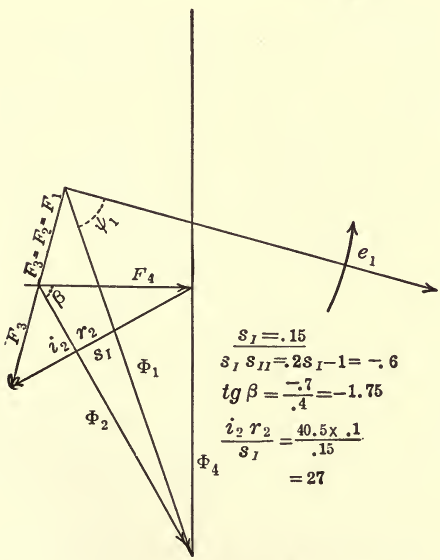


FIG. 131.—Concatenation: Resistance only, no leakage.

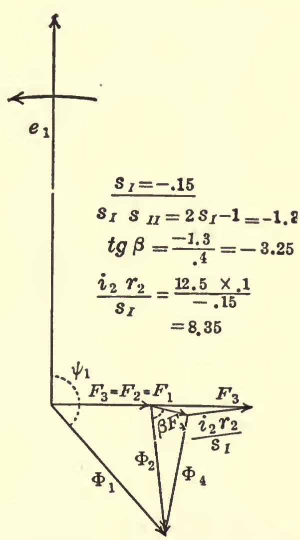


FIG. 132.—Concatenation: Resistance only, no leakage.

effect becomes equivalent to an open circuit of the primary of Motor II and the primary current of Motor I drops to the value of its magnetizing current.

To bring out this very interesting but somewhat involved process more clearly, we shall consider two specific cases. First, two equal motors in concatenation, *without* leakage, but *with* resistance r_2 and r_3; and, secondly, two equal motors *with* leakage but *without* resistance r_2 and r_3. Neither case corresponds to the

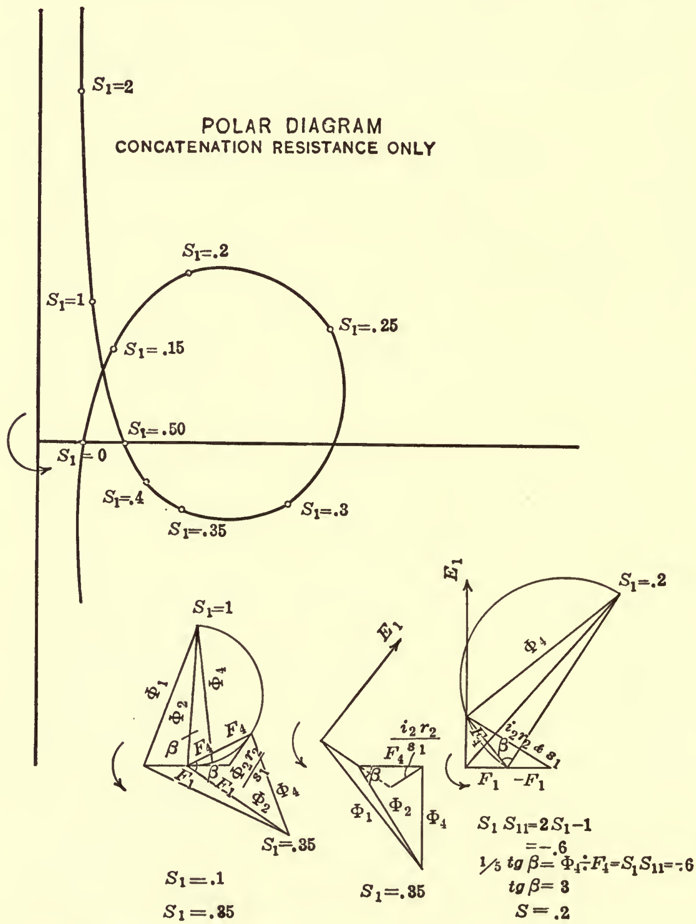


FIG. 133.—Concatenation: Resistance only, no leakage. Locus of primary current.

case treated fully in this chapter, but as "boundary" solutions, they afford a good insight into the operation of concatenation.

We assume $r_2 = r_3$ and draw F_4. (Figs. 129 and 130.) Determine $\Phi_4 = bd$ as before, and draw $ad = \Phi_2 = \Phi_3$. The resistance drop field $f_3^r = f_2^r$ is shown as $bc = 2f_3^r$. Join $ac = F_2 = F_3$ and

$Oa = F_1 = F_2$. The primary voltage e_1 is in quadrature with F_1, while $Od = \Phi_1$ represents the phase and magnitude (including a proportionality factor) of the primary current i_1. Angle $bad = \theta$, and $tg\theta = \Phi_4 \div F_4 = s_{\mathrm{I}}s_{\mathrm{II}}$. The resistance drop field $bc = 2f_2^r$ is also equal to

$$2f_2^r = K\frac{i_2r_2}{s_{\mathrm{I}}} \tag{154}$$

where K is a proportionality factor readily obtained as before.

Figure 133 shows the polar diagram of the primary current for different slips. Its relation to the fourth degree curve of Fig. 127 is striking. Figures 131 and 132 are auxiliary diagrams used in the development of the diagram. As in the ordinary induction motor without leakage, the branches of the curve are asymptotic to the ordinate axis.

Concatenation of Two Equal Motors

Resistance Only

No Leakage

s_{I}	$tg\beta$	$\cos\psi_1$	i_1
2.000	7.500	0.964	143.5
1.000	2.500	0.785	65.5
0.900	2.000	0.706	59.0
0.800	1.500	0.620	54.0
0.700	1.000	0.480	50.0
0.600	0.500	0.300	48.5
0.500	0.000	0.093	49.0
0.400	0.500	−0.105	56.0
0.300	−1.000	0.200	72.5
0.250	−1.250	−0.166	87.0
0.200	−1.500	−0.895	106.0
0.175	−1.630	0.340	106.0
0.150	−1.750	0.506	94.6
0.125	−1.877	0.690	74.5
0.100	−2.000	0.700	53.5
0.050	−2.250	0.530	30.8
0.000	0.000	0.000	25.0
−0.100	−3.000	−0.535	30.6
−0.200	−3.500	−0.760	44.0
−0.300	−4.000	−0.820	53.0
−0.600	−5.500	−0.920	78.5
−1.000	−7.500	−0.960	110.5

Figure 134 shows the current curves in polar coördinates for this limiting case, and Fig. 135 the equivalent electric circuits which are always instructive.

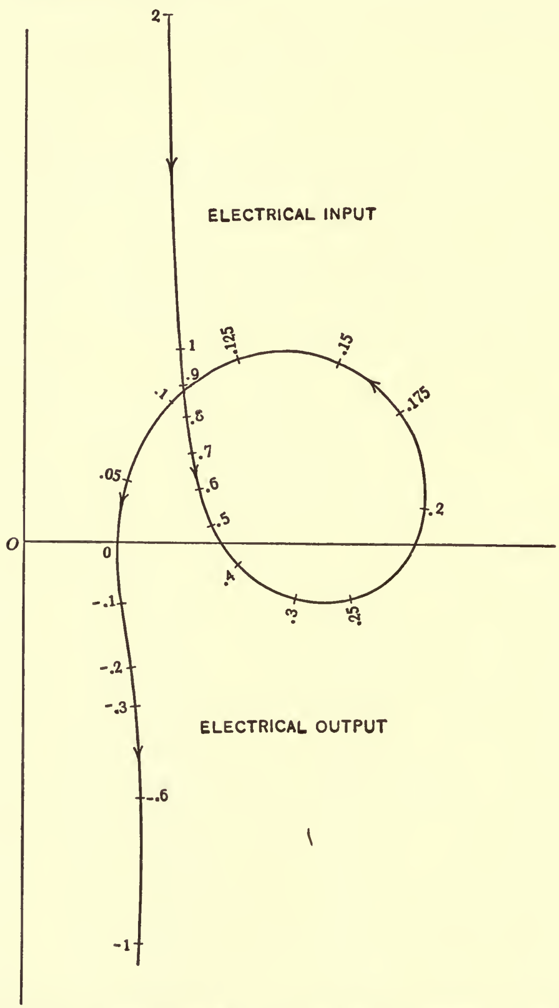


Fig. 134.—Primary current locus. Concatenation. Resistance only. Numerals represent slip s_I.

Secondly, we shall now consider the action of the group of the two motors if the resistances r_2 and r_3 are neglected, the leakage, however, being fully taken into account. This is a very important case as it will be shown that, for a range of slip from

$s_{\text{I}} = 1$ to $s_{\text{I}} = 0.4$, which is the important range for practical purposes, this diagram becomes very simple, the locus of the primary current being again a circle.

We begin again with the secondary flux, F_4 of Motor II. It induces an e.m.f.

$$e_4 = 2.12(s_{\text{I}}s_{\text{II}})\sim_1 z_4 F_4 10^{-8} \text{ volts} \tag{155}$$

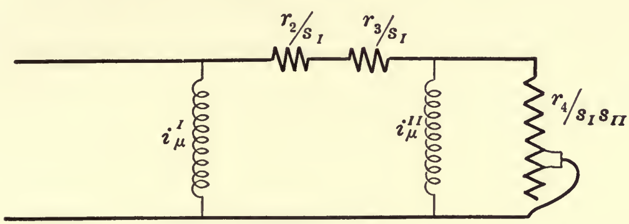


FIG. 135.—Concatenation: Equivalent circuits resistance only; no leakage.

This e.m.f. produces a current

$$i_4 = \frac{e_4}{r_4} \tag{156}$$

This current produces a leakage field $f_4 = ab = (v_2 - 1)\Phi_4$. Φ_4 and F_4 are combined into Φ_3 and $f_3 = bc$ is the leakage field in magnitude and phase of the primary m.m.f. of Motor II.

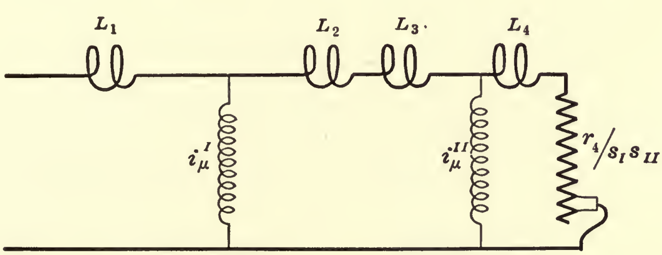


FIG. 136.—Concatenation: Equivalent circuits leakage reactance only; no resistance.

As our premise was the assumption of a negligible resistance of the windings between the two motors, see Ffg. 134 which assumption we know to be permissible only if the slip s_{I} is not less than 60 per cent, we can directly combine the field F_2, which is the resultant primary magnetic field of Motor II, with its secondary fictitious flux Φ_1, and we obtain the fictitious primary field whose leakage field is $f_1 = mu$.

The diagram is developed exactly as the flux diagram of the induction motor, and every step should be carefully thought over by the reader. (Fig. 137.)

An examination of this geometrical figure now shows the interesting and remarkable fact that, with constant impressed voltage on Motor I, neglecting its primary resistance which is permissible, the point of intersection f remains fixed for all speeds of Rotor II, and the primary flux Φ_1 of Motor I moves under this condition upon the periphery of the circle around D as center

This can readily be proved as follows:

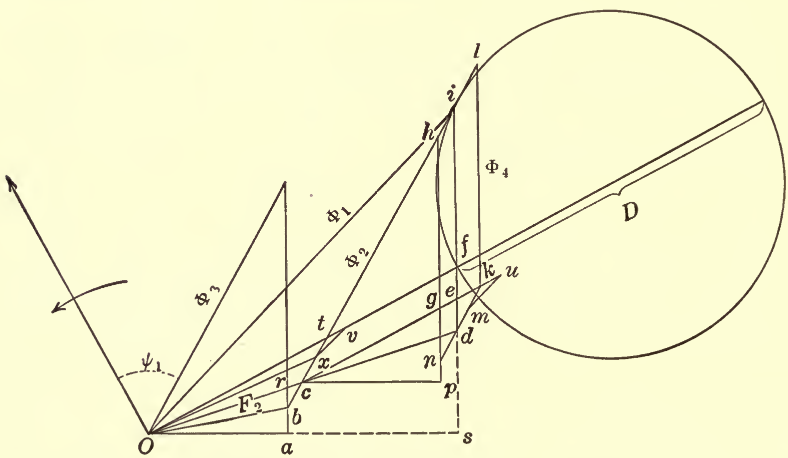


FIG. 137.—Flux diagram of two motors in concatenation. No primary and secondary resistance in motor I, and no primaary resistance in motor II.

Draw as and sd and obtain an expression for $sf = sd + df$.

$$\Delta Odf \sim \Delta cde$$
$$ed : df :: F_2 : 2F_2$$
$$\therefore df = 2ed \tag{157}$$
$$nd = \Phi_2(v_1 - 1)$$
$$dm = \Phi_2(v_2 - 1)$$
$$mk = \Phi_2\left(1 - \frac{1}{v_1}\right)$$
$$cl = \Phi_2 + hl$$
$$hl = nd + dm + mk$$
$$\therefore cl = \Phi_2\left[v_1 + v_2 - \frac{1}{v_1}\right] \tag{158}$$
$$cl = \Phi_2\lambda \tag{159}$$

$$\lambda = \left[v_1 + v_2 - \frac{1}{v_1}\right] \tag{160}$$

$$ei:kl::ci:cl$$
$$ei:\Phi_4::\Phi_2 v_1:\Phi_2\lambda$$

$$\therefore ei = \Phi_4 \frac{v_1}{\lambda} \tag{161}$$

$$\therefore ed = \Phi_4\left(1 - \frac{v_1}{\lambda}\right) \tag{162}$$

$$if = kl - 2ed$$

$$if = \Phi_4\left(2\frac{v_1}{\lambda} - 1\right) \tag{163}$$

$$ab = \Phi_4(v_2 - 1)$$

$$br = \Phi_4\left(1 - \frac{1}{v_1}\right)$$

$$\therefore ar = \Phi_4\left(v_2 - \frac{1}{v_1}\right)$$

$$Or = \frac{F_2}{v_1}$$

$$ar:ds::\frac{F_2}{v_1}:2F_2$$

$$\therefore ds = 2\Phi_4(v_1v_2 - 1) \tag{164}$$

$$df = 2ed = 2\Phi_4\left(1 - \frac{v_1}{\lambda}\right) \tag{165}$$

$$\therefore sf = 2\Phi_4\left(v_1v_2 - \frac{v_1}{\lambda}\right) \tag{166}$$

This equation shows that sf is proportional to Φ_4 and it is therefore a measure of Φ_4.

$$Of:ce::2:1$$
$$\therefore Of = 2ce$$
$$ce:ck::ei:kl$$

$$ce:\frac{F_1}{v_1}::\Phi_4\frac{v_1}{\lambda}:\Phi_4$$

$$\therefore ce = \frac{F_1}{\lambda} \tag{167}$$

But $$Of = 2ce$$

$$\therefore Of = 2\frac{F_1}{\lambda} \tag{168}$$

From similar triangles:

$$Of:D::sf:if$$

$$\frac{Of}{D} = 2v_1\frac{\left(v_2 - \frac{1}{\lambda}\right)}{\left(2\frac{v_1}{\lambda} - 1\right)}$$

$$\frac{Of}{D} = \frac{2v_1(v_2\lambda - 1)}{(2v_1 - \lambda)} \tag{169}$$

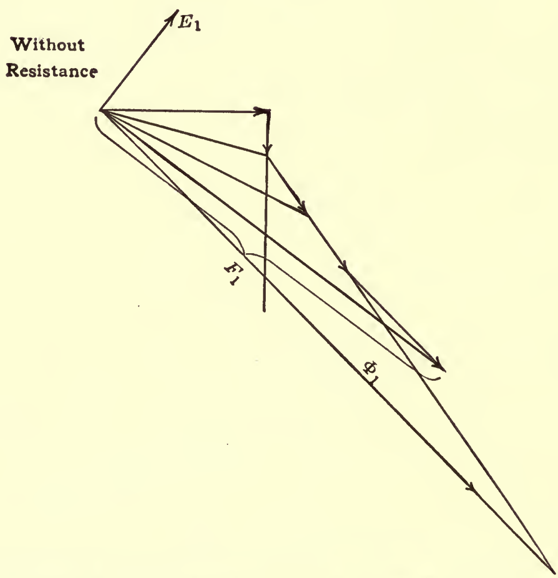


FIG. 138.—The fluxes of concatenated motors. Leakage only (not considering resistance). The group acting as generator.

The Fig. 137 is drawn for $v_1 = v_2 = 1.1$, and $F_1 = 49$ and therefore

$$\lambda = v_1 + v_2 - \frac{1}{v_1}$$

$$\lambda = 1.29$$

$$Of = 2\frac{F_1}{\lambda}$$

$$= 76$$

$$\frac{Of}{D} = 1.016$$

$$\therefore D = 74.8$$

Figure 138 shows an auxiliary diagram in which both motors are operating as induction generators.

Figure 139 shows a comparison of a single motor, as one of the concatenated group, with the concatentated group, with stray coefficients $v_1 = v_2 = 1.04$, or $\sigma = .082$.

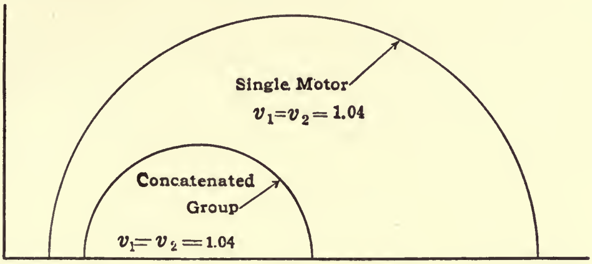


Fig. 139.—Concatenation: The influence of leakage. Approximate primary current loci of group and of single motor.

$v_1 = v_2 = 1.04$

Figure 140 shows the same groups of motors with stray coefficients $v_1 = v_2 = 1.1$, or $\sigma = .21$.

The greater the leakage, the less advantageous appears to be the grouping of the motors in concatenation as their joint capacity and maximum output are enormously reduced.

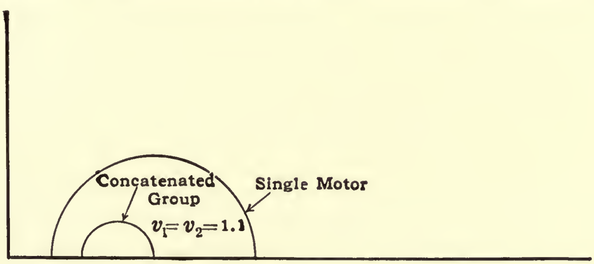


Fig. 140.—Concatenation: The influence of leakage. Approximate primary current loci of group and of single motor.

$v_1 = v_2 = 1.1$

B. THE POLY-PHASE MOTOR WITH SINGLE-PHASE SECONDARY

H. Goerges discovered in the early nineties that in an induction motor of the slip-ring type if, after starting, one brush is lifted on the rotor, the rotor will accelerate until it has reached half speed, at which speed it will run idle. This is a very curious and interesting result.

In view of the fact that heavy currents must be induced in a uni-axial rotor winding at half synchronous speed by the resultant rotating field in respect to which the rotor has a slip of 50 per cent, it is apparent that at half speed such a poly-phase motor with single-phase secondary must have a larger magnetizing current by the amount necessary to compensate for the rotor currents.

If we resolve the single-phase m.m.f. belt which exists in the rotor into two m.m.f. belts of half the amplitude, rotating in opposite directions, then we obtain the following:

Secondary frequency $\sim_1 - \sim_2$, to which corresponds clockwise an angular velocity in space $\omega_2 + \omega_1 - \omega_2 = \omega_1$, if the

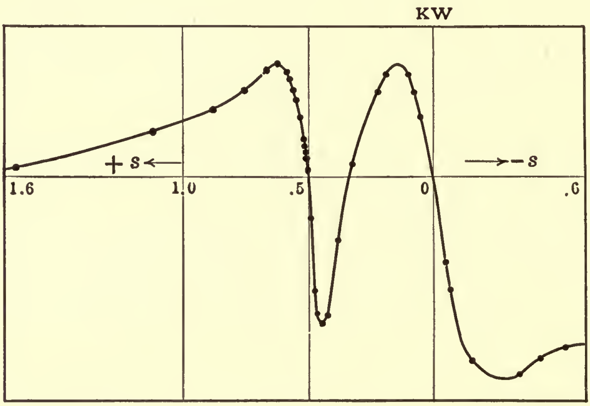


FIG. 141.—Torque curve of poly-phase motor with uni-axial rotor winding. (From Arnold, vol V, part 1.)

rotor revolves clockwise. Counter-clockwise there appears $\omega_2 - (\omega_1 - \omega_2) = -(\omega_1 - 2\omega_2)$ as the angular velocity in space of the second component of the resolved single-phase m.m.f. belt.

The first component rotating in space with angular velocity ω_1 may be combined with the impressed m.m.f. belt of the primary.

The second component rotating in space with angular velocity $-(\omega_1 - 2\omega_2)$ may be looked upon as the impressed m.m.f. belt of a motor in which the rotor is the primary and in which the stator, short-circuited as it were through its supply circuit, forms the secondary.

Thus we obtain the identical conditions of internal concatenation, and the general theory of concatenation may with propriety be applied to this subject.

For a similar statement of the theory of this phenomenon, see especially E. Arnold, "Les Machines d'Induction," Paris, Ch. Delagrave, 1912, p. 184, where an experimental diagram is given of the torque of such a motor as a function of the slip, see Fig. 141, which is like Fig. 125 in the Chap. XII. A. Mr. B. G. Lamme in "Electrical Engineering Papers," p. 519, has given the same general explanation. We differ, however, from him as to the propriety of deriving the resultant torque of such a motor from two constant-potential torque curves for different slips. It is evident that, at half speed, the currents in the rotor are almost totally watt-less; therefore, there can be no torque at that speed. The method reprinted by F. Eichberg in his "Collected Papers," p. 82, is open to the same criticism. The two fictitious motors of this combination do not operate, either under constant potential, or at all similarly to the standard induction motors with short-circuited rotor as it must have become clear in discussing concatenation in the previous chapter.

The reflection of low-frequency currents into the primary of the motor through induction from the second hypothetical motor whose secondary is the main primary circuit, short-circuited through the supply circuit as such, leads to disturbances in the supply circuit which make a practical application of this ingenious scheme most undesirable. Reference to this is found also in Arnold's book, *loc. cit.*, p. 185.

CHAPTER XIII

METHODS OF SPEED CONTROL (*Continued*)

C. CONCATENATION OF AN INDUCTION MOTOR WITH THE COMMUTATOR TYPES FOR THE INDUCTION OF A SLIP FREQUENCY E.M.F.

All the methods devised for obtaining a change in speed by means of concatenation depend upon the generation of currents of slip frequency properly injected into the rotor of the motor which is to be controlled.

(*a*) Poly-phase commutator motors of the series or shunt or compound type may be mounted on the same shaft with the induction motor which is to be regulated. The series commutator motor may have salient poles to obtain increased stability like the

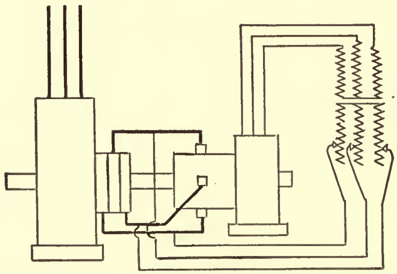


Fig. 142.—"Kraemer System," Induction motor mechanically concatenated to poly-phase commutator induction motor.

machines of F. Lydall, A. Scherbius, and Miles Walker. This system is called the "Kraemer System." (Fig. 142.)

(*b*) Instead of combining the regulating machine with the main motor into one mechanical unit, they may be separated as was done by A. Scherbius. (Fig. 143.)

(*c*) A rotary converter may be mounted on the same shaft with the induction motor, the combination operating at half speed for equal numbers of poles, and direct current power becoming available. Method proposed by J. L. la Cour. (Fig. 144.)

(*d*) The rotary converter may be separated from the shaft of the induction motor and a d.c. machine may be controlled on the same shaft as the induction motor by this rotary converter. (Fig. 145.)

(*e*) The d.c. machine on the shaft of the induction motor may be replaced by an independently running set of d.c. motor and induction generator. (Fig. 146.)

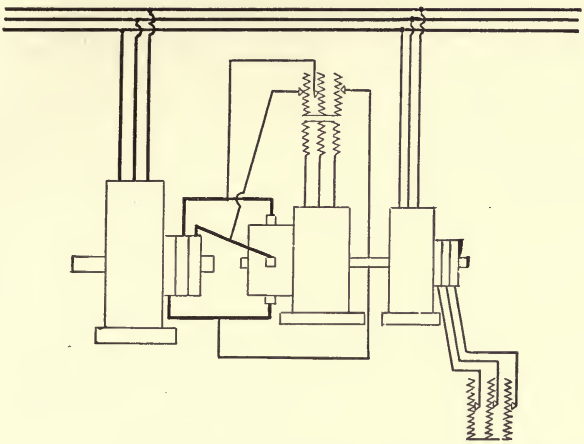


FIG. 143.—"Scherbius System," Induction motor electrically concatenated through poly-phase commutator motor which delivers current back into the line.

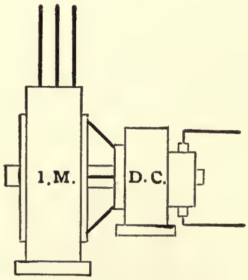


FIG. 144.—Combination of induction motor with rotary converter. (J. L. Lacour).

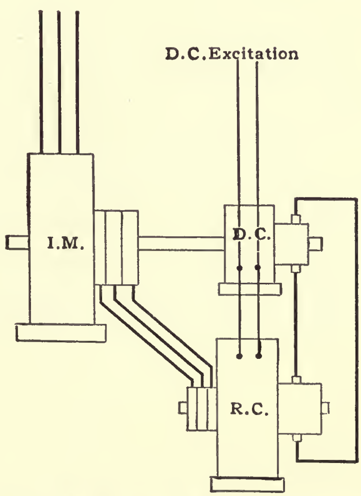


FIG. 145.—Speed regulation of induction motor by means of rotary converter and direct connected D. C. machine.

(*f*) Another method was devised by A. Heyland and R. Ruedenberg and it is shown in Fig. 147.

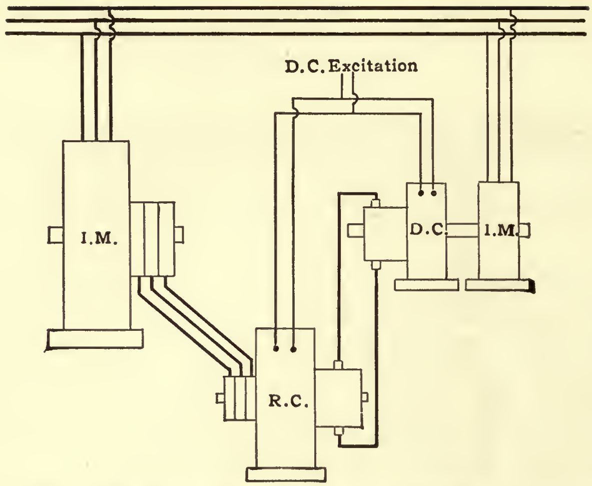


Fig. 146.—Speed regulation of induction motor by means of rotary converter and separately driven direct current induction motor generator set.

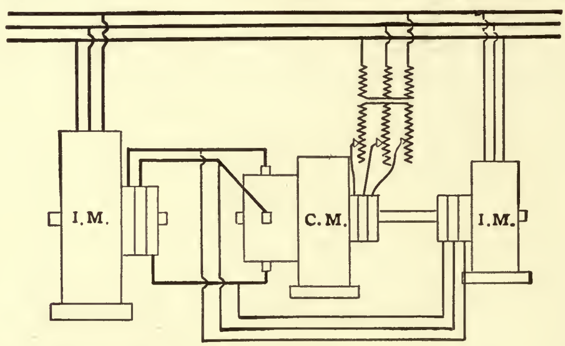


Fig. 147.—Combination of induction motor with commutator motor and small induction motor. (Heyland-Ruedenberg).

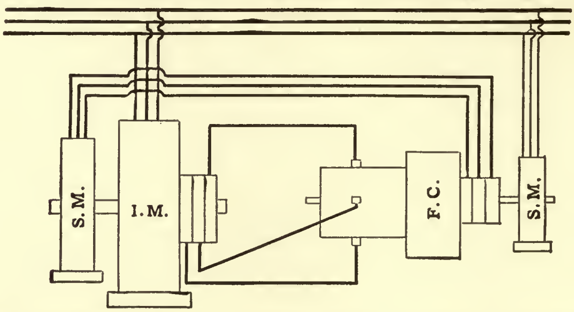
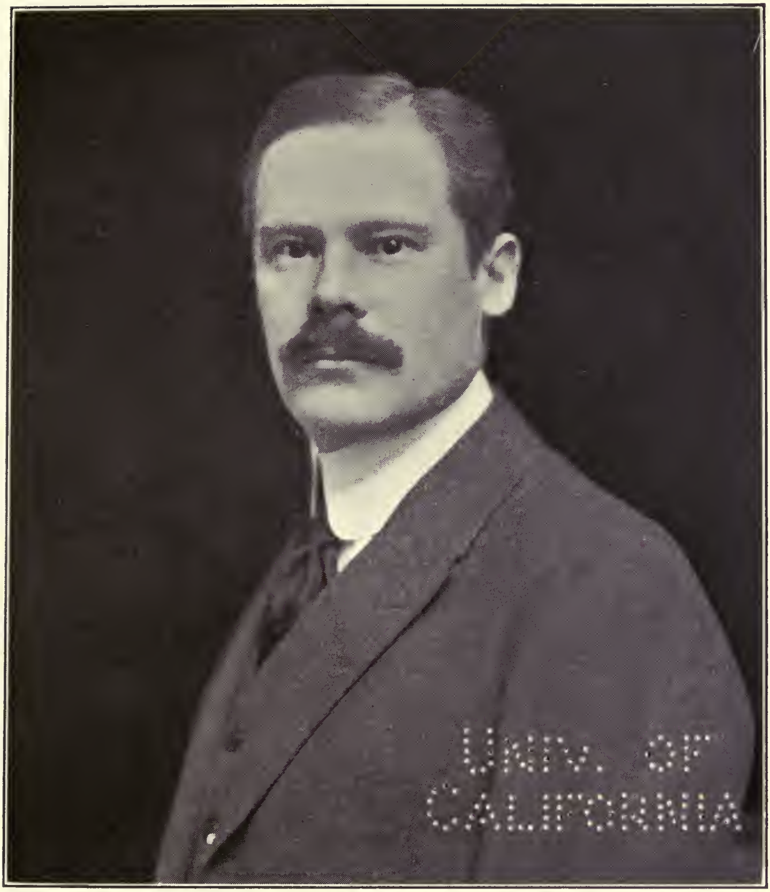


Fig. 148.—Speed regulation of induction motor by means of synchronous motor direct connected and fed from frequency changer driven by small synchronous motor. (Brown, Boveri system).



Yours sincerely
E. F. W. Alexanderson

(*Facing page* 178)

(*g*) The latest and perhaps principally the simplest method is described in the German patent No. 264,673, July 24, 1910, taken out by Brown Boveri & Company. It is shown in Fig. 148, where *S. M.* represents a synchronous motor mounted on the shaft of the induction motor, *F* is a frequency changer of the type first suggested by B. G. Lamme, and *S* is again a small synchronous motor.

D. CHANGE OF SPEED BY CHANGING THE NUMBER OF POLES

A great many windings have been devised to obtain different numbers of poles with one winding. Two or more entirely separate windings wound for different numbers of poles have been used in the same slots, either with corresponding rotor windings, or with a squirrel-cage rotor.

By utilizing a winding of a fractional pitch, it is possible to split it in such a manner as to arrange it for two numbers of poles, and even for four numbers. These windings are ingenious but intricate and their field of application is limited.

It is important to bear in mind in designing such machines that the leakage factor of a winding of twice the number of poles is very nearly twice as great. Great attention must also be paid to the magnetizing current, otherwise it might become excessive in view of the reduction of the active conductors per pole. The subject is too broad to be treated here at length, but I shall indicate one or two methods to outline the general principle and describe one of the most effective methods used to obtain this change in the number of poles.

1. The oldest method consists in the use of two or more sets of stator coils, the pitch of which is such as to give a number of windings with a different number of poles[1] for each. Such a scheme is impracticable as it is wasteful of space and material.

2. A single winding can be used which, due to different lap caused by the coil-pitch being less than the pole-pitch, may be so connected as to produce two different numbers of poles.

3. The ingenious winding of Alexanderson,[2] by which four numbers of poles, as 6, 8, 12, and 24, may be obtained with a

[1] See, for instance, B. G. Lamme, U. S. Patent No. 660,909, Oct. 30, 1900.

[2] E. F. W. Alexanderson. U. S. Patents Nos. 841,609 and 841,610, Jan. 15, 1907.

winding the individual coils of which are all alike but the connections of which, by means of 30 leads brought out from the motor, are connected through a drum controller. Six, eight, and twelve poles are obtained by the use of only 12 leads. The

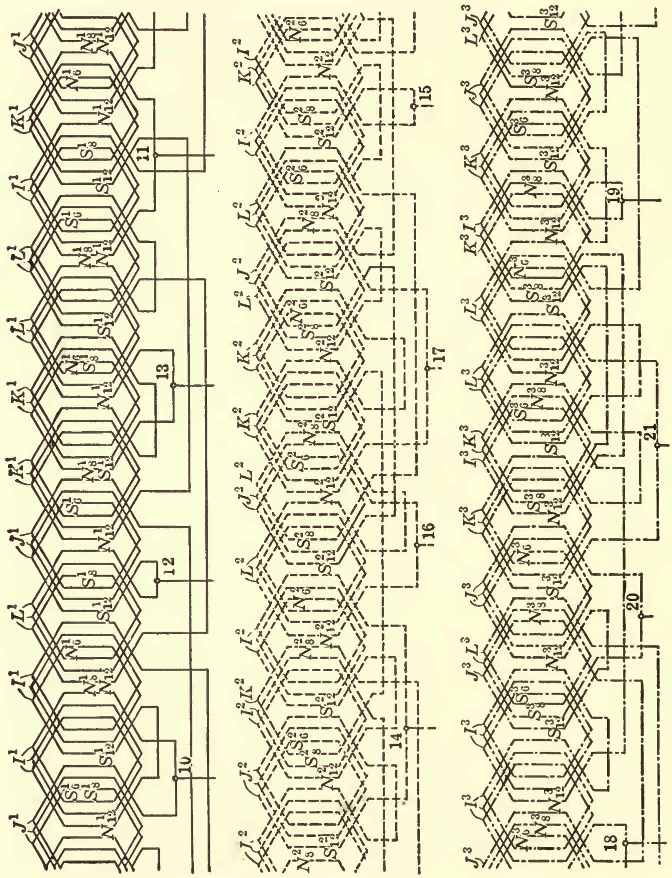


FIG. 149.—E. F. W. Alexanderson's stator winding for three or four different numbers of poles. (From U. S. patent No. 841,609, Jan. 15, 1907.)

accompanying winding diagrams, taken from Alexanderson's patent, show clearly the arrangement of the circuits. (Figs. 149 and 150.)

The Oerlikon Company[1] has built motors of this kind with a

[1] See *E. T. Z.*, 1914, No. 31.

double rotor, one inside the other, and each motor arranged for pole-changing. The motors are so arranged that one motor drives the other and with two sets of poles on the main motor

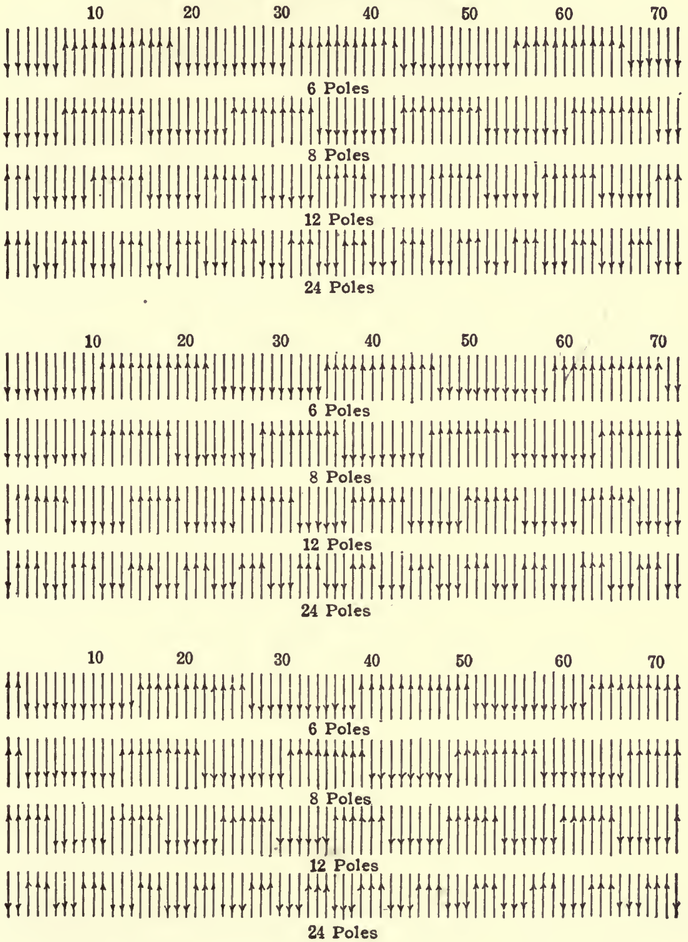


Fig. 150.—E. F. W. Alexanderson's stator winding for three or four different numbers of poles. Arrows show direction of currents in the different phases per pole. (From U. S. Patent No. 841,609, Jan 15, 1907).

and six sets of poles on the auxiliary motor, it can be seen that $2 \cdot 2 \cdot 6 + 2 = 26$ different speeds can be obtained.

CHAPTER XIV

TYPES OF VARIABLE SPEED POLY-PHASE COMMUTATOR MOTORS

Having explained at great length in previous chapters the principles upon which is based the speed control of induction motors, we shall consider briefly three types of motor which embody these principles in one unit.

A. The Plain Shunt Motor.—This motor in its simplest form is represented by Fig. 151. The voltage on the rotor brushes as well as their phase can be varied. The brushes can be shifted.

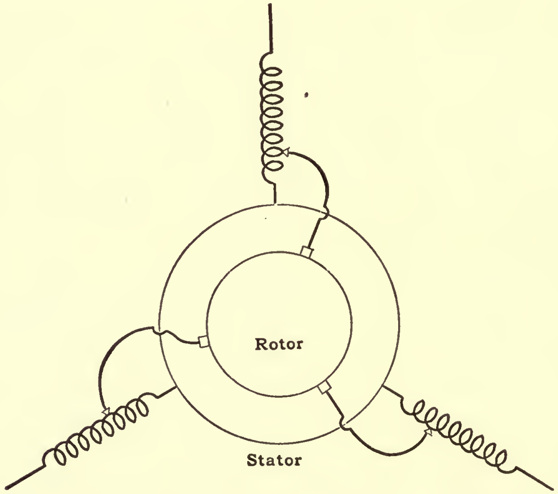


FIG. 151.—Poly-phase shunt motor. Without transformer and requiring mechanical brushshift. (This motor is identical with Heyland's "Compensated. Motor," excepting for the shunts between commutator bars, later introduced and abandoned.)

Both speed regulation and power factor regulation may be obtained. This is the Goerges motor in its simplest form. Interposing of transformers to give different voltages and phases suggests itself and innumerable combinations can be effected. (Fig. 152.)

B.-J. L. la Cour has added a second stator winding in order to obviate the mechanical shifting of the brushes which is detrimental, as it incurs higher harmonics which are very serious in

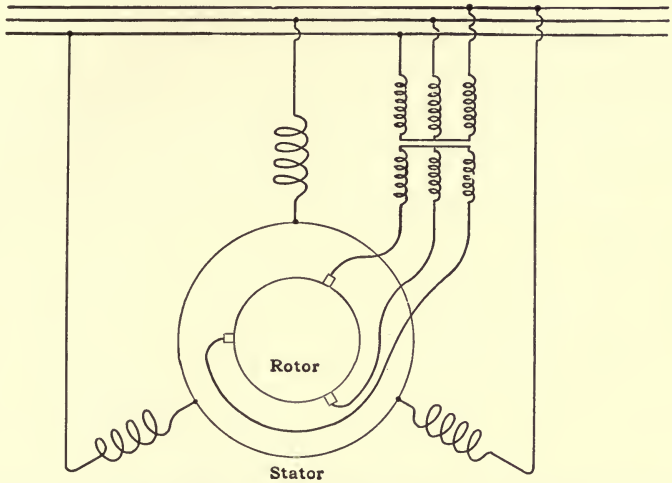


Fig. 152.—Polyphase shunt motor with transformer, requiring mechanical brush shift.

connection with commutation. Separate regulation of this winding through a transformer is necessary. (Fig. 153.)

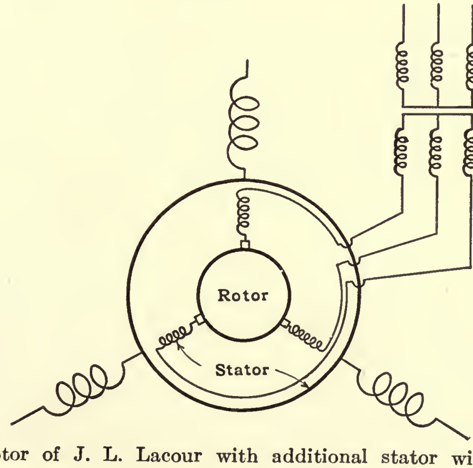


Fig. 153.—Motor of J. L. Lacour with additional stator winding to avoid mechanical brush shift.

C.-M. Osnos, *E. T. Z.*, Dec. 11, 1902, describes a motor in which the rotor is used as the primary and the stator as the secondary.

The rotor is built with a commutator on one side and slip rings on the other. The groups of coils between the brushes carry currents of the slip frequency, therefore, to the stationary, forming the secondary, then the combination lends itself directly to speed regulation, the proper voltage necessary being obtained by a pair of brushes for each phase of the secondary. (Fig. 154.)

D.-H. K. Schrage, in U. S. Patent No. 1,079,994 Dec. 2, 1913, describes a similar motor in which he has added another

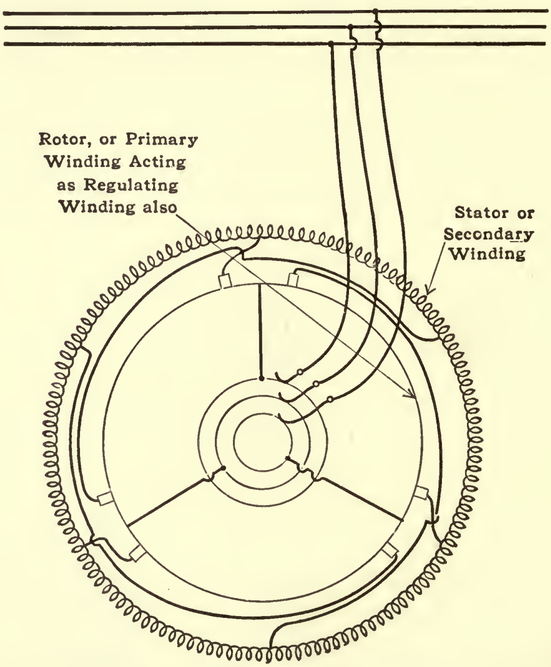


Fig. 154.—The variable speed commutator induction motor of Osnos.

winding, called regulating winding, which is connected to the commutator while the slip rings are connected to an independent primary winding. Through this modification of the Osnos motor greater freedom in the choice of secondary voltages is obtained. The drawback of these arrangements seems to be that, while in the shunt-motor type with stationary primary the e.m.f. short-circuited under the brushes varies proportionally to the slip, and disappears at synchronism, in the Osnos and Schrage motors it remains constant over the entire range at all speeds. (Fig. 155.)

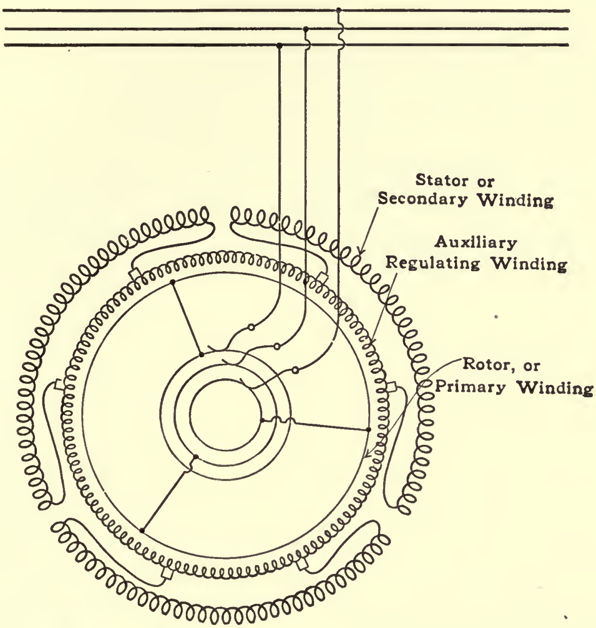


FIG. 155.—The variable speed commutator induction motor of H. K. Schrage. (U. S. Patent 1,979,994, Dec. 2, 1913.)

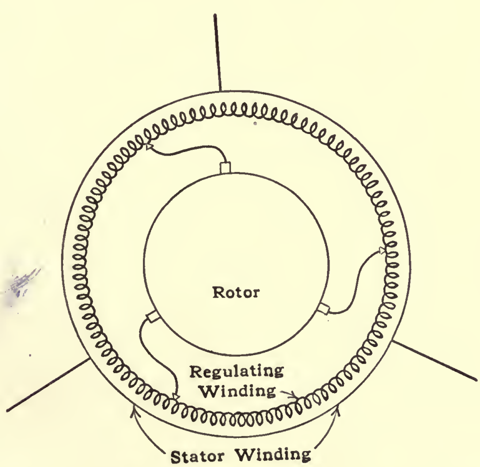


FIG. 156.—Shunt poly-phase commutator motor with auxiliary regulating winding on stator. (Inversion of Schrage motor.)

The Schrage motor is in reality a kind of frequency transformer like that described in Chap. IX, F.

E. If a regulating winding were added to the motor described under A, but placed on the stator instead of the rotor we obtain the inversion of the Schrage motor, Fig. 156.

NOTE.—In connection with the subject of commutation in these motors, briefly referred to in this chapter, see the very clear exposition by B. G. LAMME, *Journal, A. I. E. E.*, 1920, "The Alternating Current Commutator Motor."

(*Facing page* 187)

CHAPTER XV

METHODS OF RAISING THE POWER FACTOR OF INDUCTION MOTORS

A. THE METHOD OF LEBLANC USING COMMUTATOR MACHINES FOR SECONDARY EXCITATION

To the genius of Maurice Leblanc we owe the methods for raising the power factor of induction motors by introducing e.m.fs. of proper phase and frequency into the rotor. In U. S. Patent No. 613,204, Oct. 25, 1898, he describes a method of using two or three single-phase commutator machines excited with slip frequency in such a manner that leading currents are induced in the rotor. As a matter of historical interest, there is reproduced in facsimile the illustration from the patent specification. (Fig. 157.)

The theory of these interesting machines has been treated in Chap. IX, in which it was shown how a secondary phase lag or lead affects the primary current locus, and we therefore need not repeat the subject.

B. THE USE OF A POLYPHASE COMMUTATOR FOR THE GENERATION OF LEADING CURRENTS

As indicated in Chap. VIII, B, a commutator machine without excitation, fed with polyphase currents, generates at a proper speed an e.m.f. which lags behind the exciting current. Thus the arrangement of Leblanc may be replaced by a single polyphase commutator armature without excitation, operated only at a high enough speed above synchronism to obtain the effect required. This device has been used by Leblanc, Scherbius, Brown Boveri & Company, etc., and it is as ingenious as it is simple. Its theory has been given above.

An ingenious modification of this device is used by the Brown Boveri & Company. As the stator is evidently needed solely to close the magnetic circuit, it may be made integral with the rotor, without an air-gap, and the stator—forgetful of its name and connotation—revolves integrally with the rotor.

The saturation of the iron is high so that, after high currents are reached as the result of increasing load, the compensating

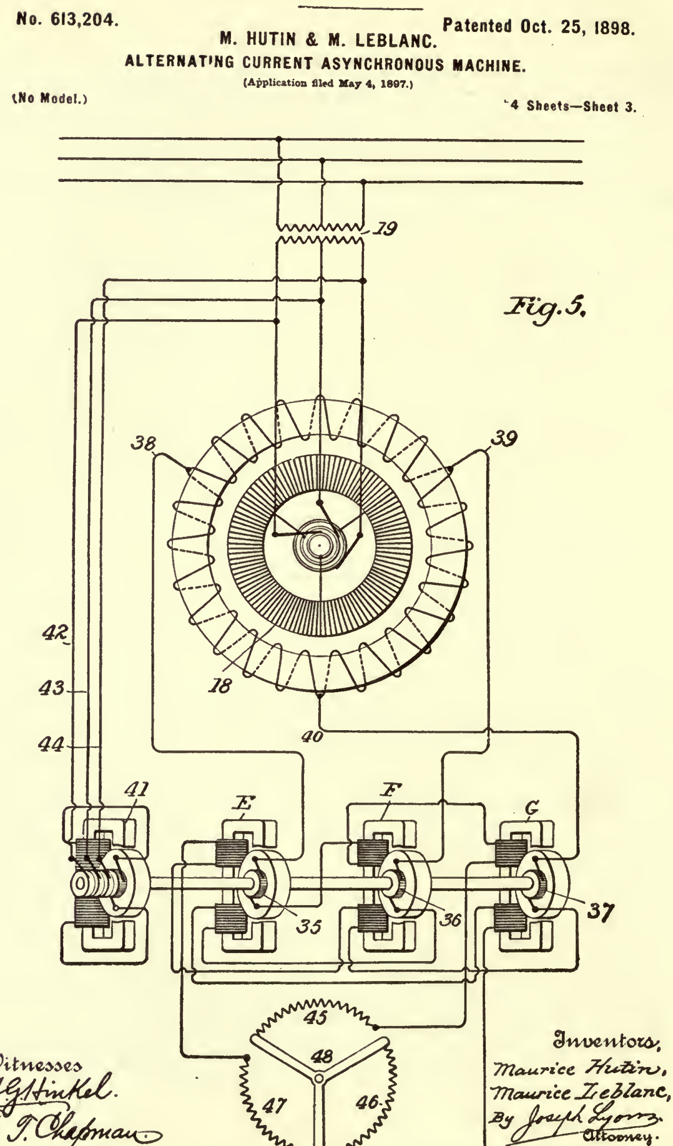
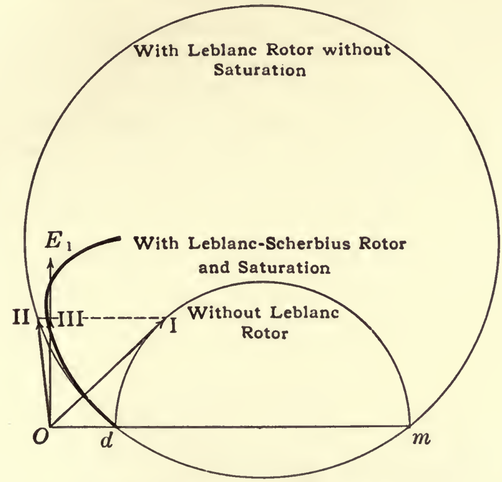


Fig. 157.—The method of Leblanc for raising the power factor of induction motors. (Facsimile of the American patent specification.)

effect diminishes resulting in a polar diagram as indicated approximately in Fig. 158. Thus the power factor is practically constant and near unity over a wide range.



I Primary Current without Leblanc Rotor
II " " with " " without Saturation
III " " " " " with "

FIG. 158.—The primary current of the induction motor with or without Leblanc-Scherbius rotor.

C. THE METHOD OF LEBLANC INDUCING LEADING CURRENTS THROUGH RAPID OSCILLATION OF AN ARMATURE IN A MAGNETIC FIELD

Maurice Leblanc described an ingenious scheme for the generation of leading currents in U. S. Patent No. 644,554, Feb. 27, 1900, with the suggestion that it be used in the secondary of a slip-ring type of induction motor. Figure 159 shows the principle.

A coil $a - a$ is suspended between the poles N and S where it can swing freely in the magnetic field established in the air-gap. The low-frequency alternating current upon which an e.m.f. producing a leading current is to be impressed by the oscillation of this device, called by the inventor a "recuperator," traverses the coil $a - a$.

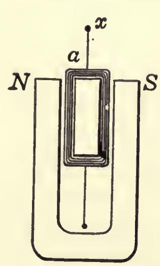


FIG. 159.—Device illustrating the principle of the Leblanc "Recuperator."

The stronger the magnetic field and the lighter the frame of the coil and the smaller the frequency of the current passing through the coil $a - a$, the greater will be the effective e.m.f. due to the oscillation causing a leading current to be induced in the circuit $a - a$. If the moment of inertia of the coil is great, lagging current may be induced.

An interesting, but not very practical, method of giving concrete shape to this idea is shown in Fig. 160, taken from Leblanc's patent. A disc swings in a strong magnetic field into which current is conducted by means of the ring R and collected at the circumference through a mercury-trough M.

The theory is interesting and it is clearly stated by Leblanc and Kapp (see later). The fundamental idea consists in making a device, like a coil spring, which through its oscillations induces an e.m.f. in the circuit from which originates the forced frequency. The effect of such a coil spring is like the effect of the "recuperator."

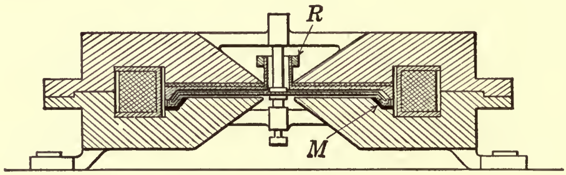


Fig. 160.—A possible practical form suggested by Leblanc for his "Recuperator."

In a general way it is apparent that we can devise two types of dynamic systems. One, in which a heavy mass is set into oscillation; another, in which a very small mass attached to a powerful spring, oscillates.

It is shown in the theory of dynamics that the velocity of the oscillating system is a maximum in the first case when it is a minimum in the second. If, therefore, e.m.fs. can be induced by the swinging coil, in the former case it may be expected that a lagging current will result, while in the latter case a leading current will be induced.

In one-half of a period, the low-frequency exciting current rises from zero to a maximum and declines to zero; during this same interval of time the magnet NS swings from its position of equilibrium in the plane YY to its extreme right position and back again to its position of equilibrium. (Fig. 161.)

The maximum velocity with which the magnet sweeps by the low-frequency exciting winding is reached when the magnet passes its plane of equilibrium. A maximum of kinetic energy is stored in the magnet at this time which is given up to the low-frequency exciting winding during the swing of the magnet to its extreme right position. Thus, energy is transferred from the moving coil to the exciting circuit, the induced e.m.f. decreasing from a maximum to zero while the exciting current increases from zero to a maximum.

On the return swing the low-frequency exciting circuit transfers energy to the magnet which had yielded up all its energy when it reached its extreme right position. Thus, the magnet induces

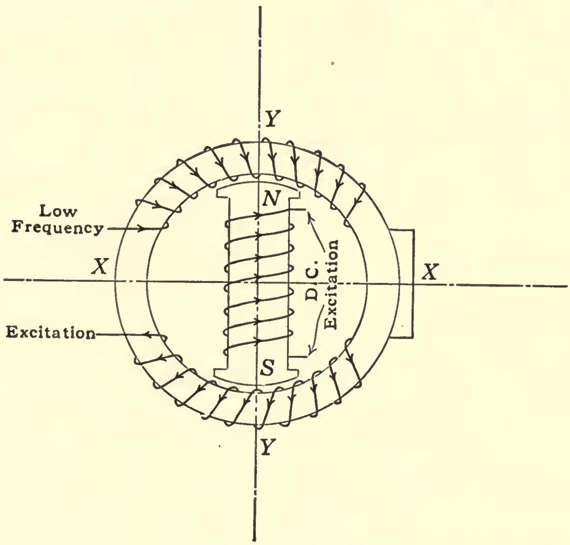


FIG. 161.—Principle of Leblanc "Recuperator" and Kapp "Vibrator."

an e.m.f. in the exciting winding which is in leading quadrature with the exciting current.

If the magnet were allowed to swing beyond the x-axis, it would operate eventually as a synchronous motor.

In fact, the "recuperator" *is* identical with an over-excited synchronous motor. If the mass of the magnet was great and the field weak, it would act like an under-excited synchronous motor. Rotation is merely a special case of the phenomenon of oscillation.

The elementary theory, omitting the effect of mechanical or electrical damping, in the oscillations of the magnet, may be given as follows:

Let the moment of inertia of the magnet in *cm* units be I. Let θ be the angle which at any moment of time the magnetic axis of the oscillating body forms with the magnetic axis of the stator. The stator is excited with low-frequency currents obtained from the slip rings of the induction motor in the circuit of which the "recuperator" is connected.

Let $$\omega = \omega_1 - \omega_2 = 2\pi(\sim_1 - \sim_2)$$

Let $\frac{d\theta}{dt}$ be the angular velocity of the swinging magnet. This magnet swings in such a manner that it moves through a total angle of $4\theta_{max}$ in the time T, in which a complete cycle is passed through by the low-frequency exciting currents.

The mean angular velocity of the rotor is therefore immediately obvious since

$$\left(\frac{d\theta}{dt}\right)_{mean} T = 4\theta_{max}$$

$$T = \frac{2\pi}{\omega}$$

Hence,

$$\left(\frac{d\theta}{dt}\right)_{mean} = \frac{4\theta_{max}}{2\pi}\cdot\omega$$

Also, as the oscillations are assumed to take place according to a simple sine law,

$$\left(\frac{d\theta}{dt}\right)_{max} = \theta_{max}\cdot\omega$$

D'Alembert's principle applied to a rigid body rotating about an axis is

$$I\,\frac{d^2\theta}{dt^2} = P_{max}\cdot\sin\omega t\cdot d$$

where $\frac{d^2\theta}{dt^2}$ is the angular acceleration, P_{max} the maximum value of the force exerted by the low frequency winding upon the swinging magnet, and d the arm of the couple producing the angular acceleration.

θ varies according to a simple sine law, hence

$$\theta = \theta_{max}\cdot\sin\omega t$$

$$\frac{d\theta}{dt} = \theta_{max}\cdot\omega\cdot\cos\omega t$$

$$\frac{d^2\theta}{dt^2} = -\theta_{max}\cdot\omega^2\cdot\sin\omega t$$

Therefore,

$$I\frac{d^2\theta}{dt^2} = -I\cdot\theta_{max}\cdot\omega^2\cdot\sin\omega t = P_{max}\cdot\sin\omega t\cdot d$$

$$\therefore \theta_{max} = -\frac{P_{max}\cdot d}{I\omega^2} \tag{170}$$

P_{max} can be calculated from the known mechanical force acting upon a conductor in a magnetic field. Let B be the density of the flux in the air-gap, i the effective low-frequency exciting current, l the length of the conductor in the field, and z the total number of conductors in the low-frequency exciting winding exposed to the flux of the magnet in the air-gap.

Then

$$P_{max} = \frac{1}{2}B\frac{i\sqrt{2}}{10}\cdot l\cdot z\frac{1}{981} \text{ grammes}$$

Therefore, by substitution of this value into (170) we obtain

$$\theta_{max} = -\frac{1}{2}\cdot\frac{Bi\sqrt{2}\,l\cdot z\cdot d}{10\cdot 981\cdot I\cdot\omega^2}$$

The maximum value of the e.m.f. induced by the oscillating magnet in the low-frequency exciting winding is

$$E = \left(\frac{d\theta}{dt}\right)_{max}\cdot\frac{d}{2}\cdot B\cdot l\cdot z\cdot 10^{-8} \text{ volts}$$

$$\left(\frac{d\theta}{dt}\right)_{max} = \theta_{max}\cdot\omega$$

Substituting we obtain

$$E = \theta_{max}\cdot\omega\cdot\frac{d}{2}\cdot B\cdot l\cdot z\cdot 10^{-8} \text{ volts}$$

$$= -\frac{i\sqrt{2}\cdot B^2\cdot l^2\cdot z^2\cdot d^2}{4\cdot 9810\cdot I\cdot\omega}10^{-8} \text{ volts}$$

And the effective e.m.f.

$$e = -\frac{i\cdot B^2\cdot l^2\cdot z^2\cdot d^2}{4\cdot 9810 I\cdot\omega}10^{-8} \text{ volts} \tag{171}$$

This equation shows that the magnitude of the injected e.m.f. increases directly as the product of the square of the flux to which the low-frequency winding is exposed into the value of the low-frequency exciting current, and inversely as the product of the moment of inertia of the magnet into the low frequency.

Write for (171)

$$e = \frac{i\Phi^2}{I(\sim_1 - \sim_2)}\cdot K \text{ volts}$$

and we see that the secondary phase angle varies, as the e.m.f.

induced in the rotor to overcome the resistance drop is proportional to the slip frequency $\sim_1 - \sim_2$.

Let C be the capacity in farads of a condenser, then

$$i = \omega \cdot C \cdot e \text{ farads} \tag{171}$$

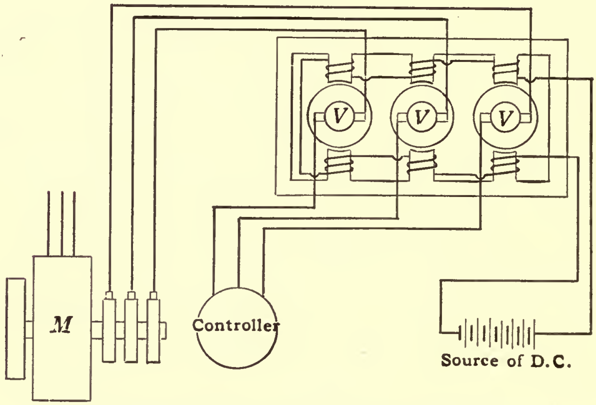


FIG. 162.—Connections of Kapp "Vibrator."

Comparing this formula with equation (171) we obtain for the capacity effect in farads of this "recuperator" the value

$$C = \frac{4\cdot9810\, I}{B^2 l^2 z^2 d^2} 10^{-8} \text{ farads}$$

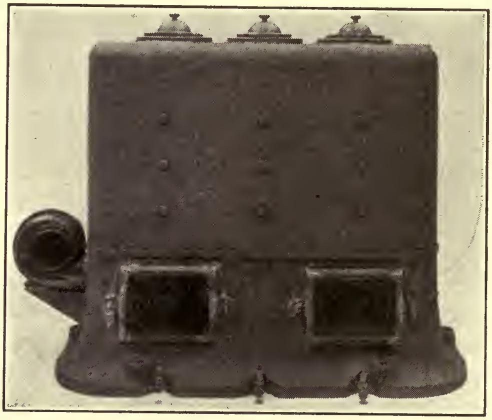


FIG. 163.—Kapp "Vibrator" for 1,500 h.p. three-phase induction motor. (Built by Westinghouse Company of Pittsburgh.)

and the "elastance," or the ability to let current pass

$$\frac{e}{i} = \frac{B^2 l^2 z^2 d^2}{4\cdot9810 \cdot I} 10^{-8}$$

It is interesting to note that the effect of this mechanical condenser is such that the secondary phase angle decreases with increasing slip. But, as it also depends upon the secondary current for its action, it is not very efficient at low slips. At synchronism it acts as an impedance.

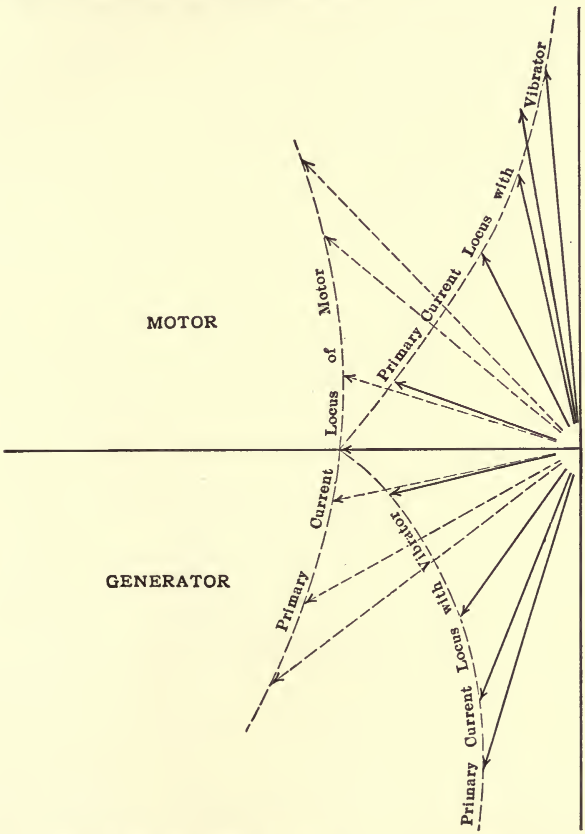


Fig. 164.—Polar diagram of 1,500 h.p. induction motor, 2,200 volts, three-phase, 60 cycles, 20 poles. With and without Kapp vibrator.

D. THE SAME METHOD AS ELABORATED BY G. KAPP

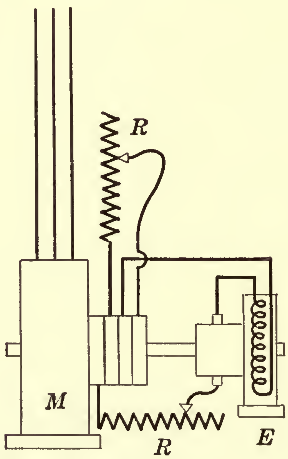
Mr. Kapp re-invented the Leblanc scheme and he applied it in practical form. He called the device a "vibrator." The general theory is identical with that of the "recuperator" of Leblanc's.

Figure 163 shows a photograph of one of these three-armature "vibrators" built by the Westinghouse Company in connection with a 1,500 hp. induction motor. The general principle of the "vibrator" is given in Figs. 161 and 162, which are taken from Mr. Kapp's paper.[1] The magnet consists of an armature of small diameter wound like a direct-current armature. Brushes convey current to a commutator.

The polar diagram of this motor with and without the "vibrator" is given in Fig. 164. The field of application of these ingenious devices appears limited.

BIBLIOGRAPHY

Horace Lamb: "Dynamics." Cambridge University Press, 1914, p. 282. Par. 95, "Forced Oscillations."



M Main Motor
E Exciter with Laminated Field
R Rheostat

Fig. 165.—The Danielson-Burke method of changing an induction motor into a synchronous motor.

Horace Lamb: "The Dynamical Theory of Sound." London, Edward Arnold, 1910, p. 16. Par. 8, *et seq.*

John Perry: "The Calculus for Engineers." London, Edward Arnold, 1899, p. 213. Par. 148, *et seq.*

J. Prescott: "Mechanics of Particles and Rigid Bodies." London, Longmans, Green & Company, 1913, p. 328. Par. 340.

Edward John Routh: "Dynamics of a System of Rigid Bodies." London, Macmillan & Co., Ltd., 1897, p. 344. Par. 432.

[1] G. Kapp, *The Electrician*, May 17 and 24, 1912. "On Phase Advancers for Non-synchronous Machines." Also U. S. Patents No. 1,236,716, Aug. 14, 1917 and No. 1,258,577, March 5, 1918.

Henri Bouasse: "Cours de Mécanique Rationelle." Paris, C. Delagrave, Chap. VII.

A. Guillet: "Propriétés Cinématiques Fondamentales des Vibrations." Paris, Gauthier-Villars, 1913.

E. THE DANIELSON-BURKE METHOD OF CHANGING THE INDUCTION MOTOR INTO A SYNCHRONOUS MOTOR

A modification of the Danielson idea, already mentioned by Tesla, of exciting a non-synchronous motor with direct current when it approaches synchronous speed, is due to Mr. James Burke.

An exciter with laminated field is connected as indicated in Fig. 165. When the starting resistance is cut out, the exciter automatically functions as a direct-current generator.

CHAPTER XVI

THE MAGNETIC PULL WITH DISPLACED ROTOR

Before closing the subject of poly-phase induction motors, we shall briefly refer to an important mechanical relation. The small air-gaps necessary with these motors make it important to be able to forecast the amount of unbalanced magnetic pull due to excentric position of the rotor within its stator.

A. THE FORMULA OF B. A. BEHREND

The author developed a very simple mathematical relation, giving the magnetic pull for a displaced rotor, in a paper[1] presented before the American Institute in November, 1900.

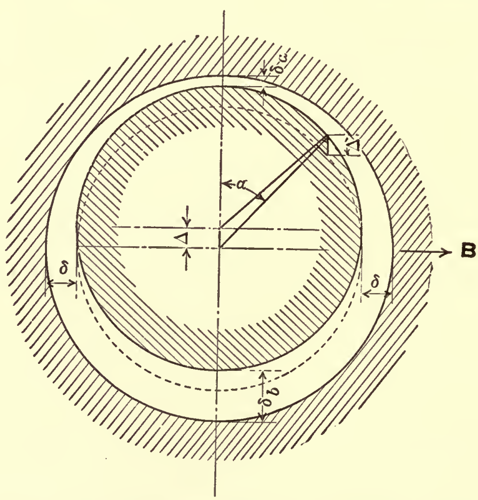


FIG. 166.—The magnetic pull with displaced rotor.

Consider two cylinders placed eccentrically as shown in Fig. 166. All around the circumference acting along a radius vector, we postulate the same m.m.f. and assume the lines of induction

[1] B. A. BEHREND, "On the Mechanical Forces in Dynamos Caused by Magnetic Attraction." *Trans., A. I. E. E.*, Nov. 23, 1900, p. 617.

to follow the shortest path. Their density is then inversely proportional to the length of the gap.

From Fig. 166

$$\delta_a = \delta - \Delta \cdot \cos \alpha$$

$$\Delta = \frac{\delta_b - \delta_a}{2}$$

Hence,

$$\frac{B}{B_a} = \frac{\delta_a}{\delta} = \frac{\delta - \Delta \cos \alpha}{\delta}$$

$$B_a = B \frac{1}{1 - \frac{\Delta}{\delta} \cdot \cos \alpha} \qquad (172)$$

The magnetic attraction acting along a radius vector upon a surface element of the area $bRd\alpha$, in which b is the width of the laminations, is

$$df_c = \frac{1}{8\pi} B_a^2 \cdot b \cdot R \cdot d\alpha$$

As the horizontal components of this force on either side of the line of symmetry balance each other, the remaining vertical component is

$$df = \frac{1}{8\pi} B_a^2 \cdot b \cdot R \cdot d\alpha \cdot \cos \alpha$$

Substituting for B_a its value above

$$df = \frac{1}{8\pi} bRB^2 \frac{\cos \alpha d\alpha}{\left(1 - \frac{\Delta}{\delta} \cos \alpha\right)^2} \qquad (173)$$

$$= \frac{1}{8\pi} bRB^2 \frac{\cos \alpha d\alpha}{1 - \frac{2\Delta}{\delta} \cos \alpha + \left(\frac{\Delta}{\delta} \cos \alpha\right)^2} \qquad (174)$$

Neglecting $\left[\frac{\Delta}{\delta} \cos \alpha\right]^2$ as a small quantity, we obtain

$$df = \frac{1}{8\pi} bRB^2 \frac{\cos \alpha d\alpha}{1 - \frac{2\Delta}{\delta} \cos \alpha}$$

$$= \frac{1}{8\pi} bRB^2 \frac{1 + \frac{2\Delta}{\delta} \cos \alpha}{1 - \left[\frac{2\Delta}{\delta} \cos \alpha\right]^2} \cos \alpha d\alpha$$

$$= \frac{1}{8\pi} bRB^2 \left[1 + \frac{2\Delta}{\delta} \cos \alpha\right] \cos \alpha d\alpha \qquad (175)$$

By integrating this expression between the limits 0 and $\frac{\pi}{2}$ we find the vertical component of the magnetic attraction in one quadrant.

$$\int_0^{\frac{\pi}{2}} df = \frac{1}{8\pi} bRB^2 \int_0^{\frac{\pi}{2}} \left(1 + \frac{2\Delta}{\delta}\cos\alpha\right)\cos\alpha d\alpha \tag{176}$$

$$= \frac{1}{8\pi} bRB^2 \left[\int_0^{\frac{\pi}{2}} \cos\alpha d\alpha + \frac{2\Delta}{\delta}\int_0^{\frac{\pi}{2}} \cos^2\alpha d\alpha\right] \tag{177}$$

$$= \frac{1}{8\pi} bRB^2 \left[\sin\alpha\Big|_0^{\frac{\pi}{2}} + \frac{2\Delta}{\delta}\cdot\frac{1}{4}\sin 2\alpha\Big|_0^{\frac{\pi}{2}} + \frac{2\Delta}{\delta}\cdot\frac{\alpha}{2}\Big|_0^{\frac{\pi}{2}}\right]$$

$$f_a = \frac{1}{8\pi} bRB^2 \left[1 + \frac{2\Delta}{\delta}\cdot\frac{\pi}{4}\right] \tag{178}$$

For the vertical component directed downwards, we find similarly the expression

$$f_b = \frac{1}{8\pi} bRB^2 \left[1 - \frac{2\Delta}{\delta}\cdot\frac{\pi}{4}\right] \tag{179}$$

Hence,

$$f_a - f_b = \frac{1}{8\pi} bRB^2 \frac{4\Delta}{\delta}\cdot\frac{\pi}{4} \tag{180}$$

The total force is twice as great, as we have integrated only over a quadrant, therefore

$$Z = \frac{1}{8\pi} B^2 A \frac{2\Delta}{\delta} \text{ dynes} \tag{181}$$

$$\left.\begin{aligned} A &= \pi Rb \\ \Delta &= \frac{\delta_b - \delta_a}{2} \end{aligned}\right\} \tag{182}$$

Numerical Example:

$$\begin{aligned} B &= 5{,}900 \text{ c.g.s.} \\ A &= 9{,}350 \text{ cm.}^2 \\ \Delta &= 0.1 \text{ cm.} \\ \delta &= 0.36 \text{ cm.} \\ Z &= \frac{5{,}900^2 \times 9{,}350}{8}\ \frac{2 \times 0.1}{0.36} \text{ dynes} \end{aligned}$$

$$\begin{aligned} 9.81 \times 10^5 \text{ dynes} &= 1 \text{ kg.} \\ &= 7{,}400 \text{ kg.} \end{aligned}$$

B. THE ACCURATE SOLUTION BY J. K. SUMEC

J. K. Sumec[1] has given the integral of equation (174) and obtained an elegant solution.

He starts with the author's equation (174).

$$df = \frac{1}{8\pi} \frac{B^2}{\left(1 - \frac{\Delta}{\delta}\cos\alpha\right)^2} br\cdot d\alpha \qquad (174)$$

The vertical component is

$$\int_0^\pi df = \frac{B^2}{8\pi}\cdot\frac{A}{\pi} \int_0^\pi \frac{\cos\alpha\cdot d\alpha}{\left(1 - \frac{\Delta}{\delta}\cos\alpha\right)^2} \qquad (183)$$

Taking points diametrically opposite and simplifying

$$\frac{\cos\alpha}{\left(1 - \frac{\Delta}{\delta}\cos\alpha\right)^2} - \frac{\cos\alpha}{\left(1 + \frac{\Delta}{\delta}\cos\alpha\right)^2} = \frac{4\frac{\Delta}{\delta}\cos^2\alpha}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2\cos^2\alpha\right]^2}$$

$$\int_0^\pi df = \frac{B^2}{8\pi}\cdot\frac{A}{\pi}\,\frac{\Delta}{\delta}\,4 \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos^2\alpha d\alpha}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2\cos^2\alpha\right]^2}$$

$$= \frac{B^2}{8\pi}\,\frac{A}{\pi}\,\frac{\Delta}{\delta}\cdot 8 \int_0^{\frac{\pi}{2}} \frac{\cos^2\alpha d\alpha}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2\cos^2\alpha\right]^2} \qquad (184)$$

Substituting

$$u = tg\alpha$$
$$du = (1 + tg^2\alpha)d\alpha = (1 + u^2)d\alpha$$
$$\cos^2\alpha = \frac{1}{1 + tg^2\alpha} = \frac{1}{1 + u^2}$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2\alpha d\alpha}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2\cos^2\alpha\right]^2} = \int_0^\infty \frac{du}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2 + u^2\right]^2} \qquad (185)$$

This integral is a rational algebraic function of the form

$$\int_0^\infty \frac{du}{(a + u^2)^2}, \text{ where } a = 1 - \left(\frac{\Delta}{\delta}\right)^2 \qquad (186)$$

[1] J. K. Sumec, *Zeitschrift für Elektrotechnik*, Vienna, Dec. 18, 1904.

which can be solved readily by trigonometric substitution

$$x = \sqrt{a} \tan \theta \tag{187}$$

$$dx = \sqrt{a} \frac{d\theta}{\cos^2\theta} \tag{188}$$

$$\int_0^\infty \frac{\sqrt{a} \cdot d\theta}{\left(a + a\dfrac{\sin^2\theta}{\cos^2\theta}\right)^2 \cos^2\theta} = \int_0^\infty \frac{\sqrt{a} \cdot \cos^2\theta \cdot d\theta}{a^2} \tag{189}$$

$$= \frac{1}{a\sqrt{a}} \cos^2\theta \cdot d\theta$$

$$= \frac{1}{a\sqrt{a}} \left\{\frac{1}{2}\theta + \frac{1}{4} \cdot \sin 2\theta\right\}$$

$$= \frac{1}{2a}\left\{\frac{1}{\sqrt{a}} \tan^{-1} \frac{x}{\sqrt{a}} + \frac{x}{a + x^2}\right\}\Bigg|_0^\infty \tag{190}$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2\alpha \cdot d\alpha}{\left[1 - \left(\dfrac{\Delta}{\delta}\right)^2 \cos^2\alpha\right]} = \frac{1}{2a\sqrt{a}} \frac{\pi}{2} \tag{191}$$

Substituting this evaluated integral in (184)

$$\int_0^\pi df = \frac{B^2}{8\pi} \cdot A \cdot \frac{2\Delta}{\delta} \cdot \frac{1}{\left\{1 - \left(\dfrac{\Delta}{\delta}\right)^2\right\}^{3/2}} \tag{192}$$

$$\left.\begin{aligned} A &= \pi R b \\ \Delta &= \frac{\delta_b - \delta_a}{2} \end{aligned}\right\} \tag{193}$$

This is Prof. J. K. Sumec's very elegant result which but for the last term is identical with the author's formula (181).

The factor

$$\frac{1}{\left[1 - \left(\dfrac{\Delta}{\delta}\right)^2\right]^{3/2}} \tag{194}$$

is evaluated for different ratios of $\frac{\Delta}{\delta}$ and the following table is obtained which is also taken from Prof. Sumec's paper.

$\frac{\Delta}{\delta}$	0.1	0.2	0.3	0.4	0.5
$\frac{1}{\left[1 - \left(\frac{\Delta}{\delta}\right)^2\right]^{3/2}}$	1.015	1.063	1.152	1.30	1.54

For eccentricities less than 25 per cent the errors in using the author's formula are negligible. At 30 per cent eccentricity the error is 15 per cent, and at 50 per cent eccentricity, which is not at all unusual in induction motors, the error is 54 per cent. On this account this subject has been included in this book.

BIBLIOGRAPHY

The paper by E. Rosenberg, *Trans. A. I. E. E.*, 1918, Vol. XXXVII, p. 1417, with a full bibliography by Alexander Miller Gray and J. G. Pertsch, Jr., entitled "Critical Review of the Bibliography on Unbalanced Magnetic Pull in Dynamo-electric Machines," should be consulted by the reader. Also the papers by B. A. Behrend, "On the Mechanical Forces in Dynamos Caused by Magnetic Attraction," *Trans. A. I. E. E.*, Nov. 23, 1900, p. 617. And J. K. Sumec, *loc. cit.*

CHAPTER XVII

THE SINGLE-PHASE INDUCTION MOTOR

"The single-phase motor has been the subject of perhaps more theoretical speculation than any other dynamo-electric machine, and the reason for this is, undoubtedly that it is in its functioning, the most complicated of all dynamo-electric machines, although in its structure it is the simplest of them all."[1]

The simplified and improved transformer diagram which has served us well in the understanding of the phenomena in polyphase motors, serves our purpose equally well in the treatment of the single-phase motor. We shall employ two methods of dealing with this problem.

A. Galileo Ferraris and André Blondel have made use of Fresnel's theorem that an alternating or oscillating field of force may be replaced by two equal and oppositely rotating fields the amplitude of each of which being equal to one-half the maximum amplitude of the alternating field. These two fields rotate in opposite directions at an angular velocity equal to 2π times the frequency of the alternating field, assuming a two-pole field as has been done throughout this volume.

Employing Fresnel's theorem, the author[2] developed the following simple vector diagram which has proved amply accurate for all the purposes of engineering applications.

A two-pole rotor, revolving at an angular velocity corresponding to a frequency $\sim_2$ in an oscillating magnetic field whose frequency of oscillation is $\sim_1$, has relative to field I of Fresnel's two fields a slip $\frac{\sim_1 - \sim_2}{\sim_1} = s$ and relative to the other field II a slip $\frac{\sim_1 + \sim_2}{\sim_1} = 2 - s$.

Consider the field II. At the great slip $\frac{\sim_1 + \sim_2}{\sim_1}$ the primary and secondary ampere-turns act almost in space opposition. They would act exactly in space opposition if the ohmic rotor

[1] E. F. W. Alexanderson, *Trans. A. I. E. E.*, Part I, p. 691, 1918.

[2] B. A. Behrend, "Asynchronous Alternating Current Motors," *E. T. Z.*, March 25, 1897.

resistance could be neglected. In order to simplify the understanding of the theory of this motor we shall assume, which is admissible without great error, that so far as the field II is concerned the rotor resistance is negligible and rotor and stator ampere-turns are in phase in space, the stator ampere-turns being just enough larger than the rotor ampere-turns to magnetize the core to the extent of producing a field which balances the voltage required to pass through the stator of field II the current the ampere-turns of which we have considered here.

The primary ampere-turns of the single-phase motor have been resolved into two oppositely rotating components of one-half the amplitude. *This resolution is equivalent to two poly-phase motors whose stator windings are connected in series while the rotor windings are common to both stators.* The rotor being integral, the rotor torques act in opposite directions.

The field I tries to turn the rotor in a clockwise direction, while the field II tries to turn the rotor in a counter-clockwise direction. The motor I takes the larger share of the voltage since the apparent reactance of an induction motor is high at a small slip, and low for a large slip. Hence, as the stators of I and II carry the same current, *viz.*, one-half the total current, the voltage impressed on the stator of motor II is very small.

If we neglect in the operation of motor II the rotor resistance, it has been pointed out that the effect of motor II consists mainly in the reactance effect of this motor. The voltage impressed upon motor I is diminished by the voltage required by motor II and this voltage is always proportional to the stator current of motor I and in time quadrature with it.

Therefore, the effect of motor II may be taken into account by correspondingly increasing the primary leakage of motor I. The resultant vector diagram of a single-phase induction motor may now be developed.

a. THE MAGNETIZING AND NO-LOAD CURRENTS

At synchronism, the slip of motor II is $\frac{\sim_1 + \sim_2}{\sim_1} = 2$. The *same current* passes through the stators of motor I and II. With no leakage and no resistance in the rotor and stator of motor II, the voltage impressed upon motor II would be zero and therefore, the full voltage would be impressed upon motor I. Hence, the magnetizing current would be exactly doubled.

As the magnetizing current is proportional to the impressed voltage it is admissible to write for the case above, designating the magnetizing current of the single-phase induction motor by i_μ and its no-load current by i_0

$$i_0 = 2i_\mu + 0 = 2i_\mu \tag{195}$$

where i_μ is the magnetizing current of either motor I or II.

However, taking *leakage* into account, we have the current $\frac{i_0}{2}$ passing through each stator I and II. The impressed voltage

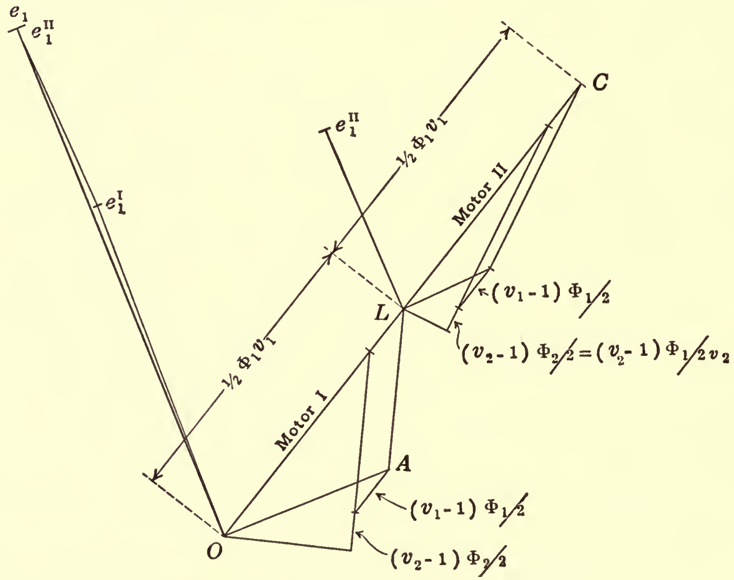


FIG. 167.—Vector diagram of the two-motor theory of the single-phase induction motor. (The two motors in series.)

on motor I at no load is proportional to $\frac{i_0}{2}$, while the impressed voltage on motor II is proportional to i_μ^{II}, where i_μ^{II} is the magnetizing current of motor II. Both magnetizing currents must equal the magnetizing current of the single-phase induction motor.

We shall draw a flux diagram and a current diagram to make this clear. These diagrams may be compared advantageously with the corresponding diagrams for the equivalent circuits.

An examination of the flux and current diagrams reveals the following simple relations: (Figs 167, 168, 169, 170, 171, and 172).

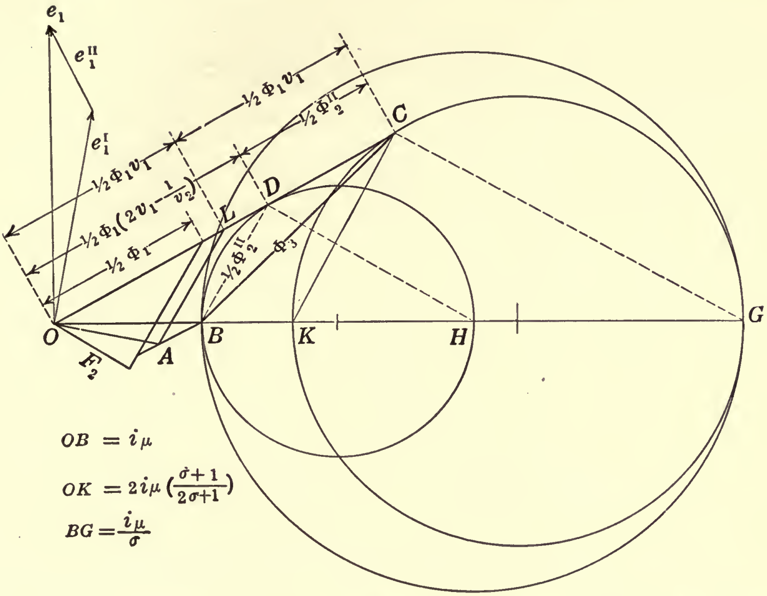


FIG. 168.—The time-phase vector diagram of the fluxes of the single phase induction motor. The two motor theory.

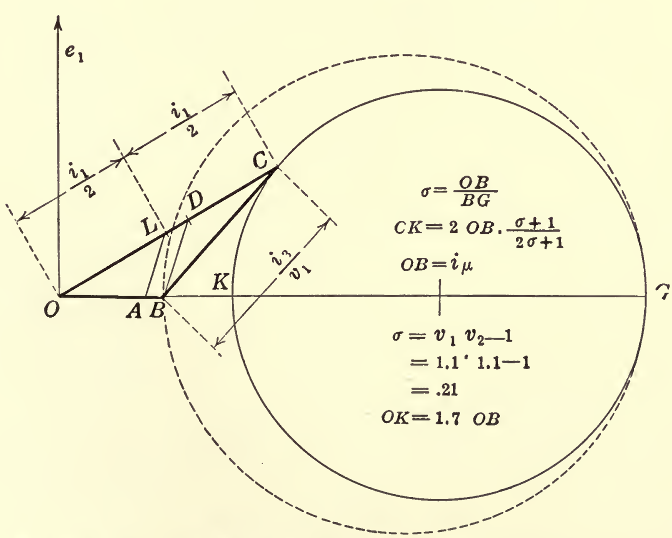


FIG. 169.—The circle diagram of the single-phase induction motor. The two-motor theory (after Behrend).

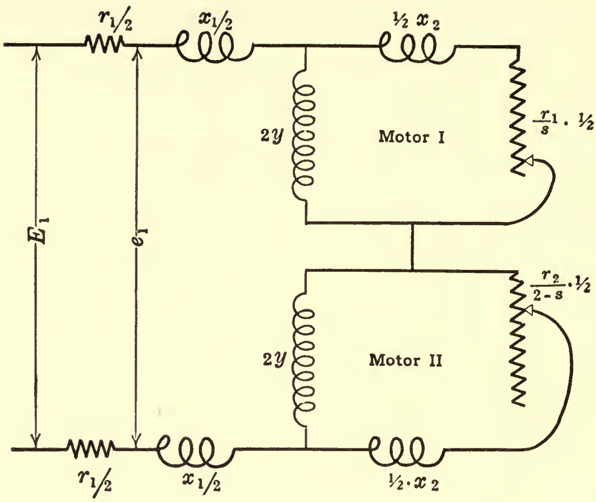


FIG. 170.—The complete and theoretically exact equivalent circuits of the single-phase induction motor in the two motor theory.

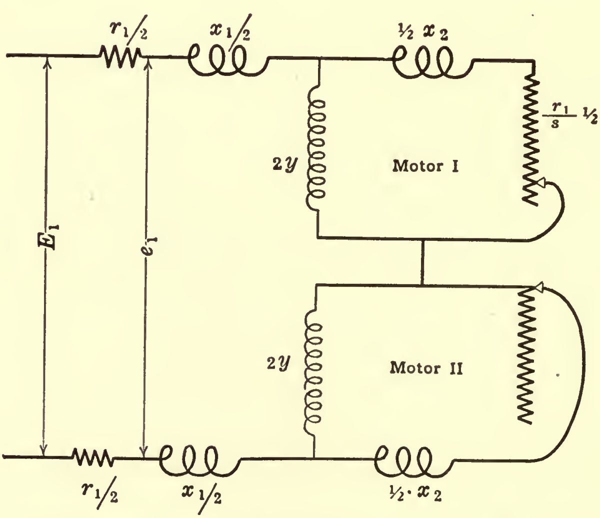


FIG. 171.—First approximation of equivalent circuits of single-phase induction motor in the two motor theory.

The ratio of the magnetizing current of the second motor i_μ^{II} to the total current of this motor at no load is $i_\mu^{II} \div \frac{i_0}{2}$. Now, $i_\mu^{II} : \frac{i_0}{2}$ is equal to the ratio of the sum of the leakage fields of the second motor to the total flux $\frac{1}{2}\Phi_1 v_1$. Hence,

$$i_\mu^{II} \div \frac{i_0}{2} = (f_1^{II} + f_2^{II}) \div \frac{1}{2}\Phi_1 v_1 \tag{196}$$

$$f_2^{II} = \frac{1}{2}(v_2 - 1)\Phi_2 = \frac{1}{2}(v_2 - 1)\frac{\Phi_1}{v_1} \text{ approximate}$$

$$f_1^{II} = \frac{1}{2}(v_1 - 1)\Phi_1 \tag{197}$$

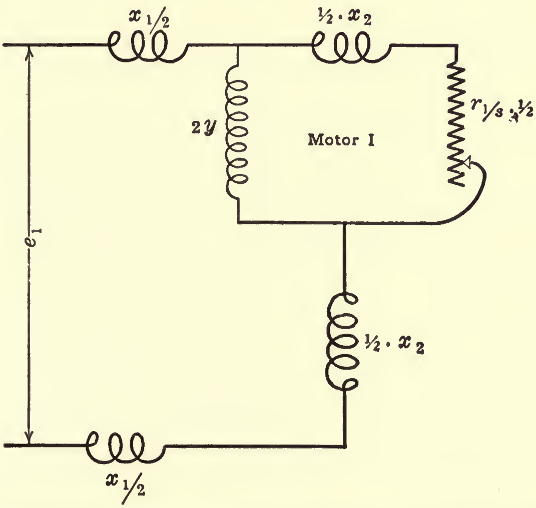


FIG. 172.—Second approximation of equivalent circuits of single-phase induction motor in the two motor theory. Admittance y negligible in comparison with $1/x_2$.

Therefore,

$$i_\mu^{II} \div \frac{i_0}{2} = \frac{\frac{v_2}{v_1} - \frac{1}{v_1} + v_1 - 1}{v_1} = \frac{v_1^2 - 1}{v_1^2} \tag{198}$$

assuming $v_1 = v_2$ which is permissible for the second motor. A simple transformation now yields

$$i_\mu^{II} \div \frac{i_0}{2} = \frac{\sigma + 1}{\sigma} \tag{199}$$

The total magnetizing current of the single-phase motor is the sum of the magnetizing currents of motor I and motor II.

$$i_\mu = \frac{i_0}{2} = i_\mu^{II} \tag{200}$$

$$i_\mu = \frac{i_0}{2}\left(\frac{2\sigma + 1}{\sigma + 1}\right) \tag{201}$$

In speaking loosely of "adding magnetizing currents" of two motors in series, we mean of course that the voltages impressed upon each motor may be added and to the sum of these voltages corresponds a magnetizing current equal to the sum of the magnetizing currents of each motor.

The same result is readily read off the Fig. 169 if we divide OD by OC. We obtain

$$OD \div OC = \left(2v_1 - \frac{1}{v_2}\right)\frac{1}{v_1} = (2v_1v_2 - 1) \div v_1v_2$$

$$i_\mu \div i_0 = \frac{1}{2}\left(\frac{2\sigma + 1}{\sigma + 1}\right) \tag{202}$$

For $\sigma = O$, that is, for a motor without leakage, we have:

A common value for σ is 0.05 when

$$i_\mu \div i_0 = \frac{1}{2}$$

The point G, Figs. 168 and 169, is determined in the same manner as in the theory of the general alternating current transformer, from the consideration that the ratio between the magnetizing current, proportional to OB, and BG, is equal to $\sigma = v_1v_2 - 1$.

It is clear that the point C moves also on a circle as the point D, which moves on the circumference of the circle BHD, divides OC in a constant ratio. (Fig. 168.)

Thus the locus of C is the circle KGC, where

$$OK = 2i_\mu\left(\frac{\sigma + 1}{2\sigma + 1}\right) \tag{203}$$

and

$$OB = i_\mu \tag{204}$$

and

$$BG = i_\mu \div \sigma \tag{205}$$

We have re-drawn the current diagram for these conditions in order to impress the picture more vividly upon the mind, Fig. 169.

b. THE CURRENTS IN THE ARMATURE

The advantages for popular use of this somewhat loosely-knit and so much misunderstood theory of two poly-phase motors connected in series to simulate the single-phase induction motor, may now be discussed in more detail. All manner of errors and mistakes have been made, even by leading writers, in the interpretation of the theory by means of the two-motor method. Twenty-six years ago, in a famous and otherwise brilliant book, an author assumed that the two poly-phase motors were connected in parallel and this same curious mistake has recently been reproduced in a noted textbook and also in handbook of wide circulation. But such misapprehensions are not inherent in the theory.

The current in the armature of the single-phase motor is equal to the vector sum of the secondary currents $v_1 \cdot BD$ and $v_1 \cdot DC$ of the two poly-phase motors, hence, it is equal to $v_1 \cdot BC$. A glance at Fig. 169 shows that only about one-half of the energy dissipated in the armature can be utilized for the production of the torque. At all loads the secondary currents represented by BD and DC remain very nearly equal and, as only one motor is doing useful work, the armature currents in motor II represent a loss very nearly equal to the loss in the armature of the working motor I. *The slip in the single-phase motor indicates, therefore, only about one-half of the energy dissipated in the armature,* hence, the loss in the rotor of a single-phase motor is twice as large as that in a poly-phase motor, provided the slip be equal in the two motors.

It is also interesting to note that, at synchronism, the rotor carries a current of double frequency. At speeds lower than synchronism a fundamental of slip frequency is superimposed upon the frequency of $\frac{\sim_1 + \sim_2}{\sim_1}$.[1]

c. THE TORQUE AND SLIP

The torque can be calculated as follows: Suppose the armature is wound in three phases, each having the resistance r_2. The output of the motor is then:

$$P = e_1 i_1 \cdot \cos\psi_1 - i_1^2 r_1 - 3 i_2^2 r_2 \qquad (206)$$

[1] M. I. Pupin is right, *Trans. A. I. E. E.*, 1918, p. 686, that it is not necessary to "assume two rotary fields produced by the stator current at all; in fact they have no physical existence; but the presence of two rotary magnetic fields produced by the rotor current is a fact."

from which follows

$$61.6D_{mkg} = \frac{p}{\sim_2} \cdot P_{watts} \tag{207}$$

in which equation D_{mkg} is the torque in *mkg* and p the number of north or south poles.

The following reasoning yields a value for $\sim_2$:

We have for motor I,

$$9.81 \cdot D_{\text{I}}(\omega_1 - \omega_2) = \frac{3i_2{}^2r_2}{2} \tag{208}$$

and for motor II

$$9.81 \cdot D_{\text{II}}(\omega_1 + \omega_2) = \frac{3i_2{}^2r_2}{2} \tag{209}$$

where $\omega_1 = 2\frac{\sim_1}{p}$ and

$\omega_2 = 2\pi \cdot n$ the angular velocity of the rotor at n revolutions per second.

Hence,

$$9.81(D_{\text{I}} - D_{\text{II}})\omega_2 = 3i_2{}^2r_2 \cdot \frac{1}{\left(\frac{\omega_1}{\omega_2}\right)^2 - 1} \tag{210}$$

$$3i_2{}^2r_2 = P_{watts}\left[\left(\frac{\omega_1}{\omega_2}\right)^2 - 1\right] \tag{211}$$

$$3i_2{}^2r_2 = P_{watts}\left[\left(\frac{\sim_1}{\sim_2}\right)^2 - 1\right] \tag{212}$$

Writing S for $\frac{\sim_2}{\sim_1}$ we obtain

$$3i_2{}^2r_2 = P_{watts}\left[\frac{1}{S^2} - 1\right] \tag{213}$$

To illustrate, Let us assume $\sim_1 = 50$, and $\sim_2 = 45$, then we have

$$\begin{aligned} 3i_2{}^2r_2 &= P_{watts}\left[\left(\frac{50}{45}\right)^2 - 1\right] \\ &= 0.23\,P_{watts} \end{aligned}$$

In words, if the slip is 10 per cent, the loss of energy in the armature amounts to 23 per cent of P or approximately twice the percentage of the slip, if figured upon $P + 3i_2{}^2r_2$.

It is instructive to compare (212) with the similar one in the poly-phase induction motor, which develops after some transformations:

$$3i_2{}^2r_2 = P_{watts}\frac{\sim_1}{\sim_2}\left(\frac{\sim_1 - \sim_2}{\sim_1}\right) \tag{214}$$

In words the slip in per cent is equal to the ratio of the armature loss to the entire secondary power $\frac{\sim_1}{\sim_2} \cdot P$.

In Fig. 173 the output and torque in watts and synchronous watts as a function of the rotor speed expressed in cycles per second have been represented as calculated from the results given in this chapter. The heavy lines represent a rotor with small resistance, while the broken lines represent a rotor with fairly large resistance. These curves represent actual conditions.

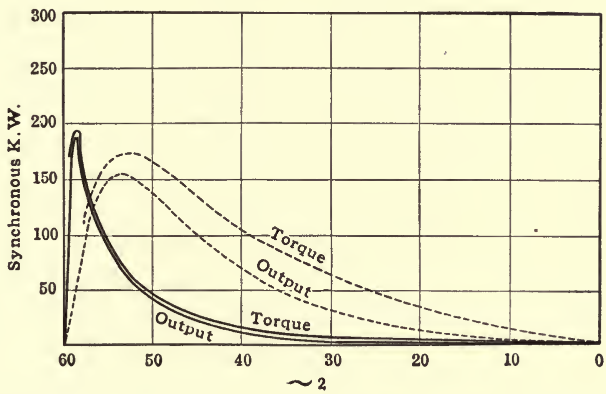


Fig. 173.—Output and torque curves of single-phase induction motor as a function of the speed.

d. EXPERIMENTAL DATA

There are reproduced here, from the first edition of this book, the characteristics of a 10-hp. single-phase motor for 110 volts, 50~, and 1,500 r.p.m. The total number of conductors in the field was 120; the total number of conductors in the armature was 312. The resistance of the field was 0.015 ohm; of each of the three phases of the armature, it was 0.08 ohm.

It is instructive to calculate the armature loss in watts for the greatest load. We find this to be $55^2.024 = 730$ watts, corresponding, according to equation (213), to a slip of 2.1 per cent. The discrepancy between this and the measured value of 2.66 per cent. is due probably mainly to the difficulty of measuring a small slip.

TESTS OF 10-HP. MOTOR

R.p.m.	Frequency	Slip, per cent	Amperes field	Amperes armature	Watts input	Watts output	Efficiency	Power factor
0	51.5	100.00	29.0	0	900	0	0.000	0.283
1,553	51.5	......	51.5	..	1,700	0	0.000	0.300
1,579	52.8	......	54.0	..	2,250	700	0.310	0.379
1,572	52.4	......	58.0	..	3,300	1,770	0.536	0.518
1,554	52.4	......	90.0	..	8,100	6,300	0.778	0.820
1,547	51.4	......	112.0	21	10,500	8,100	0.770	0.850
1,525	51.5	1.30	138.0	29	13,500	10,600	0.785	0.890
1,517	50.8	0.52	150.0	31	14,700	11,400	0.775	0.890
1,483	50.2	1.80	169.0	35	16,500	13,000	0.788	0.888
1,430	48.8	2.40	200.0	..	19,400	14,860	0.765	0.882
1,460	49.9	2.66	245.0	55	23,500	16,830	0.716	0.873

e. CALCULATION OF THE MAGNETIZING CURRENT OF THE SINGLE-PHASE MOTOR

We must now calculate the counter e.m.f. of the single-phase motor induced upon itself by the oscillating resultant field in the stator. We shall consider two cases, *viz.*, *first*, the case in which

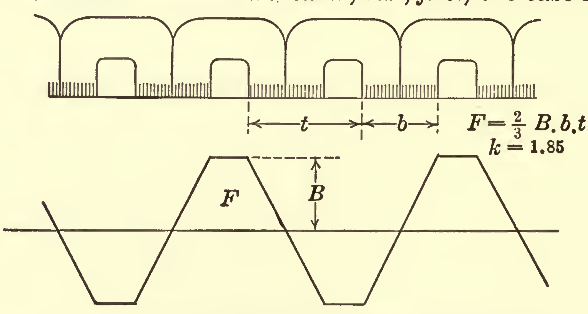


FIG. 174.—The field belt of the single-phase motor.

the field coil is spread over two-thirds of the pole-pitch; *secondly*, the case in which the field coil covers the entire pole-pitch.

First.—We saw in Chap. I that the e.m.f. induced by a field F in a coil of a certain width is two-thirds as large as the e.m.f. which would be induced by the same field in a coil which is not distributed but lodged in one slot. For the latter case, we have

$$e = 2.22 \sim \cdot z \cdot F \cdot 10^{-8} \text{ volts}$$

Figure 174 shows the form of the field for a coil the width of which is equal to two-thirds of the pole-pitch. Call the number of active conductors per pole n, then we have for the induction in the air

$$B = \frac{ni_\mu\sqrt{2}}{1.6\Delta} \tag{215}$$

In this formula i_μ is the effective value of the magnetizing current, Δ the air-gap on one side. If b is the width of the iron of the motor in *cm.*, t the pole-pitch, then we have,

$$F = \frac{2}{3}b \cdot t \cdot B \tag{216}$$

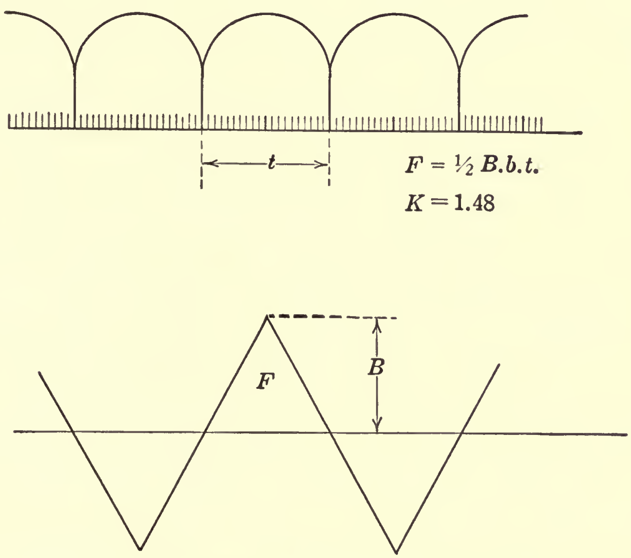


FIG. 175.—The field belt of the single-phase motor.

Instead of a coefficient of 2.22 we obtain, as will easily be seen, a coefficient 1.85. Thus, we have the equation

$$e = 1.85 \cdot z \cdot F \cdot 10^{-8} \text{ volts} \tag{217}$$

Secondly.—Figure 175 shows this case. The total flux is

$$F = \frac{1}{2}b \cdot t \cdot B \tag{218}$$

For the e.m.f. we have

$$e = \frac{2}{3} \cdot 2.22 \cdot \sim \cdot z \cdot F \cdot 10^{-8}$$

$$e = 1.48 \cdot \sim \cdot z \cdot F \cdot 10^{-8} \text{ volts} \tag{219}$$

It is evident from these equations that it is not advantageous to use too wide a coil spread.

CHAPTER XVIII

THE SINGLE-PHASE INDUCTION MOTOR, CONTINUED

B. THE CROSS FLUX THEORY

In 1894 Alfred Potier[1], and in 1895 and 1903 H. Goerges, discussed the theory of the single-phase induction motor on the basis of a cross field, or speed field, generated by the rotation of the rotor conductors in the resultant primary field of the main circuit. Dr. A. S. McAllister also developed this theory and he has always been with justice the advocate and protagonist of this theory in America versus the two-motor theory. J. K. Sumec, basing his development of the theory on the Blondel flux diagram and following the present author's method of reasoning in connection with the circle diagram of the poly-phase motor, derived, in a brilliant article in the *Zeitschrift für Elektrotechnik*, No. 36, Vienna, 1903, the correct circle diagram of the single-phase induction motor. In this chapter we shall follow mostly the clear analysis of the phenomena as given by Sumec and McAllister. Though the theory is not easy to master, I believe its study will repay amply as Dr. McAllister's contention is doubtless correct that the physical phenomena[2] are accounted for more logically in this theory than in the "two-motor" theory.

(a) A General Consideration of the Theory

A general consideration of this theory, based on Fig. 176 with slight modifications is taken from Dr. McAllister's book and it is a good introduction to the more complex analysis. The vertical set of poles may represent the exciting winding on the stator, in

[1] *Bulletin de la Société internationale des Electriciens*, Paris, May 1894.

[2] See also, J. Slepian, *Trans. A. I. E. E.*, 1918, p. 661. Referring to the two-motor theory Dr. Slepian says: "Of course, the instantaneous torque, heating, etc., cannot be obtained in this way. This points to one difference between the mechanically connected series machines and the single-phase current motor. In the former, the torque in each machine is constant in time, so that the same is true of their sum. In the latter the torque is pulsating, vanishing generally four times per cycle. The mean value, however, is the same in the two cases."

reality of course it is a distributed drum winding. It sets up a field which is vertical in space. If we consider the short-circuited rotor conductors, then the combined effect of the impressed m.m.f. and of the induced m.m.f. produces a resultant field F_2 which we shall consider here as in the transformer and induction motor.

It is perfectly clear that, so long as the rotor does not move, there cannot be any other resultant rotor field than the field F_2. However, as soon as the rotor cuts through the resultant field F_2, an e.m.f. of rotation will be impressed upon its conductors. The time-phase of this impressed e.m.f. must coincide

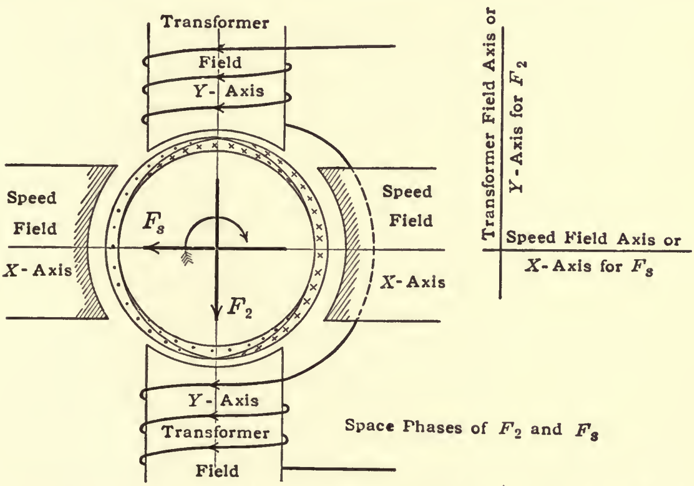


FIG. 176.—Physical representation of the cross flux theory of the single-phase induction motor. (After McAllister).

with the time-phase of F_2. Hence, if any current were to flow in the short-circuited rotor conductors, this current would produce a field whose space-phase was horizontal. Therefore, it is now clear that the rotation must produce such a field which is called the "cross field" or the "speed field," and that the magnitude of this field, as well as its time-phase, depend upon the rotor resistance, the rotor leakage, and the reluctance of the magnetic circuit of the speed field.

Let it be observed at the outset that, whatever the time-phase of these currents, on account of their space quadrature in relation to the main field they no more react upon this than does one phase of a two-phase motor react upon the other.

The induction produced by a certain magnetization is

$$B = \frac{0.4\pi n_2 i_x \sqrt{2}}{\rho_x}$$

where i_x is the current in the "x-system," *i.e.*, the speed field, and ρ_x the total reluctance of the speed field including its leakage field. The total flux follows directly from the area and distribution of the induction.

If there is no resistance in the rotor, or if the reluctance of the speed field circuit were zero, then the time-phase of the speed flux F_s would be in quadrature with the impressed e.m.f. of rotation,

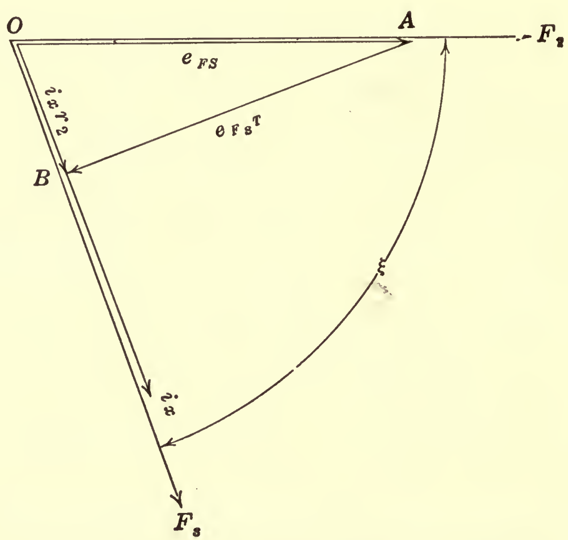


FIG. 177.—The e.m.f.'s in the speed field and their time-phases. Production of the speed field.

e_{FS}, lagging 90 degrees behind this e.m.f. (We use the subscripts $_{FT}$, $_{FS}$, $_{FsS}$ and $_{FsT}$ to indicate that an e.m.f. is due to the flux F or the flux F_s, and produced by transformation $_T$ or by speed rotation $_S$.)

Exactly as in an open-circuited transformer—*and the speed field is always in the open-circuited condition*—by impressing upon the speed field circuit the e.m.f. e_{FS} a current flows which, through its rate of change, induces a counter e.m.f. in time quadrature and lagging 90 degrees behind the flux produced by the current which is in time-phase with the flux. (See Fig. 177.) F_s is the speed field in time-phase with i_x which is the current. We designate the current with the subscript $_x$ as its m.m.f. acts in space in the X-axis.

Similarly, as in the transformer on open circuit,

$OA = e_{FS}$

is the impressed e.m.f. generated by rotation in the main field F_2

$AB = i_x r_2$ is the ohmic drop in the rotor.

$AC = e_{F_sS}$ is the counter e.m.f. produced by the rate of change of i_x. This counter e.m.f. may be considered as composed of two e.m.fs. if we wish to resolve the total field in the speed field or x-axis produced by i_x into a leakage field and an air-gap field. However, this is entirely unnecessary as the speed-field circuit is in the position of a choke coil.

Let x_0 be the total reactance of the speed-field circuit including its leakage effects and let it be inversely proportional to its reluctance p_x, then

$$AB = e_{F_sT} = i_x x_0$$

Hence,

$$OB + AB = i_x r_2 \div i_x x_0$$

$$\therefore tg\xi = \frac{x_0}{r_2} = \frac{K}{r_2 \rho_x} = \text{Constant} \qquad (220)$$

It appears, therefore, from this very general inspection, that the time-phase angle ξ between the speed e.m.f. $OA = e_{FS}$ and the speed flux F_s is constant.

Since OA must be in time-phase with F_2, it appears that the time-phase angle between the main field and the speed field is constant at all loads and speeds.

Considering now the main field, or the Y-system of the rotor in space, we note at once that there will be induced in it *first*, an e.m.f. of transformation e_{FT} and, *secondly*, an e.m.f. of rotation produced by the cutting of the rotor winding through the speed field F_s. This e.m.f. we denote by e_{F_sS}. The former is in time quadrature with F_2, while the latter is in time-phase with F_s. (Fig. 178.)

As e_{FT} is due to the rate of change of F_2, it lags in time 90 degrees behind F_2, while e_{F_sS}—produced by rotation in the speed flux—must be opposed to e_{FT}, their vector difference being equal to the drop due to the resistance and local leakage reactance of the rotor.

In the theory of the poly-phase induction motor we have replaced with advantage the local leakage reactances by their respective leakage fluxes. The same will be done here.

We shall consider from now on, unless specifically stated otherwise, that F_2 represents—as in the theory of the induction motor—the *net* flux actually passing through the rotor winding so that, vectorially, $F_2 = F - f_2$, where f_2 is the local secondary leakage flux in the transformer field or Y-axis.

$$e_{F_sT} = e_{FS} \cdot \sin \xi$$
$$e_{F_sT} = \omega \cdot F_s$$
$$e_{FS} = S \cdot \omega F_2$$
$$\therefore \omega F_s = S \cdot \omega \cdot F_2 \cdot \sin \xi$$
$$\therefore F_s = S \cdot F_2 \cdot \sin \xi$$
$$tg\, \xi = \frac{e_{F_sT}}{E_x} = \frac{\omega F_s}{r_2 i_x} = \frac{\omega \frac{i_x}{\rho_x}}{r_2 i_x}$$
$$tg\, \xi = \frac{\omega}{\rho_x r_2} = \text{constant}$$

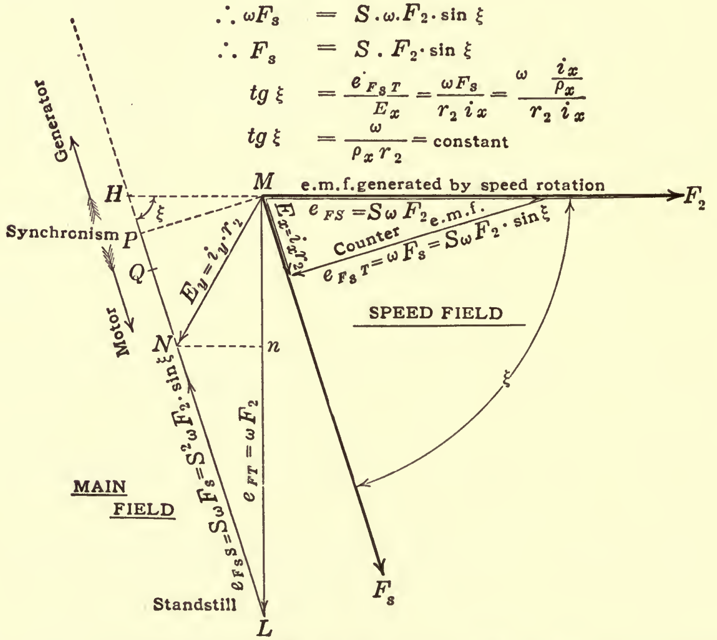


FIG. 178.—Single-phase induction motor. Time-phase diagram of the main field and the speed field and their e.m.f's.

If this is done, then, as in the theory of the induction motor, we may draw the following diagram in which the resultant of e_{FS} and e_{F_sT} is used to overcome only the resistance of the rotor winding the same applying to the resultant of e_{FT} and e_{F_sS}.

Very simple relations can now be derived for the speed flux in terms of the main flux. These follow directly from the geometrical figure. At standstill NL is zero and N lies at L. At synchronism N lies at P and above synchronism the motor turns into a generator. (Fig. 178.)

$$S \text{ is the ratio of } \omega_2 \div \omega_1$$
$$\omega_1 = 2\pi\sim_1$$

The reader is cautioned that we have designated the slip $s = \dfrac{\sim_1 - \sim_2}{\sim_1}$ with a small s, and the "speed" $\dfrac{\sim_2}{\sim_1}$ with a large $S = 1 - s$.

If the direction of rotation be reversed, *i.e.*, if S is made negative, S^2 remains positive and, therefore, the range from L to N and to P will be resumed. This is strikingly illustrated by the fact that a single-phase induction motor runs in either direction dependent on the direction of the impulse which is necessary to make it start.

(b) The Derivation of the Circle Diagram and the Locus of the Primary Current

It is interesting and important to investigate the locus of the primary current under different loads and speeds. (Fig. 179)

Let ab be the secondary leakage flux in time phase with $MN = i_y r_2$ of the previous figure.

Let $OA = F_2$ be the NET transformer flux, whose space-phase is the -axis.

Let $bc = NM = \Phi_2$ be the "fictitious" secondary flux proportional to i_y (i_y corresponds to i_2 in the theory of the transformer and induction motor).

$$ab \div bc = (v_2 - 1)$$
$$ac = v_2\Phi_2 \tag{221}$$

Then $Oc = \Phi_1$, the "fictitious" primary flux, proportional to i_1.

$$cM \div Oc = (v_1 - 1)$$
$$OM = v_1\Phi_1 \tag{222}$$

v_1 and v_2 are here as always Dr. John Hopkinson's stray coefficients.

Draw MG parallel to $Oa = F_2$, to the intersection G with the extension of $ON = F_1$.

Then the reader can readily prove that

$$\frac{ON}{NG} = \sigma = v_1 v_2 - 1 \tag{223}$$

This is our old well-known leakage coefficient of the transformer.

It is instructive now to draw the triangle LMN which is taken from our previous figure. We have chosen our scale for this triangle arbitrarily so that $NM = i_y = \frac{E_y}{r_2}$. As all quantities are entered into the figure, we shall let the reader think for himself.

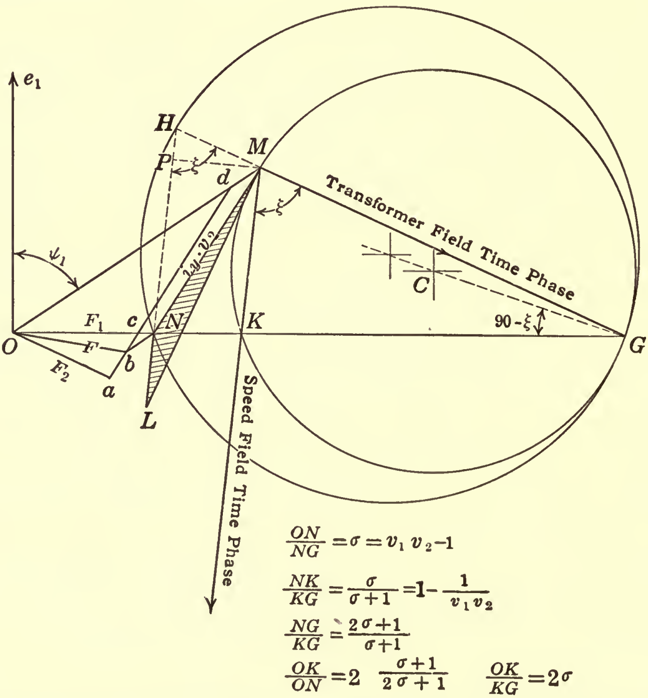


FIG. 179.—The primary current locus of the single-phase induction motor. (ML perpendicular to Oa See Fig. 178).

Draw LN to the intersection H with the extension of GM establishing at H the angle ξ. This angle we have seen to be constant and equal to

$$tg\xi = \frac{\omega}{\rho_x \cdot r_2}$$

Thus H lies on the arc of a circle with NG as a chord.

Through M draw MK parallel to HN, establishing the angle ξ at M, so that *the locus of M is the circle over KG as a chord.*

The center C of the circle KGM is readily seen to be determined by the angle

$$\sphericalangle KGC = \frac{\pi}{2} - \xi \tag{224}$$

Remembering that $E_y = i_y r_2$ and $i_y = \frac{E_y}{r_2}$ and that to i_x corresponds the total "fictitious" flux $v_2\Phi_2$ in view of the secondary leakage path being parallel with the air-gap reluctance, we obtain

$$\frac{NK}{KG} = \frac{HM}{MG} = \frac{BN}{MG}$$

From simple elementary geometry the reader will find,

$$\begin{aligned}\frac{HM}{MG} &= \frac{F_2}{v_2} \div NG\,v_1\frac{F_2}{F_1}\\ &= \frac{1}{v_1v_2}\frac{F_1}{NG}\\ &= \frac{1}{v_1v_2}(v_1v_2 - 1)\end{aligned}$$

$$\frac{HM}{MG} = \frac{\sigma}{\sigma + 1} \tag{225}$$

$$\frac{NG}{KG} = \frac{2\sigma + 1}{\sigma + 1} \tag{226}$$

$$\frac{OK}{ON} = 2\cdot\frac{\sigma + 1}{2\sigma + 1} \tag{227}$$

The last equation states that the ratio of no-load current to magnetizing current in the single-phase induction motor is equal to

$$2\,\frac{\sigma + 1}{2\sigma + 1}$$

Comparing this result with that obtained in Chap. XVII by an entirely different method, we find complete agreement, see equation (202). Also

$$\frac{OK}{KG} = 2\sigma \tag{228}$$

The no-load current of a single-phase induction motor is thus nearly twice the magnetizing current as the rotating field which exists near synchronous speed has to be supplied from one phase instead of from two- or three-phases as in the poly-phase motor.

(c) Sumec's Circles for Synchronism, No Load, and Standstill

The point P in the previous figure moves to N at synchronous speed $S = 1$. As PM is now in time quadrature with the speed field F_s the intersection with the circle KGM of a semi-circle over NK at M_s marks the synchronous speed point.

At no load, which occurs slightly below synchronism, we shall prove presently that the primary current lies at M_l determined as the intersection of a semi-circle over OK as diameter.

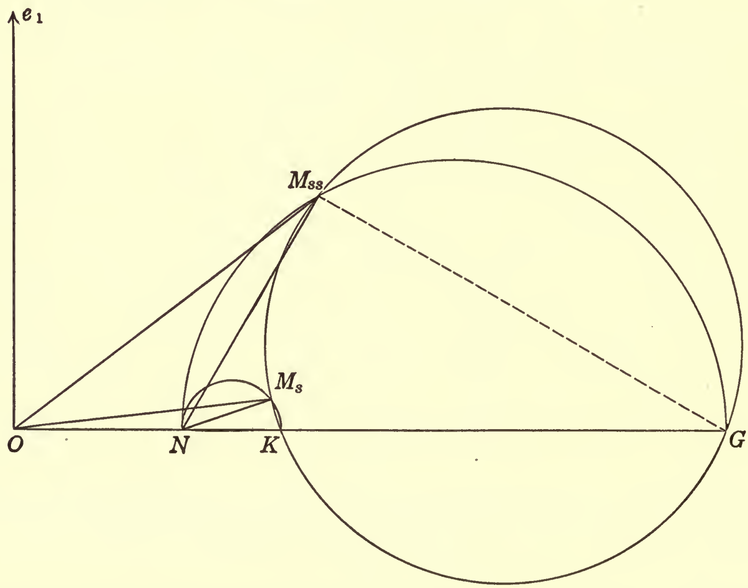


Fig. 180.—The determination of synchronous speed and standstill in the single-phase induction motor. (After Sumec).

At standstill NMG forms a right angle as in any stationary transformer, the vector of the primary current being OM_{ss}. These conditions are illustrated in the following figures. (Figs. 180 and 181.

(d) The Influence of the Rotor Resistance upon the Primary Current Locus

In very evident contradistinction from the poly-phase induction motor, the primary current locus of the single-phase induction motor depends very materially upon the rotor resistance.

It has been seen that the time-phase angle between the transformer field and the speed field, which we have designated with ξ, is a constant as long as neither the reluctance of the speed field nor the resistance of the rotor undergoes a change. We have seen that

$$tg\xi = \frac{\omega}{\rho_x r_2} \tag{220}$$

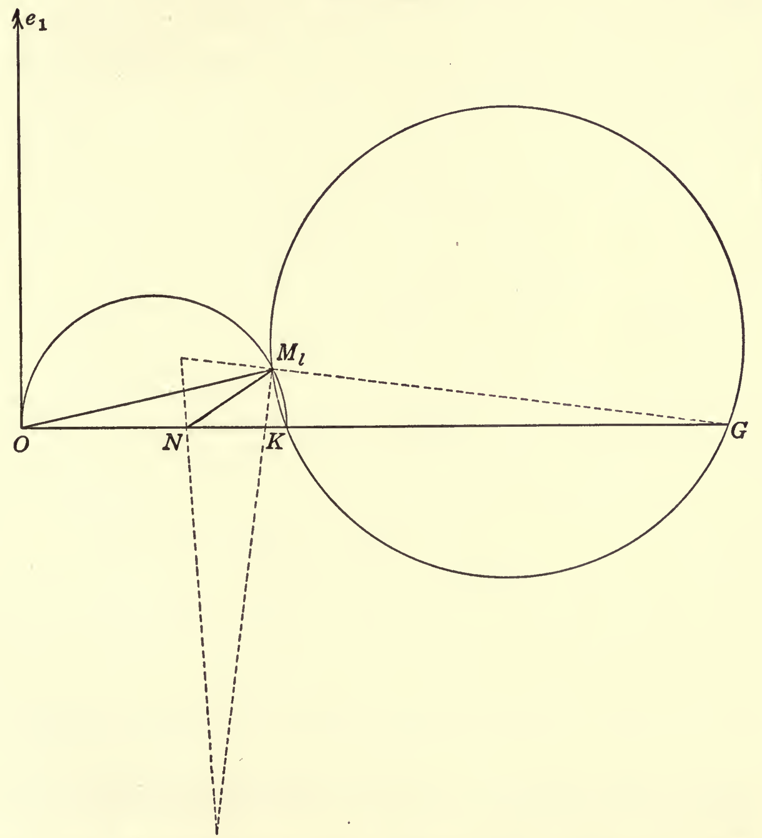


FIG. 181.—The determination of the no load point on the locus of the primary current in the single-phase induction motor. (After Sumec).

It is desirable that the angle ξ approach π_2 or that its tangent approach infinity. Hence, both *the reluctance of the speed field and the resistance of the rotor should be as small as possible.*

As the tangent of the angle KGC is

$$tg\left(\frac{\pi}{2} - \xi\right) = \cot \xi = \frac{\rho_x r_2}{\omega} \tag{229}$$

it is directly proportional to the secondary resistance and the distance of C from KG is proportional to r_2.

Thus J. K. Sumec has drawn a diagram showing at a glance the change in primary current locus as a function of the resistance of the rotor, Fig. 182.

We see at a glance, that, as the center of the circle rises from 0 to 0.2, the range of the motor measured between the "no-load" and "standstill" circles, is reduced to less than one-half. At 0.5, the motor can do no work at all. And above 0.5, the energy which is absorbed is dissipated into heat.

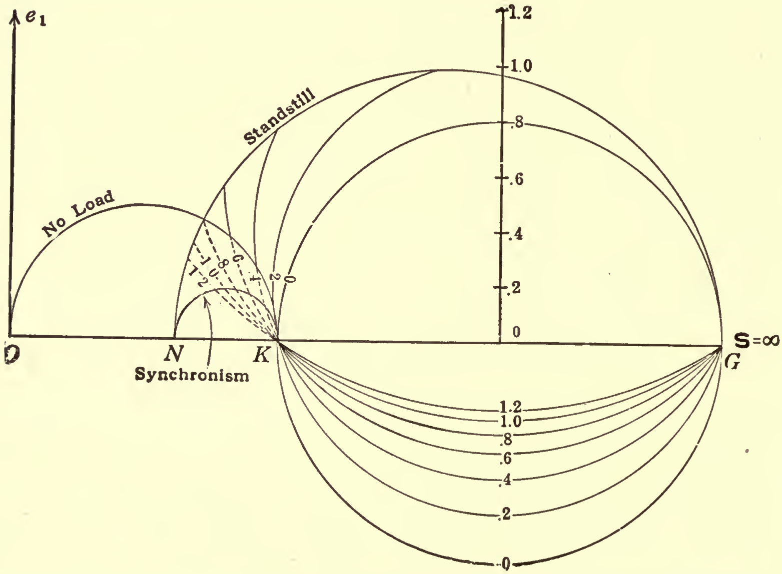


FIG. 182.—The effect of resistance in the rotor upon the locus of the primary current. (After Sumec). The single phase induction motor.

The range below the abscissa covers the operation of the induction machine as a generator.

(e) EQUIVALENT CIRCUITS

Before discussing further the properties of the single-phase induction motor, we wish to outline briefly how a system of electric circuits can be built up which will simulate the peculiar relations of phases and load in the single-phase motor. While these circuits appear more artificial here than in the poly-phase motor, yet it is interesting to develop them.

Referring to Fig. 178 $MN = E_y$ may be resolved into two components, one of which is a watt-component Mn, while the other is a watt-less component Nn.

We thus obtain the following relations:

$$\begin{aligned} Mn &= \omega_1 F_2 - \omega_1 F_2 S^2 \sin^2\xi \\ &= \omega_1 F_2 (1 - S^2 \sin^2 \xi) \qquad (230) \\ Nn &= \omega_1 F_2 \cdot S^2 \cdot \sin\xi \cdot \cos\xi \qquad (231) \end{aligned}$$

Two circuits must now be substituted in which the same currents flow, one purely a watt-current i_w, the other a watt-less current i_{wl}, but these currents, passing through a common reactance x_2, are fed from a common voltage proportional to $\omega_1 F_2$.

Hence, the resistance and reactance circuits must be composed of the following resistance and reactance:

$$Mn = i_w r_2 = \omega_1 F_2 (1 - S^2 \sin^2 \xi)$$

Also $$i_w = \frac{\omega_1 F_2}{R_w}$$

where R_w is the resistance in the equivalent circuit to produce the current i_w under an impressed voltage ωF_2.

Hence $$\therefore R_w = \frac{\omega_1 F_2}{i_w}$$

$$= \frac{r_2}{(1 - S^2 \sin^2 \xi)} \qquad (232)$$

Apply the same reasoning to the watt-less circuit.

$$Mn = i_{wl} \cdot r_2 = \omega_1 F_2 \, S^2 \sin \xi \cos \xi$$

Also $$i_{wl} = \frac{\omega_1 F_2}{X_{wl}}$$

where X_{wl} is the reactance to produce the same watt-less current i_{wl}.

Hence $$X_{wl} = \frac{\omega_1 F_2}{i_{wl}}$$

$$= \frac{r_2}{S^2 \sin \xi \cos \xi} \qquad (233)$$

And $$tg\xi = \frac{\omega_1}{\rho_x r_2} \qquad (234)$$

$$\therefore X_{wl} = \frac{\omega_1}{\rho_x} \cdot \frac{1}{S^2 \sin^2 \xi} \qquad (235)$$

We thus obtain the following circuits which simulate the characteristics of the single-phase induction motor, Fig. 183.

If the reluctance of the speed field circuit is infinite, the machine does not operate as a motor as it is permanently short-circuited.

If the reluctance of the speed-field circuit is zero, then the machine resembles somewhat a poly-phase induction motor as the extra watt-less circuit is open and $\sin^2 \xi = 1$.

Such a motor would show only a different speed characteristic from a two-phase induction motor.

Note.—The assumption that the effect of the leakage field in the rotor may be taken into account by assuming that the leakage of the m.m.f. of the currents of the Y-system only need be con-

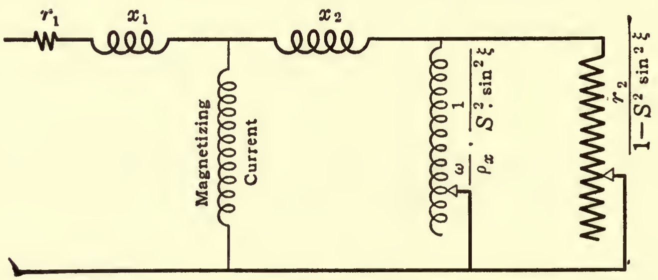


FIG. 183.—The equivalent circuits of the single-phase induction motor in the cross flux theory.

sidered, is an entirely arbitrary one. It is a very reasonable one and it appears to answer all practical considerations and to lead to a comparatively lucid picture of the phenomena within the machine.

However, as we have seen in Chap. IX, E, in discussing the secondary leakage reactance of the induction motor with slip rings and with commutator, there are here also local leakage fields which must induce e.m.fs. in the conductors of the rotor in such a manner that the leakage fields produced by the currents in the speed field affect the conductors in the transformer field.

This can most readily be seen if we study for a moment the conditions near synchronism. At synchronism there exists a nearly perfect rotating field created by the interaction of the X-currents which are in perfect space quadrature and nearly in time quadrature with the Y-currents.

That portion of this field which does not reach the primary, cuts the rotor conductors at slip frequency, thus the effect of the speed-field currents consists in diminishing the leakage reactance of the Y-system in the rotor.

(f.) Theoretical Considerations

Consider a single turn in the rotor. Denote the flux at time t in the Y-system with f_y^t and the flux at the same time in the X-system with f_x^t.

$$f_y^t = F_y \cdot \sin \omega t \tag{236}$$

$$f_x^t = F_x \cdot \sin (\omega t - \xi)^1 \tag{237}$$

Let the single turn form the angle α with the Y-axis, then the flux at time t passing through the coil is

$$\begin{aligned} f_\alpha^t &= f_y^t \cdot \cos \alpha + f_x^t \cdot \sin \alpha \\ &= F_y \cdot \cos \alpha \cdot \sin \omega t + F_x \cdot \sin \alpha \cdot \sin(\omega t - \xi) \end{aligned} \tag{238}$$

The e.m.f. induced in the coil at a rotor speed of $S\omega$ is

$$\begin{aligned} e_\alpha = -\frac{df_\alpha^t}{dt} = -\frac{\partial f_\alpha^t}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial t} \\ &= [-\omega F_y \cos \omega t - S\omega F_x \sin (\omega t - \xi)] \cos \alpha \\ &+ [S\omega F_y \sin \omega t - \omega F_x \cos (\omega t - \xi)] \sin \alpha \end{aligned}$$

which may be written

$$e_\alpha = e_y \cos \alpha + e_x \sin \alpha$$

as we may write

$$\left.\begin{aligned} e_y &= -\omega F_y \cos \omega t - S\omega F_x \sin (\omega t - \xi) \\ e_x &= S\omega F_x \sin \omega t - \omega F_x \cos (\omega t - \xi) \end{aligned}\right\} \tag{239}$$

The equations show that the mode of consideration of two space fields in each of which two e.m.fs. are active, a conclusion which we reached at the outset of this chapter on general physical principles, is consistent with a more careful mathematical analysis.

(g.) The Torque

The e.m.f. induced in a single rotor turn at the speed ωS is,

$$e_{\alpha s} = -\frac{\partial f_\alpha}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial t} = (f_y^t \cdot \sin \alpha - f_x^t \cdot \cos \alpha) S\omega$$

Through this conductor there flows a current

$$i_\alpha^t = i_y^t \cdot \cos \alpha + i_x^t \cdot \sin \alpha$$

[1] Throughout this paragraph we denote the "speed field" F_s by F_x, and F_2 by F_y.

Hence, the work done upon this conductor

$$-e_{\alpha s} i_\alpha^t \cdot dt = (f_x \cos\alpha - f_y \sin\alpha)(i_y^t \cdot \cos\alpha + i_x^t \cdot \sin\alpha) S\omega dt$$

Carrying out the trigonometrical multiplication and summing up the work over the total number of rotor conductors z_2, we obtain

$$\begin{aligned} dW &= \Sigma(f_x^t\, i_y^t \cos^2\alpha - f_y^t\, i_x \cdot \sin^2\alpha) S \cdot \omega \cdot dt \\ &= \frac{1}{2} z_2 (f_x\, i_y^t - f_y\, i_x^t) S\omega dt, \end{aligned}$$

and the *instantaneous torque*,

$$T_t = \frac{dW}{S\omega dt} = \frac{1}{2} z_2 (f_x^t\, i_y - f_y^t\, i_x) \qquad (240)$$

At any moment the instantaneous torque T_t consists of two quantities: A *positive* torque produced by the interaction of the "speed field" with the "transformer currents," and a *negative* torque produced by the interaction of the "transformer field" with the "speed field currents."

Let θ be the angle of time lag of the y-current relative to the main transformer-field F_2, then we have

$$\begin{aligned} T_t &= \frac{1}{2} z_2 [F_x i_y \sin(\omega t - \xi) \cdot \sin(\omega t - \theta) - \\ &\qquad F_y i_x \sin\omega t \cdot \sin(\omega t - \xi)] \qquad (241) \\ &= \frac{1}{4} z_2 [F_x i_y \{\cos(\theta - \xi) - \cos(2\omega t - \theta - \xi)\} - \\ &\qquad F_y i_x \{\cos\xi - \cos(2\omega t - \xi)\}] \end{aligned}$$

The resultant momentary torque, composed of a positive and a negative part, pulsates with double frequency as each part pulsates in this manner. The *mean* torque is therefore:

$$T = \frac{1}{4} z_2 [F_x i_y \cos(\theta - \xi) - F_y i_x \cos\xi] \qquad (242)$$

From Fig. 178

$$\begin{aligned} F_x &= S F_y \cdot \sin\xi \\ i_x &= \frac{S\omega}{r_2} F_y \cdot \cos\xi \end{aligned}$$

$$i_y \cos(\theta - \xi) = \frac{\omega}{r_2} F_y \sin\xi (1 - S^2)$$

By substitution in (242) above

$$\begin{aligned} T &= \frac{1}{4} \frac{z_2}{r_2} \omega F_y^2 S[(1 - S^2)\sin^2\xi - \cos^2\xi] \\ &= \frac{1}{4} \frac{z_2}{r_2} \omega F_y^2 S[(2 - S^2)\sin^2\xi - 1] \qquad (243) \end{aligned}$$

These are the equations as given by Goerges and Sumec.

From this equation follows:

1. For $S = 0$ and for

$$S = \sqrt{2 - \frac{1}{\sin^2\xi}}$$

the torque becomes zero, *viz.*, at starting and slightly below synchronism. These conditions we have already noted from a general consideration of the physical phenomena.

2. Increasing the rotor resistance diminishes ξ and therefore sin ξ thus the torque decreases rapidly as we have seen clearly in Sumec's circle diagram.

(h) Mechanical Output

From the equation for the torque T we have the output

$$\begin{aligned} P &= S\omega T \\ &= \frac{1}{4}\frac{z^2}{r_2}(\omega F_y)^2[S^2(1 - S^2)\sin^2\xi - S^2\cos^2\xi] \end{aligned}$$

The output due to the y-currents is proportional to, see Fig.178,

$$MN{\cdot}LN{\cdot}\cos(NML) = NP{\cdot}NL \tag{244}$$

The energy loss per second due to the x-currents is equal to $i_x{}^2{\cdot}r_2$.

$$E_x = S{\cdot}MP$$

$$S^2 = \frac{LN}{LP}$$

$$MP^2 = PH{\cdot}PL$$

so that

$$E_x^2 = PH{\cdot}LN \tag{245}$$

Hence the NET output is proportional to the difference of (244) and (245),

$$\begin{aligned} P &\equiv NP{\cdot}LN - PH{\cdot}LN \\ &\equiv LN(NP - PH) \\ &\equiv LN{\cdot}NQ \end{aligned} \tag{246}$$

(i) Rotor Copper Losses

In each conductor there is dissipated into heat in time dt

$$r_2 i_\alpha^{t2} dt = r_2(i_y^t \cos\alpha + i_x^t \sin\alpha)^2 dt$$

In summing up over the circumference of the rotor

$$r_2\Sigma((i_y^t)^2\cos^2\alpha + (i_x^t)^2\sin^2\alpha)dt = \frac{1}{2}z_2r_2((i_y^t)^2 + (i_x^t)^2)dt$$

Or, by integration, per unit time

$$\frac{1}{4}z_2r_2(i_y{}^2 + i_x{}^2)$$

Again from our figure

$$\begin{aligned} r_2i_y{}^2 &= PM^2 + PN^2 \\ &= (\omega F_y)^2[(1 - S^2)^2 \sin^2\xi + \cos^2\xi] \qquad (247) \\ r_2i_x{}^2 &= (\omega F_x)^2S^2 \cdot \cos^2\xi \end{aligned}$$

Hence the total rotor loss

$$\frac{1}{4}\frac{z_2}{r_2}(\omega F_y)^2[(1 - S^2)^2 \sin^2\xi + (1 + S^2) \cos^2\xi] \qquad (248)$$

CHAPTER XIX

THE SINGLE-PHASE REPULSION MOTOR

A. THE NON-COMPENSATED REPULSION MOTOR

The theory of the single-phase repulsion motor can be approached successfully in the same way as we have treated the poly-phase and single-phase induction motors. In fact, the use of the leakage fluxes shows their great effectiveness in their application to these interesting motors.

Throughout the treatment of the theory of alternating-current commutator machines it must be borne in mind that the phenomena occurring in the short-circuited coils under the brushes vitiate to a great extent all theoretical considerations. Any attempt to take the effects of short-circuit into account proves disastrous as the complications that have to be introduced into the theory befog utterly the mind that desires to obtain a general understanding of the characteristics of these motors.

We intend to limit our discussion to an ideal hypothetical motor without core losses and without losses in the coils under the brushes or other unpleasant confusing elements introduced by the process of commutation. However, the performance of this ideal motor is very instructive and it offers a clue to the understanding, and a good sign post to the designer, of the real motor.

(a) The General Theory.—A treatment of the theory of the repulsion motor consistent with the general theory which we have followed in this book, was given almost simultaneously by M. Osnos[1] and André Blondel.[2] Both papers are beautiful

[1] M. Osnos, *E. T. Z.*, Oct. 29, 1903.

[2] A. Blondel, *L'Eclairage Electrique*, Dec. 12 and 26, 1903.

contributions to the simple elucidation of the complex phenomena in the ideal repulsion motor.

Figure 184 shows diagrammatically the circuits of the repul-

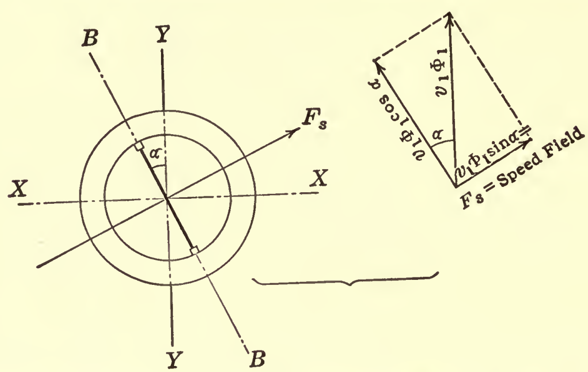


FIG. 184.—Single-phase repulsion motor and diagram of space phases.

sion commutator induction motor. The axis Y-Y is the axis of the stator field in space, while B-B is the axis of the short-circuited brushes on the commutator of the rotor.

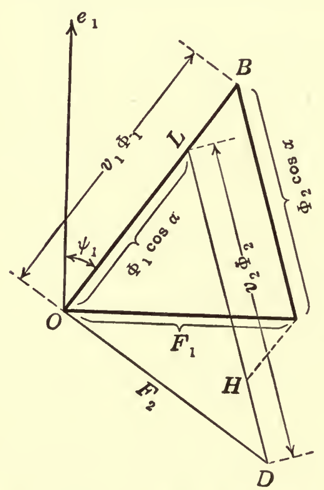


FIG. 185.—The flux diagram of the single-phase repulsion motor. (All time-phases.)

At the outset, we warn the reader that he should distinguish carefully in his mind between *space-phases* and *time-phases.*

The only way in which the short-circuited rotor can react upon the stator is by means of a field whose *space* axis lies in the Y-Y axis. Hence any rotor field reacting upon the stator is going to appear multiplied by cos α, the cosine of the angle of brush shift.

Again, the total field in the stator, appearing in the axis Y-Y, is not able to react upon the rotor. Only such part as falls into the brush axis B-B can affect the rotor.

Let, as before, $v_1\Phi_1$ be the total "fictitious" primary field, including the primary leakage fields, due to the primary m.m.f. Then, the interaction between $v_1\Phi_1$ and Φ_2 cos α from the rotor must produce, or leave over, so to speak, the real field F_1 which balances the impressed primary voltage.

Now, *all time-phases*, Fig. 185.

$$OB = v_1\Phi_1$$
$$AB = \Phi_2 \cos \alpha$$
$$OA = F_1$$

In order to find the rotor fluxes, we have to consider that the interaction of $v_1\Phi_1 \cos \alpha$ and $v_2\Phi_2$ leaves over the resultant rotor flux $F_2 = OD$.

These relations are those discussed again and again in this volume slightly altered quantitatively only by the mechanical configuration of the interacting systems.

As in the previous chapter we considered the resultant voltage produced by the action of the "transformer" field and of the "speed" field, so we consider here the similar e.m.fs.

The resultant "transformer" field in the axis B-B is F_2 which, through the periodic rate of change, induces a transformer voltage in time quadrature with F_2. This voltage we shall designate with e_{F_2T} to indicate, as in the cross-flux theory of the single-phase induction motor, that it is induced by F_2 and by the same process as that of transformation.

At right angles *in space* to the brush axis B-B there exists a "speed" field $F_s = v_1\Phi_1 \sin \alpha$ whose *time-phase* coincides with the *time-phase* of Φ_1. Through rotation at angular velocity $S\omega_1$, where S is again the *speed*, $S = \frac{\omega_1}{\omega_2}$, there is induced a "speed" e.m.f. $e_{F_sS} = S\omega_1F_s = S\omega_1v_1\Phi_1 \sin \alpha$, which if composed vectorially with e_{F_2T} gives a resultant which must be equal to the ohmic drop in the rotor all leakages having been taken into account through their respective leakage fields. (Fig. 186.)

It is now evident, and we remind the reader of the procedure in the previous chapter, that the angle at K which is equal to $90 - \xi$, is constant as $DK:OD$ is constant.

Hence the angle at L which is equal to $180 - \xi$ is constant.

Draw $O'D$ parallel to OA, then $O'L$ is proportional to OB and therefore to i_1, the primary current. As L lies on the arc $O'LG$ with C as center, the primary current locus is the arc $O'LG$.

$$O'L:LD::OB:AB$$
$$O'L:v_2\Phi_2::v_1\Phi_1:\Phi_2 \cos \alpha$$

$$\therefore O'L = v_1 v_2 \frac{\Phi_1\Phi_2}{\Phi_2 \cos\alpha} = v_1 v_2 \frac{\Phi_1}{\cos\alpha}$$

$$\therefore i_1 : LD :: v_1\Phi_1 : \Phi_2 \cos\alpha$$

$$i_1 : i_2 \frac{v_1}{\cos\alpha} :: v_1\Phi_1 : \Phi_2 \cos\alpha$$

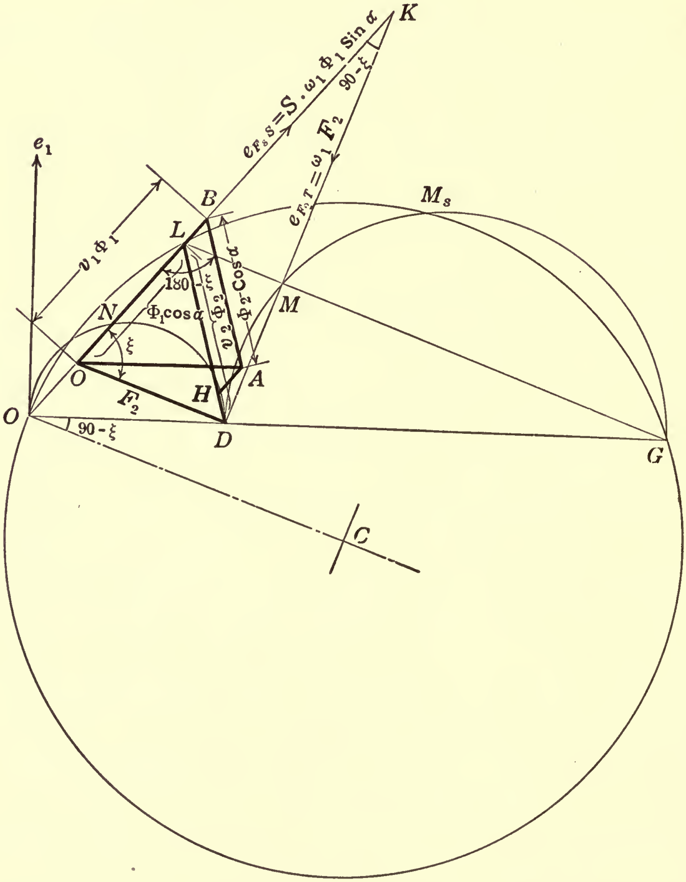


FIG. 186.—The diagram of the single-phase repulsion motor.

NOTE.—In order not to lose in simplicty of the diagram, we have assumed $r_2 = 1$, so that the scale for e_{F_sS} and e_{F_2T} and LD could be made the same.

$$O'L = i_1 \tag{249}$$

$$LD = \frac{i_2 v_1}{\cos\alpha} \tag{250}$$

From simple proportionalities follows

$$\frac{O'D}{O'G} = \left(1 - \frac{\cos^2\alpha}{v_1 v_2}\right) \tag{251}$$

Also

$$\frac{O'D}{DG} = \frac{v_1 v_2}{\cos^2\alpha} - 1 \tag{252}$$

Hence, the leakage coefficient of the repulsion motor is similar to that of the standard induction motor excepting that it depends also upon the brush-shift angle α.

If $\alpha = O$, then

$$\frac{O'D}{DG} = v_1 v_2 - 1$$

as is to be expected.

(b) The Speed in the Diagram.—We have found that $i_2 r_2$ is the vector resultant of $e_{F_s S}$ and $e_{F_2 T}$.

$$e_{F_s S} = LK = S\omega_1 \Phi_1 \sin\alpha$$

Now, $\Phi_1 \cos\alpha \cdot \sin\xi = MD$

Also $LK \cdot \cos\xi = LM$

Hence

$$S\cdot\omega_1 = \frac{LK}{\Phi_1 \sin\alpha}$$

$$= \frac{LM}{\cos\xi \cdot \dfrac{MD}{\cos\alpha \sin\xi} \cdot \sin\alpha}$$

$$S\omega_1 = \frac{LM}{MD} \cdot tg\xi \cdot \cot\alpha \tag{253}$$

The *speed* of the motor is therefore equal to the tangent of the angle $\angle LDM$.

This is zero at the point M_s hence M_s is the standstill point.

It approaches a maximum, but *not* infinity, as the primary current decreases.

(c) The Torque.—The torque is proportional to the product of the "speed" field F_s into the rotor current component in time-phase with the time-phase of the "speed" field, this latter being equal to the time-phase of the primary current, neglecting hysteretic lag.

Hence, we may write

$$T = \text{Const.}\ \frac{O'L}{v_1} \cdot \sin\alpha \cdot LD\ \frac{v_1}{\cos\alpha} \cdot \cos DLK$$

$$\equiv O'L \cdot LD \cdot tg\alpha \cdot \cos DLK$$

Construct a semi-circle over $O'D$ as diameter, then

$$LN = LD \cdot \cos DLK$$

Hence,

$$T \equiv O'L \cdot LN \cdot tg\alpha \qquad (254)$$

M. Osnos, to whom this relation was first due, Blondel and myself having borrowed it from him, points out that, for a given primary current $O'L$, the rapid decrease in torque is due rather to the increased phase lag between the rotor current and the primary current, than to the decrease in the rotor current itself.

The torque, as determined here, vanishes at the point P, hence the maximum speed the motor is capable of obtaining, barring losses, is reached at this point. The repulsion motor, therefore, does not run away like the series motor.

(d) The Effect of the Rotor Resistance upon the Diagram.—From the physical relations and the diagram we have

$$tg(90 - \xi) = \frac{i_\mu^{\text{II}} r_2}{\omega_1 F_2} = \rho \cdot r_2 \cdot \text{ Const.},$$

where i_μ^{II} is the magnetizing current of the rotor producing the field F_2, and ρ the reluctance of the air-gap path.

Hence, the distance of the center C from the abscissa $O'G$ is a measure of, and proportional to, the rotor resistance.

This result is similar to that obtained in the previous chapter in the theory of the single-phase induction motor.

Thus a number of circles may be drawn, as in Fig. 187, showing that the primary current locus of the repulsion motor is a semi-circle *only* if the rotor resistance is zero. For an infinite rotor resistance, the locus is a straight line and no power can be developed by the machine. For resistances between zero and infinity, the loci curves are arcs of circles whose centers lie on the same vertical line CC'. Rotor resistance control can be used for speed regulation as the diagram indicates. The maximum speed and the starting torque and starting current can be regulated in the same manner.

(e) The Effect of the Brush Shift.—Rocking the brushes and changing the angle α changes the ratio of the magnetizing current to the length of the chord of the arc of the locus of the primary current. It does not affect the angle ξ.

We have seen that

$$\frac{O'D}{DG} = \frac{v_1 v_2}{\cos^2\alpha} - 1$$

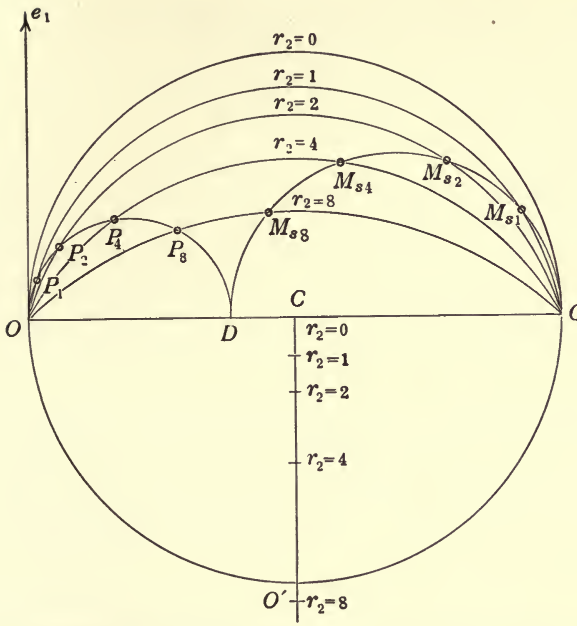


FIG. 187.—The influence of rotor resistance upon the primary current locus of the repulsion motor. Fixed brush position. (After Osnos).

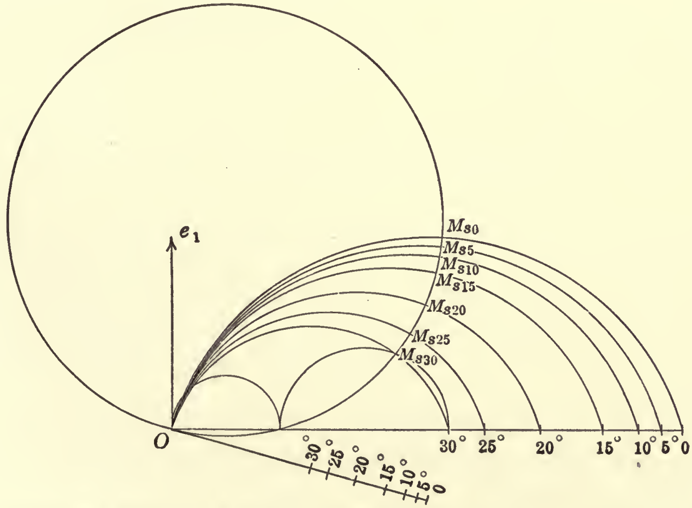


FIG. 188.—The influence of the brush shift on the primary current locus of the repulsion motor fixed rotor resistance.

This expression is a minimum, and therefore DG a maximum the smaller the angle α and the nearer $\cos \alpha = 1$.

For $\alpha = 0$, we obtain for this ratio $v_1 v_2 - 1$.

For $\alpha = \frac{\pi}{2}$, we obtain for this ratio 0.

Figure 188 shows these circles very much as given in the brilliant paper by M. Osnos already repeatedly referred to. The points M_s marking the starting point lie on a circle as indicated. The diagram suggests the mode of regulating the speed and torque by means of mechanical brush-shift used at one time extensively by the Brown Boveri Company for railway motors.

(f) **Commutation.**—The repulsion motor develops an elliptical rotating field as it gains in speed. At or near synchronous speed this field rotates with little or no slip relative to the rotor, hence the injurious effect of commutation through short-circuit currents induced in the coils under the brush is less in these motors than in motors of the series type.

However, in starting, there is no advantage in the repulsion motor over the series motor and it is in starting that the transformer effects in the coils under the brush are most injurious. It is doubtless on this account that the career of the repulsion motor as a railway motor has been rather brief.

A performance curve of an actual motor will be given in the Chapter on Commutator Motors.

B. THE COMPENSATED REPULSION MOTOR OF WIGHTMAN, LATOUR, AND WINTER-EICHBERG

(a) While the compensated type of repulsion motor in which the "speed" field is produced by passing the primary current through the rotor by means of an additional set of brushes, and leaving off the field coil, has ceased to be of practical importance, it still offers an object of great interest from the point of view of its theory.

It should be entirely clear what this motor really is, but to avoid misunderstanding let the two types of repulsion motor be placed side by side.

Supposing we wish to eliminate the field coil F in the connection diagram, Fig. 189, of the ordinary repulsion motor in order to get rid of the self-inductive effect which causes a voltage drop and a lowering of the power factor of the motor. The idea immediately suggests itself to utilize the armature itself for the purpose of exciting the "speed" field and to conduct the primary

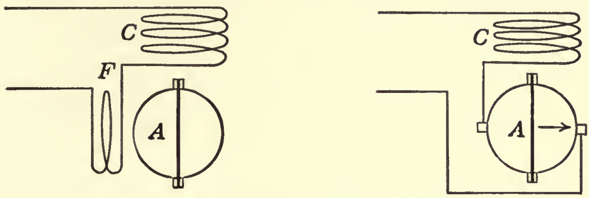


FIG. 189.—Repulsion motor and compensated repulsion motor.

current into the armature through a set of brushes in mechanical quadrature with the power brushes.

It remains for us to take stock of the gain or loss resulting from the abolition of a field coil on the stator and the addition of an extra set of brushes on the rotor.

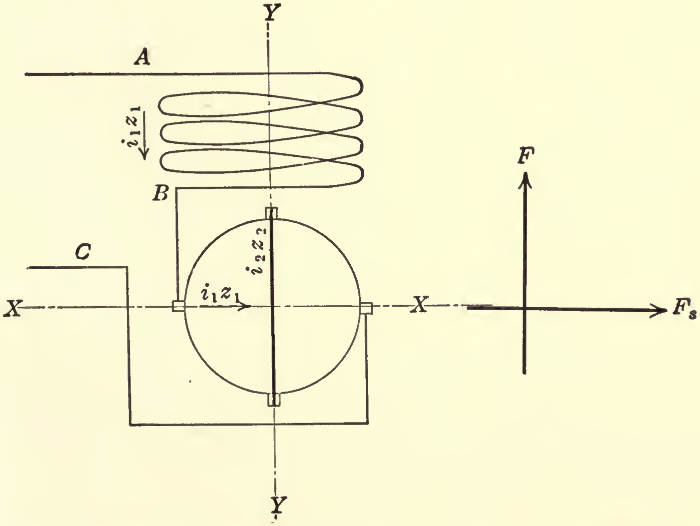


FIG. 190.—Connections and space phases of compensated repulsion motor.

Let us neglect leakage provisionally in order to get under way and to obtain a general idea of the phenomena within the motor.

We make our usual assumption that the active number of conductors on the stator and rotor is equal, $z_1 = z_2$. Then we have again in *space* two fields, at right angles to each other, one being the "transformer" field F in the Y-axis, the other being the "speed" field F_s in the X-axis. *Neither can react upon the other.* (Fig. 190.)

It follows that the resultant magnetization in the Y-axis which is due to the m.m.f. of the stator i_1z_1 and the opposing m.m.f. i_2z_2 of the rotor, must produce a field which balances the part of the impressed voltage between the points A and B. There must be added to AB the voltage drop BC, in order to obtain AC, which is the total impressed voltage. Thus AH is the primary m.m.f. and HM the secondary m.m.f. resulting in AM the magnetizing m.m.f.

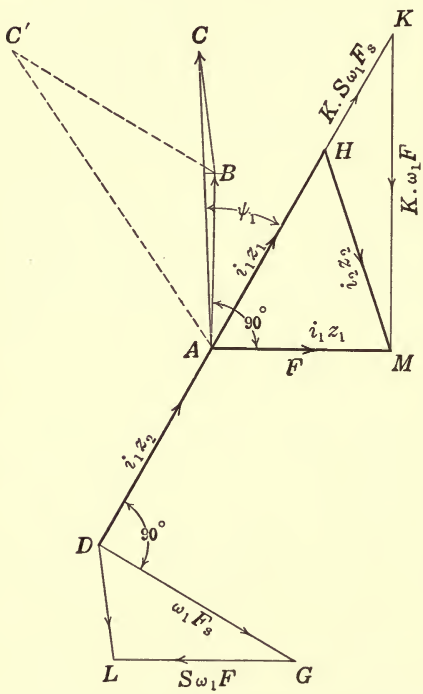


FIG. 191.—Time-phase diagram of compensated repulsion motor. (No leakage.)

However, the current creating the "speed" field in the X-axis induces through transformation an e.m.f. ω_1F_s which lags in the time-phase diagram Fig. 191 a quarter-phase behind the field. Let DG be this e.m.f.

When the motor stands still we have to add the e.m.f. DG with sign reversed to AB, obtaining BC' the total impressed e.m.f.

Now, assume the rotor to turn as a result of the torque produced between the current i_2 and the "speed" field F_s

There will be induced in the rotor four e.m.fs. acting in two pairs, in mechanical space quadrature.

First, as in the case of the repulsion motor, there are two e.m.fs. set up in the rotor in the Y-axis. The e.m.f. of transformation proportional to ω_1F and independent of the speed, and the speed e.m.f. $S\omega_1F_s$ due to the rotor cutting through the "speed" flux F_s produced by the primary current flowing through the exciting brushes in the X-axis. The resultant e.m.f. i_2r_2

must fall in the direction HM and, if we assume $r_2 = 1$, then HM is a measure of this e.m.f.

Secondly, between the exciting brushes in the rotor in the X-axis there is set up by rotation in the "transformer" field F a "speed" e.m.f. proportional to and in time-phase with F. The direction of this e.m.f. must follow Lenz's law and be opposed to the transformer e.m.f. DG.

Let $GL = S\omega_1 F$ be this "speed" e.m.f., then DL is the resultant e.m.f. which must be made up by an equal and opposite e.m.f. BC so that AC is the total impressed e.m.f. upon the motor.

Now, following a suggestion made by M. Osnos,[1] we may introduce a hypothetical or imaginary "transformer" field, to the rate of change of which there is due the "real speed" e.m.f. GL which is produced through rotation in the field F. (Fig. 192.)

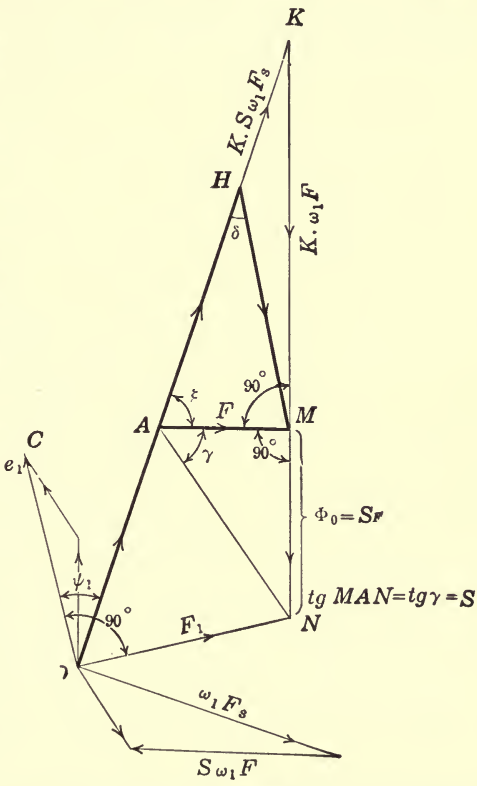


FIG. 192.—Time-phase diagram of compensated repulsion motor. (No leakage.) (After Osnos.)

As we wish to simulate a field Φ_0 which would induce, if it existed, by transformation the e.m.f. $S\omega_1 F$, we have

$$S\omega_1 F = \omega_1 \Phi_0 \qquad \text{or}$$
$$\Phi_0 = SF$$

Hence, make $MN = \Phi_0 = SF$, so that the tangent of angle $MAN = tg\gamma = S$, then in the force polygon $DNMA$ we find DN as the resultant field which we have usually designated

[1] *E. T. Z.*, November 12, 1903.

with F_1 and with which the impressed e.m.f. $A\,C$ is in time quadrature.

It is important to note that, if z_1 is not equal to z_2, as would be the rule in an actual motor, then, as AD represented the "speed" *field* in time-phase the "speed" *voltage* being measured by its magnitude, with, for instance, $z_2 = \frac{1}{2}z_1$, the speed voltage would be $S\omega_1 F_s \cdot \frac{1}{2}z_1$, and F_s being itself proportional to z_2, it follows

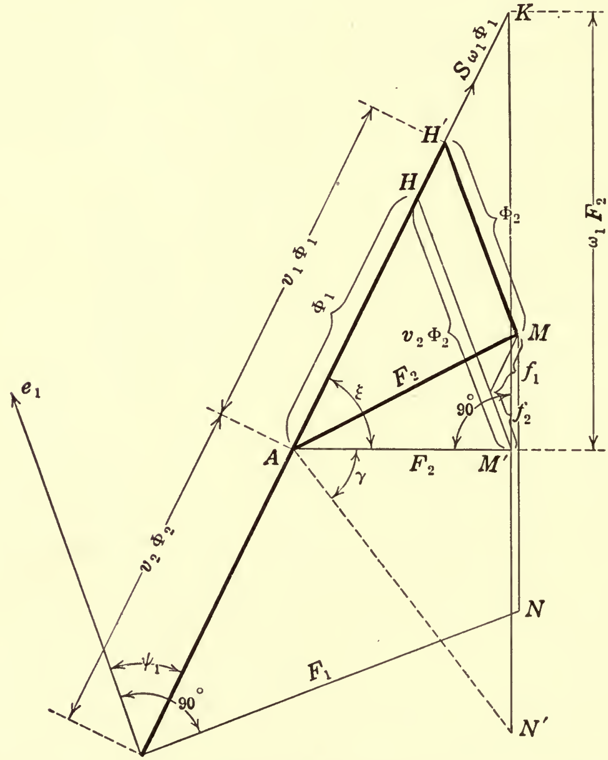


FIG. 193.—Polar, or time-phase, diagram of compensated repulsion motor. (Including leakage.)

that the "speed" voltage is proportional to $\left(\frac{z_2}{z_1}\right)^2$. Hence, AD must be multiplied with this ratio in order that it can be composed with AM and with the hypothetical field MN into *one resultant* DN in time quadrature with e_1.

(b) The Torque.—The motor has two torques, a positive and a negative one.

The positive torque is

$$T_1 \equiv DA \cdot HM \cdot \cos \delta \tag{256}$$

The negative torque is

$$T_2 \equiv -DA \cdot AM \cdot \cos \xi \tag{257}$$

as is readily seen from Fig. 192. As the speed increases, these torques become more and more equal to one another. The motor cannot run away and it is in this respect like the repulsion motor.

(c) **Performance.**—As will be shown from an actual performance curve in Chap. XX, the motor acts like a repulsion motor with the same advantages as to commutation, approaching a rotating field at speeds near synchronism. Its power factor increases with the speed.

(d) **Leakage.**—The leakage is taken into account in Fig. 193. As usual we lay down the total primary "fictitious" flux which would be produced if the primary m.m.f. acted alone on the magnetic circuit. This is $AH' = v_1 AH$.

$$HM' = v_2 HM$$
$$AM = F_1'$$
$$AM' = F_2$$
$$M'N' = \Phi_0 = SF_2$$

$DN = F_1$ which is *not* a real field, the real primary field in the Y-axis being $AM = F_1'$, $DN = F_1$ being a hopothetical field as the "speed" voltage induced by rotation in the field F_2 is not represented by a field in the Y-axis. To obtain the real physical conception of the phenomena in the motor, it is well to adhere to Figs. 191 and 192.

CHAPTER XX

SINGLE-PHASE COMMUTATOR MOTORS

A CONDENSED REVIEW

With the appearance in 1902 of Mr. B. G. Lamme's paper on the application of single-phase commutator motors to railway work,[1] a new impetus was given to the inventors and engineers the like of which had not been witnessed since Tesla's great invention. Though the particular installation referred to in the paper was never actually executed, though the motor was, in Steinmetz's[2] expression, "our old friend" the single-phase commutator motor, yet there was infused new hope and energy into the railway field. The feverish activity which absorbed the engineering community for 10 years after the reading of Mr. Lamme's paper resulted in the creation of innumerable types which, while they varied only slightly in the manner of their operation, attracted attention altogether beyond their intrinsic value and interest.

Nineteen years have passed and a more sober frame of mind has superseded the fond dreams of that early period. A few types have survived among them particularly the conductively compensated series motor with interpoles in shunt connection and the repulsion induction motor already treated in Chap. XIX.

[1] B. G. Lamme, "Washington, Baltimore & Annapolis Single-phase Railway," *Trans. A. I. E. E.*, September, 1902.

[2] "I believe we can congratulate ourselves then that here is published the record of some work done in the direction of developing apparatus, giving the proper characteristics for alternating current railway work. I must confess, however, that I have been somewhat disappointed in reading this paper, by seeing that after all this new motor is nothing but our old friend the continuous current series motor adapted to alternating currents by laminating the field. Now, I remember this type of motor very well because I was associated with Mr. Eickemeyer in 1891 and 1892, and we spent a very great deal of time in building alternating current series motors, investigating their behavior, and trying to cure them of their inherent vicious defects." Mr. C. P. Steinmetz's discussion of Mr. Lamme's paper, *Trans., A. I. E. E.*, Sept. 6, 1902.

Sincerely Yours, - B. G. Lamme

(*Facing page* 246)

A. VARIETY OF TYPES OF SERIES A. C. COMMUTATOR MOTORS

In the accompanying diagrams a number of connections are shown representing a few of the very large number of schemes which have been brought to light as a result of the feverish inventive activities in this direction.

Many of these connections are quite old and interest in them was revived when single-phase motors again commanded attention as a result of the work of Mr. Lamme and Mr. Westinghouse.

For instance, the interesting compensated motor later reinvented by Marius Latour and Winter and Eichberg, is described fully in the U. S. patent by M. J. Wightman, No. 476, 346, dated June 7, 1892, and assigned to the Thomson-Houston Electric Company.

Mr. Lamme's first motors were not compensated. They had a field winding of few turns and a magnetic frame of low reluctance. To handle the transformer current so troublesome in starting and at low speeds, induced in the coils short-circuited under the brushes, Mr. Lamme used resistance leads ingeniously embedded in the slots side by side with the conductors. We refer the reader to U. S. patents No. 758, 667, May 3, 1904, and to No. 758,668 of the same date.

It is very easy and very human and natural to remark, as was done at the time when Mr. Lamme read his epoch-marking paper that there was little that was new in the system he described. That statement may be admitted. Yet no one had used these old ideas and no one had designed a workable single-phase commutator motor. I think the present author may claim fairly for himself that he never became a single-phase enthusiast. The limitations of the system were too deeply forced upon his attention in his work done in the middle nineties. At no time did he share the enthusiasm expressed, for instance, by Mr. Steinmetz in the following telling passage:[1]

> "If I may be permitted to take a look into the future, although we do not know what to-morrow will bring, I think the system of the future will be the single-phase system. Where the power is transmitted over a long distance by an overhead wire, the ground can be used as the return conductor. . . . But if we use a single-phase current in the power transmission of the future, then we will have to learn many things."

[1] "Proceedings of the International Electrical Congress held in the City of Chicago. Published by the *A. I. E. E.*, New York, 1894, p. 437.

The present writer sounded a note of scepticism and caution in a paper in *Cassier's Magazine*, May, 1907, where he said:

"There is, in the realm of ideas, a distinct difference between 'natural' ideas and 'forced' ideas. The natural idea may be likened to a plant growing under favorable conditions and adapting itself to its environment; the forced idea may be likened to a plant raised in a hothouse, with the exclusion of such conditions as might have a tendency to prevent its development. The natural idea will survive; the forced idea will go to the wall; but it is often only after extended experiments conducted on a large scale have been laboriously completed that we realize that an idea has been followed out which could have lived only under particularly favorable conditions, such as are not usually found in practical operation. In contemplating the history of the development of the utilization of alternating currents, the single-phase system has appeared to be an almost ideally simple system. It is only too obvious that, if power could be safely transmitted and utilized in an economical manner, and by means of simple mechanical apparatus, the generation and utilization of single-phase currents would soon replace the poly-phase system. Such attempts were made 15 years ago, and, after considerable effort, most engineers abandoned hope in developing a practical system of transmission of energy by means of single-phase currents. The experience gained during the past 15 years with polyphase currents and the many opportunities afforded the engineer for comparing the single-phase machinery, generators, and motors, with their poly-phase cousins, have led to an attitude of skepticism towards single-phase current. The very much reduced output of both generators and motors if operated single-phase; the reduced efficiency; the impaired regulation; the increased heating, and the lesser stability of single-phase motors and generators, connected with the increased cost as produced by the greater amount of material required; these form the main reasons for inducing me to call the recent attempts which have been made in the utilization of single-phase current, *a forced idea*."

And later in the same paper the present author remarked:

"Those gigantic experiments to be conducted on the New York, New Haven & Hartford Railroad are being watched with the respect due an enterprise of such magnitude, and with the hope that, even if the system should not be all that its ingenious designers had expected, it may yet lead on to ideas which will finally solve the problem of electrically operating the present steam railways of the world."

Starting with the non-compensated series motor (1) we proceed to the type with conductively compensated armature (2) and then the inductively compensated connection suggests itself (3). (Fig. 194.)

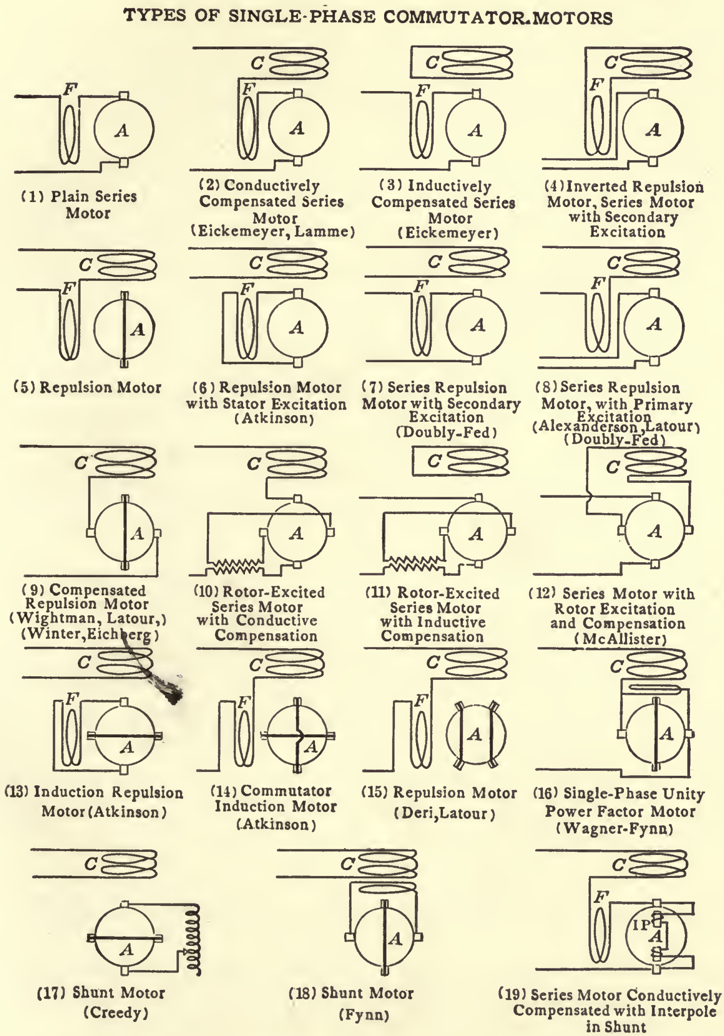


Fig. 194.—Single-phase commutator motors. A variety of connections.

As the next step we may excite both the field and the compensating coil inductively (4).

By exciting the field and compensating windings from the bus bars and short-circuiting the rotor we obtain one of the many types[1] of repulsion induction motor (5).

By splitting field and compensating winding, we obtain Figs. (6) and (7).

By feeding both the field and compensating windings in series from one potential and the armature from another we obtain the interesting "doubly-fed" motor of Latour[2] and Alexanderson.[3]

By exciting the field, which is located on the stator in the repulsion motor (5), in the rotor proper, by means of an additional set of brushes at right angles to the short-circuited brushes, we arrive at the Wightman-Latour-Winter-Eichberg compensated motor (9) the chief advantage of which lies in better commutation and higher power factor than in the repulsion induction motor. The extra brushes are a grave mechanical drawback and they would seem to be too high a price to pay for the advantage obtained.

(10), (11), (12) the McAllister connection, and (13) are modifications of (9).

(14) is a single-phase commutator induction motor, also due to Atkinson, I believe.

Deri and Latour suggested (15), Fynn suggested (16), Creedy suggested (17), and (18) is again a Fynn motor.

(19) appears to represent the chief survivor of this great host of types. It is a plain series conductively compensated single-phase commutator motor with interpoles excited from the impressed voltage so as to produce a commutating field of the right time-phase. If motors of this type are started with direct current and operated on single-phase alternating current, then commutation will be very satisfactory.

B. OPERATING CHARACTERISTICS OF DIFFERENT TYPES

In order to give a general idea of the performance of some of the types of single-phase commutator motor which have been enumerated here we reproduce from tests the following:

[1] See Llewelyn Birchall Atkinson, *Proceedings*, Institution of Civil Engineers of Great Britain, Feb. 22, 1898. "The Theory, Design and Working of Alternate-Current Motors."

[2] M. C. A. Latour, U. S. Patent No. 841,257, Jan. 15, 1907.

[3] E. F. W. Alexanderson, U. S. Patent, No. 923,754, June 1, 1909.

Figure 195 shows the characteristics of a repulsion motor like (5).

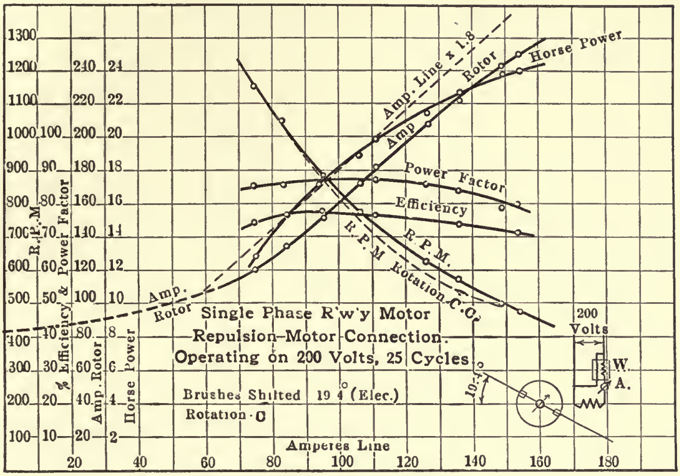


FIG. 195.—Operating characteristics of a single-phase repulsion motor.

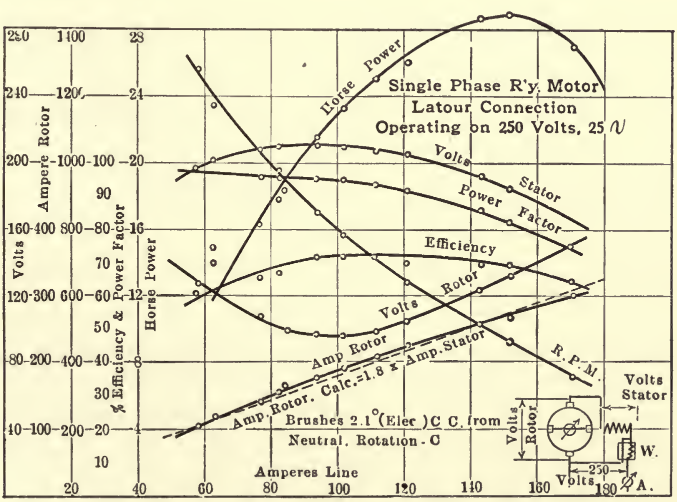


FIG. 196.—Operating characteristics of a single-phase repulsion motor. Latour connections.

Figure 196 shows the performance characteristics of a compensated repulsion motor with Latour connections like (9).

Figure 197 shows the performance characteristics of the same motor connected as suggested by A. S. McAllister in (12).

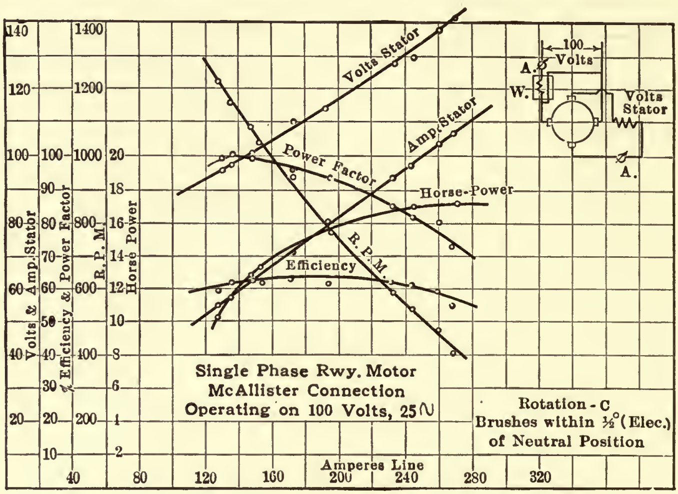


FIG. 197.—Operating characteristics of a single-phase commutator motor. McAllister connections.

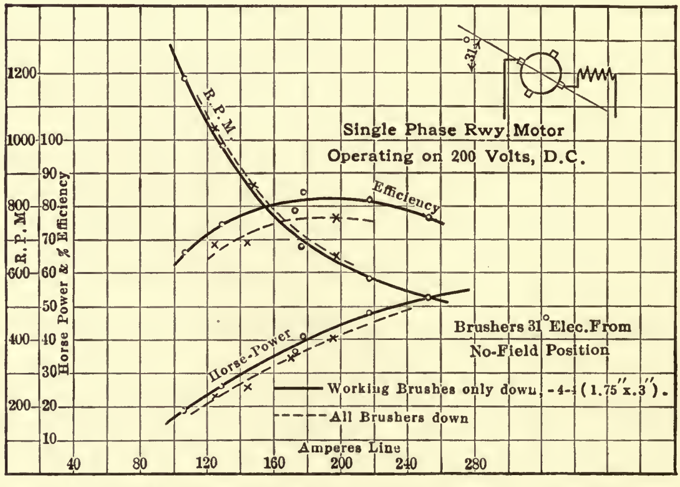


FIG. 198.—Operating characteristics of a single-phase commutator motor on direct current.

Figure 198 shows the performance of the same motor operating as a direct current motor.

Examination and comparison of these curves are left to the reader.

Figure 199 shows the performance of one of the latest types according to (19) designed by the Westinghouse Company.

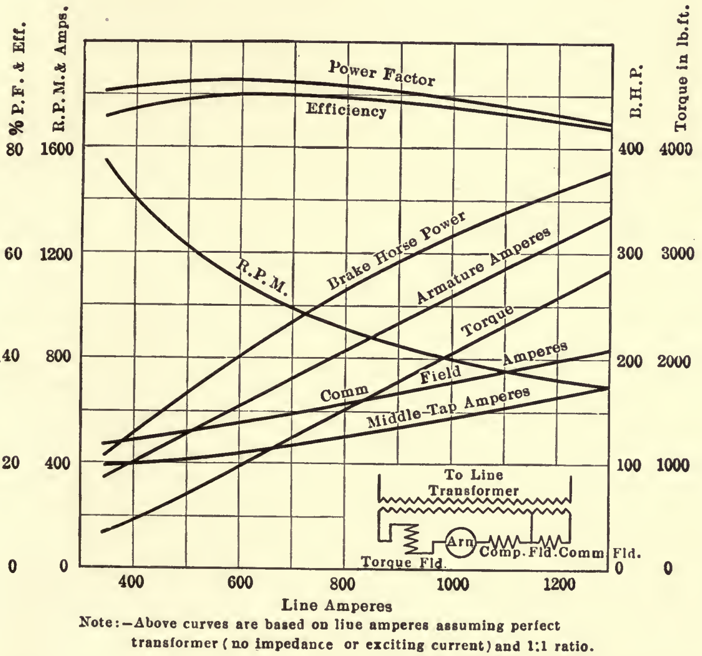


FIG. 199.—Operating characteristics of conductively compensated series commutator motor. One of the latest types of the Westinghouse Company.

C. METHODS OF IMPROVING COMMUTATION[1]

(a) Resistance Leads and Limits of Voltage for Commutation. The re-introduction by Mr. Lamme of resistance leads, already known in connection with the development of the direct-current generator, to take care of the transformer current under the brushes, appeared at first as a happy solution of a very troublesome problem. In spite of the ingenious mechanical disposition of these resistance leads, they became a source of great trouble

[1] See a valuable paper by B. G. LAMME, "The Alternating-Current Commutator Motor," *Journal, A. I. E. E.*, April, 1920. Also, E. ARNOLD, Vol. V, Part II.

on account of mechanical break-down, doubtless due to vibration difficult to cope with. These resistance leads have been abandoned.

In the paper referred to below, Mr. Lamme gives as a practicable average voltage under the brush, which will still permit satisfactory operation in railway motors operating under average conditions not exceeding one hour full load, a maximum of 10 to 12 volts. No general rules can be laid down.

(b) (1) The object of the interpole in a direct-current machine consists in generating an e.m.f. in the coil under commutation equal and opposite to the e.m.f. of self-induction which tends to prevent the rapid rate of change of direction necessary for sparkless commutation.

The same object is obtained in the alternating-current series motor by a series coil placed upon an interpole so that the interpole flux is in time-phase with the "speed" field and hence with the primary current.

(2). In addition to the creation of an e.m.f. which takes care of the e.m.f. of self-induction, it is necessary to compensate for the e.m.f. induced through transformation in the coil under commutation. As this e.m.f. has to be produced by rotation in order to have the same phase, it must be produced by a flux in time quadrature with the "speed" field. However, the "speed" field is in time-phase—barring hysteresis—with the primary current, hence the exciting current of the interpole must be in time quadrature with the primary current.

Dr. Behn-Eschenburg and Latour shunt the interpole with a non-inductive resistance to obtain the right phase relation to take care of both (1) and (2).

As pointed out by Arnold, the time phase of the transformer e.m.f. can be compensated by a connection of the interpole to the rotor, while the interpole winding should be connected to the field coil whose voltage is in quadrature to the current in order that it should compensate the e.m.f. of self-induction. Thus, if the interpole coil is connected across the impressed potential of the motor, a good general effect may be obtained.

As the compensation of the "transformer" voltage by means of a "speed" voltage requires rotation, compensation at starting when it is most needed, cannot be obtained in this manner.

D. THE SHUNT EXCITED A. C. COMMUTATOR MOTOR[1]

Referring to our compilation of diagrams of connections the types (13), (14), (15), (16), (17) and (18) may be considered as shunt types in contradistinction to the series types.

A plain shunt connection like Fig. 200 cannot be effective as the time-phase of the field flux lags approximately a quarter of a period behind the impressed e.m.f. and therefore there can be no torque developed by the rotor as the field flux and the rotor current would be in time quadrature.

(*a*) It has been suggested to use condensers for obtaining time-phase equality between the flux and the impressed e.m.f.

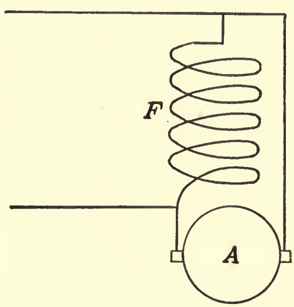


Fig. 200. — Single-phase shunt motor. (Inoperative).

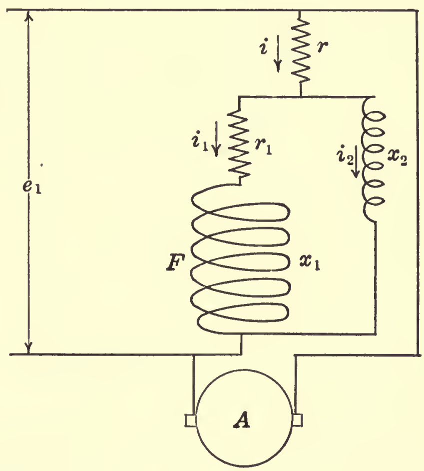


Fig. 201.—Single-phase shunt motor with Behn-Eschenburg connection.

(*b*) Another method perhaps equally undesirable has been suggested by Dr. Behn-Eschenburg, which we wish to mention here because of its theoretical interest. (Fig. 201.)

In series with the exciting coil x_1, which is to produce the field of the motor, there is connected a resistance r_1 and the field coil x_1 and resistance r_1 are shunted by an impedance x_2r_2 and in series with these parallel circuits there is connected a resistance r. In this manner the field current i_1 can be brought in phase with the impressed e.m.f. E.

[1] See F. Creedy, "Single-phase Commutator Motors." New York, D. Van Nostrand Co., 1913.

E. Arnold, "Die Asynchronen Wechselstrommaschinen," Vol. V, Part II, by E. Arnold, J. L. la Cour and A. Fraenckel. Berlin, J. Springer, 1912, p. 484 *et seq.*

Let us take the general case of two shunted impedances with a resistance in series. It is interesting to investigate under what conditions the current i_1 passing through one of these impedances is in time-phase with the impressed e.m.f. applied at the points A and B.

The vector diagram for these circles is easily drawn for the condition to be investigated, *viz.*, phase equality of i and e_1 Figs. 202 and 203,

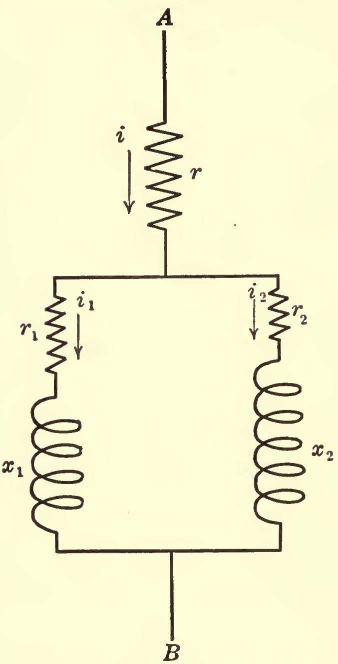


Fig. 202.—Combination of resistances and reactances to produce desired time-phases in the field and armature of single-phase shunt motor.

$$OA = i \sin \alpha = i\frac{i_1 x_1}{ir} = i_1\frac{x_1}{r}$$

$$OC : OA :: i_2 z_2 : i_1 r_1$$

$$\therefore OC = i_2 z_2 \frac{i_1 \frac{x_1}{r}}{i_1 r_1} = i_2 z_2 \frac{x_1}{rr_1}$$

$$OB = i_2 \cos \gamma = i_2 \frac{x_2}{z_2}$$

$$BC : BD :: AC : OA$$

$$AC = OC \cdot \sin \beta = i_2 z_2 \frac{x_1}{rr_1} \cdot \frac{i_1 x_1}{i_2 z_2}$$

$$\therefore AC = i_1 \frac{x_1^2}{rr_1}.$$

$$\therefore BD = i_2 \sin \gamma = i_2 \frac{r_2}{z_2}$$

$$BC = i_1 i_2 \frac{x_1^2 r_2}{rr_1 z_2} \cdot \frac{r}{i_1 x_1}$$

$$BC = i_2 \frac{x_1 r_1}{z_2 r_1}$$

$$OB - BC = OC$$

$$i_2 \frac{x_2}{z_2} - i_2 \frac{x_1 r_2}{z_2 r_2} = i_2 z_2 \frac{x_1}{rr_1}$$

$$\frac{x_2}{z_2} - \frac{x_1 r_2}{z_2 r_1} = \frac{x_1 z_2}{rr_1}$$

Hence $$rr_1 x_2 - rr_2 x_1 = x_1 z_2^2 \qquad (258)$$

For $x_2 = z_2$, also $r_2 = 0$

Hence $$rr_1 = x_1 x_2 \qquad (259)$$

This beautiful scheme proves impracticable on account of its low efficiency. If we look into this carefully we find that under the most favorable conditions the ratio of the kw. dissipated into heat in the resistances to the exciting kva. is 5.2. Therefore, if we assume that the exciting kva. equal about one-fifth of the output, then one-fifth of 5.2 is equal to 1.04 or the kw. dissipated in the resistances to obtain phase equality between the exciting current and the impressed e.m.f. of the motor are

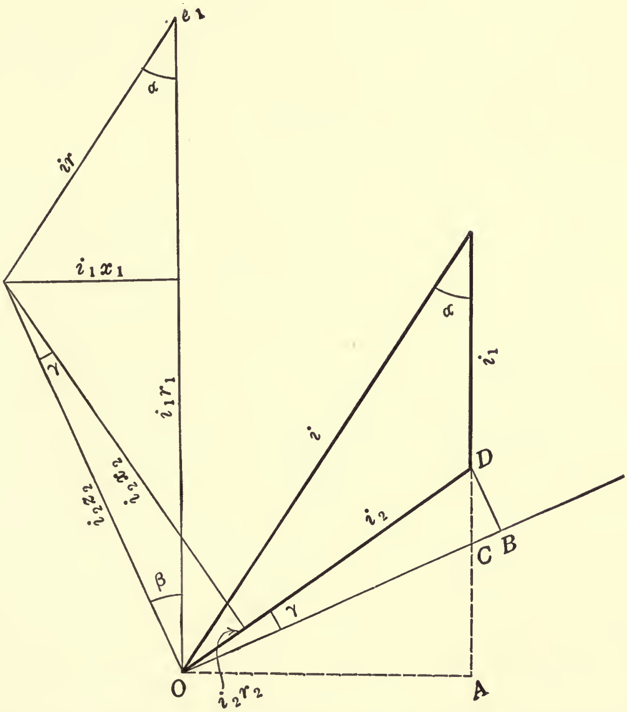


Fig. 203.—Time-phase diagram for combination of resistances and reactances.

4 per cent larger than the entire output of the motor. Such a motor, therefore, considering its other losses, would have an efficiency of about 40 per cent.

To prove this, form the ratio λ of kw. dissipated to kva. excitation.

$$\lambda = \frac{i_1^2 r_1 + i^2 r}{i_1^2 x_1} = \frac{r_1}{x_1} + \frac{r}{x_1} + 2\frac{x_1}{r_1} + \frac{x_1}{2}\left(1 + \frac{x_1}{r_1}\right)^2$$

This expression is a minimum for

$$r_1 = x_1\sqrt{3}$$

$$r = \frac{2x_1}{\sqrt{3}}$$

$$x_2 = 2x_1$$

$$i_1 = i_2$$

and

$$\lambda = \frac{9}{\sqrt{3}} = 5.2 \qquad \text{Q.E.D.}$$

E. THE SUPPLY OF SINGLE-PHASE POWER FROM THREE-PHASE SYSTEMS

With the growth of certain single-phase developments like the systems of the Pennsylvania Railroad between Broad Street Station, Philadelphia, and Paoli, and of the New York, New Haven & Hartford Railroad between Woodlawn and New Haven, single-phase power has to be purchased from public service stations generating three-phase currents.

The armature reaction in turbo generators caused by an unbalanced three-phase system due to the single-phase load is very troublesome. This is so much so that the use of single-phase substations has been considered so as to be able to generate single-phase power in machines of comparatively moderate speed built in such a manner that the effect of the double-frequency m.m.f. of the armature can be dampened out of existence by powerful damping circuits placed in the pole-faces of the generators. This is comparatively easy to do in moderate speed machines, but very difficult in turbo-generators.[1]

Some very ingenious work has been done recently by E. F. W. Alexanderson[2] and by R. E. Gilman and C. Le. G. Fortescue.[3] A very able survey has been published by Prof. Miles Walker[4] to which we must refer for want of space. However, we wish to copy from Prof. Walker's paper the simple underlying princi-

[1] See A. B. Field and B. A. Behrend, U. S. Patent No. 1,269,590, June 18, 1918.

[2] See E. F. W. Alexanderson and G. H. Hill, "Single-phase Power Production," *Trans. A. I. E. E.*, Oct., 1916, p. 1315.

[3] Ibid., p. 1329.

[4] Miles Walker, *Transactions*. "The Institution of Electrical Engineers," of Great Britain, Nov., 1918.

ple of the breaking up of an elliptically rotating field, produced by unbalanced poly-phase currents, into two unequal oppositely rotating magnetic fields.

"Consider the general case of any elliptically-rotating field. Let the ellipse be that shown in Fig. 204. Whatever be the slope of the major axis with respect to the horizontal line, it is always possible to take our abscissæ x along the major axis and our ordinates y along the minor axis, and to express the curve in the form $x^2/a^2 + y^2/b^2 = 1$, where a and b are the lengths of the major and minor axes respectively. If we now write $a = b + c$, where c can be found in any given case,

$$\frac{x^2}{(b+c)^2} = \cos^2\omega t \text{ and } \frac{x^2}{b^2} = \sin^2\omega t$$

$$x = b\cos\omega t + c\cdot\cos\omega t$$

$$y = b\sin\omega t$$

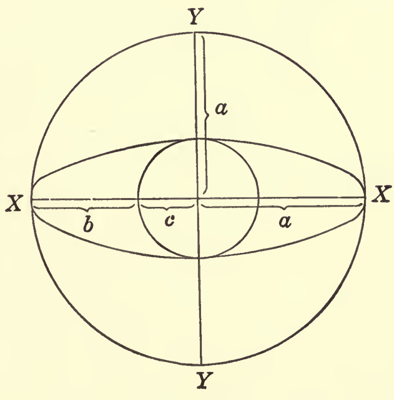


Fig. 204.

The elliptically-rotating field can be regarded as consisting of two parts: one part a simple rotating field whose vector is given by the coördinates $x_1 = b\cos\omega t$, $y_1 = b\sin\omega t$, and the other part a single-phase stationary field lying along the major axis of the ellipse, given by the expression

$$x_2 = c\cdot\cos\omega t;\ y_2 = 0$$

Now, the single-phase field can be broken up into two oppositely-rotating fields:

$$x_2 = x_4 + x_5 = \frac{1}{2}c\cdot\cos\omega t + \frac{1}{2}c\cdot\cos(-\omega t)$$

$$y_2 = y_4 + y_5 = \frac{1}{2}c\cdot\sin\omega t + \frac{1}{2}c\cdot\sin(-\omega t)$$

Adding x_1 to x_4 and y_1 to y_4, we get:

$$x_6 = x_1 + x_4 = \left(b + \frac{1}{2}c\right) \cos \omega t$$

$$y_6 = y_1 + y_4 = \left(b + \frac{1}{2}c\right) \sin \omega t$$

These form a uniform field rotating forwards, while $x_5 = \frac{1}{2}c \cdot \cos(-\omega t)$ and $y_5 = \frac{1}{2}c \cdot \sin(-\omega t)$ give us a uniform field rotating backwards. Thus we see that the magnetic field produced in a poly-phase machine having symmetrically disposed coils carrying an unbalanced load can be regarded as the resultant of two uniform fields of different amplitudes rotating in opposite direction."

The general principle enunciated in Gilman & Fortescue's paper, *viz.*, that "an unbalanced poly-phase system can be resolved into two balanced systems of positive and negative phase-rotation," does not appear to have been proved rigorously in that paper. The subject is of increasing importance and would have merited more space than this passing reference.

APPENDIX

It became incumbent upon the writer more than 20 years ago, to appear as though he gave countenance to the infringement of the fundamental Tesla patents. A large number of induction motors designed by him during the life of these patents, which constituted a plain infringement of Tesla's inventions, have no doubt been pointed to as an indication that he either did not believe in the validity of these patents or that he deliberately became a party to their infringement.

The Company of which, at the period referred to, he was Chief Engineer owed its growth and development largely to his personal efforts in the design and development of electrical machinery and to his success in organizing an effective engineering staff, consisting of a number of eminent men among whom were David Hall, A. B. Field, W. L. Waters, Bradley T. McCormick, H. A. Burson, Alexander Miller Gray, R. B. Williamson, Carl Fechheimer and others. In due course the owners of the Tesla patents proceeded against our company and in the long litigation which followed the writer's position was at times embarrassing and disagreeable. By way of epilogue, he begs leave to publish now, with the bitterness of the controversy abated, a letter addressed to the patent counsel of his Company:

Cincinnati, Ohio, May 23d, 1901.

Mr. Arthur Stem,
Patent Attorney,
City.

My dear Sir:

Enclosed please find my notes on the Record of Final Hearing in the suit of Westinghouse Electric & Mfg. Co. vs. the New England Granite Co.

You will see that I am now, even more than I have been before, of the opinion that it is not possible for us to bring forth arguments that could go to show the invalidity of the Tesla Patents in suit. While I am, as engineer in charge, perfectly willing to give you all the technical assistance in my power that you might need or ask for, I cannot undertake to give expert evidence in this case in favor of my concern, as such evidence would be against my better convictions in this case. As, during my last call at your office, you intimated my being one of the experts, I think it best to let you know as early as possible that I cannot undertake this duty.

Model maker, Mr. W. J. Schultz, called at our office yesterday and I gave him all the necessary instructions for making the devices that we had deemed advisable to make for this suit. Mr. Schultz is thus prepared to let us have his bid on them and this will be submitted to our management.

I remain,

Yours very truly,
(Signed) B. A. Behrend,
Chief Engineer, etc.

It was a matter of gratification to the writer that almost 20 years later, when he was a member of the Edison Medal Committee, he was able to propose the name of Tesla for the award of the Edison Medal and upon the occasion of the presentation of the Medal to express his great admiration for the medallist's creative work. There is published herewith, as the closing chapter of a long story, the writer's address as it was delivered on that occasion.

To Professor André Blondel
of l'École nationale des ponts et chaussées
of paris

An honorary member of our Institute, whose brilliant and inspiring work a generation ago laid the theoretical foundation of the development of alternating-current engineering practice; to whose generous support of my own modest labors, over a score of years ago, I owed recognition; in the hour of his country's trial, I beg leave to inscribe these remarks. Hands across the sea, may a happier future dawn before us!

B. A. Behrend.

Address

Mr. Chairman: Mr. President of the American Institute of Electrical Engineers: Fellow Members: Ladies and Gentlemen:

By an extraordinary coincidence, it is exactly twenty-nine years ago, to the very day and hour, that there stood before this Institute Mr. Nikola Tesla, and he read the following sentences:

"To obtain a rotary effort in these motors was the subject of long thought. In order to secure this result it was necessary to make such a disposition that while the poles of one element of the motor are shifted by the alternate currents of the source, the poles produced upon the other elements should always be maintained in the proper relation to the former, irrespective of the speed of the motor. Such a condition exists in a continuous current motor: but in a synchronous motor, such as described, this condition is fulfilled only when the speed is normal.

"The object has been attained by placing within the ring a properly subdivided cylindrical iron core wound with several independent coils closed upon themselves. Two coils at right angles are sufficient, but a greater number may be advantageously employed. It results from this disposition that when the poles of the ring are shifted, currents are generated in the closed armature coils. These currents are the most intense at or near the points of the greatest density of the lines of force, and their effect is to produce poles upon the armature at right angles to those of the ring, at least theoretically so; and since this action is entirely independent of the speed—that is, as far as the location of the poles is concerned—a continuous pull is exerted upon the periphery of the armature. In many respects these motors are similar to the continuous current motors. If load is put on, the speed, and also the resistance of the motor, is diminished and more current is made to pass through the energizing coils, thus increasing the effort. Upon the load being taken off, the counter-electromotive force increases and less current passes through the primary or energizing coils. Without any load the speed is very nearly equal to that of the shifting poles of the field magnet.

"It will be found that the rotary effort in these motors fully equals that of the continuous current motors. The effort seems to be greatest when both armature and field magnet are without any projections."

Not since the appearance of Faraday's "Experimental Researches in Electricity" has a great experimental truth been voiced so simply and so clearly as this description of Mr. Tesla's great discovery of the generation and utilization of poly-phase alternating currents. He left nothing to be done by those who followed him. His paper contained the skeleton even of the mathematical theory.

Three years later, in 1891, there was given the first great demonstration, by Swiss engineers, of the transmission of power at 30,000 volts from Lauffen to Frankfort by means of Mr. Tesla's system. A few years later this was followed by the development of the Cataract Construction Company, under the presidency of our member, Mr. Edward D. Adams, and with the aid of the engineers of the Westinghouse Company. It is interesting to recall here to-night that in Lord Kelvin's report to Mr. Adams, Lord Kelvin recommended the use of direct current for the development of power at Niagara Falls and for its transmission to Buffalo.

The due appreciation or even enumeration of the results of Mr. Tesla's inventions is neither practicable nor desirable at this moment. There is a time for all things. Suffice it to say that, were we to seize and to eliminate from our industrial world the results of Mr. Tesla's work, the wheels of industry would cease to turn, our electric cars and trains would stop, our towns would be dark, our mills would be dead and idle. Yea, so far reaching is this work, that it has become the warp and woof of industry.

The basis for the theory of the operating characteristics of Mr. Tesla's rotating field induction motor, so necessary to its practical development, was laid by the brilliant French savant, Professor André Blondel, and by Professor Kapp of Birmingham. It fell to my lot to complete their work and to coordinate—by means of the simple "circle diagram"—the somewhat mysterious and complex experimental phenomena. As this was done twenty-one years ago, it is particularly pleasing to me, upon the coming of age of this now universally accepted theory—tried out by application to several million horsepower of machines operating in our great industries—to pay my tribute to the inventor of the motor and the system which have made possible the electric transmission of energy. *His* name marks an epoch in the advance of electrical science. From *that* work has sprung a revolution in the electrical art.

We asked Mr. Tesla to accept this medal. We did not do this for the mere sake of conferring a distinction, or of perpetuating a name; for so long as men occupy themselves with our industry, his work will be incorporated in the common thought of our art, and the name of Tesla runs no more risk of oblivion than does that of Faraday, or that of Edison.

Nor indeed does this Institute give this medal as evidence that Mr. Tesla's work has received its official sanction. His work stands in no need of such sanction.

No, Mr. Tesla, we beg you to cherish this medal as a symbol of our gratitude for a new creative thought, the powerful impetus, akin to revolution, which you have given to our art and to our science. You have lived to see the work of your genius established. What shall a man desire more than this? There rings out to us a paraphrase of Pope's lines on Newton:

Nature and Nature's laws lay hid in night:
God said, "Let TESLA *be," and all was light.*

New York City
May 18, 1917.

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